Incoherent Dicke model - new type of phase transition

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Motivation 1

- ✦ *Deep strong coupling* $\longrightarrow$ Rabi ground state changes qualitatively.
- ✦ *Deep strong coupling* $\longrightarrow$ Dicke ‘superradiant’ phase transition.
- ✦ Hard to reach, classical Dicke

$$\hat{H} = \kappa \hat{S}_z + \omega \hat{S}_x$$

Not critical, not even very exciting!

- ✦ ‘Incoherent Dicke model’

$$\partial \hat{\rho} = i \left[\hat{\rho}, \omega \hat{S}_x \right] + \kappa \left(2\hat{S}^- \hat{\rho} \hat{S}^+ - \hat{S}^+ \hat{S}^- \hat{\rho} - \hat{\rho} \hat{S}^+ \hat{S}^- \right)$$

It’s getting interesting….

Motivation 2

✦ Equilibrium phase transitions (continuous):

- ***Universal*** - few critical exponents determine the physics (microscopic details irrelevant).
- ***Spontaneous symmetry breaking*** - in one phase the state does not possess the same symmetry as the full Hamiltonian.
- ***Excitations*** - energy gap closes at the critical point. Excitations in 'symmetry broken phase' either gapped (*Higgs*) or continuous (*Goldstone*).
- ***Mermin-Wagner theorem*** - the type of symmetry + the dimensionality determine which type of transition that is allowed.

How do the above results translate to phase transitions in open quantum systems?

Outlook

1. (Equilibrium) Quantum phase transitions
   - Symmetry breaking.
   - Scale invariance and universality.
2. Open quantum phase transitions
3. A new type of phase transition
4. Prospects

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Phase transitions

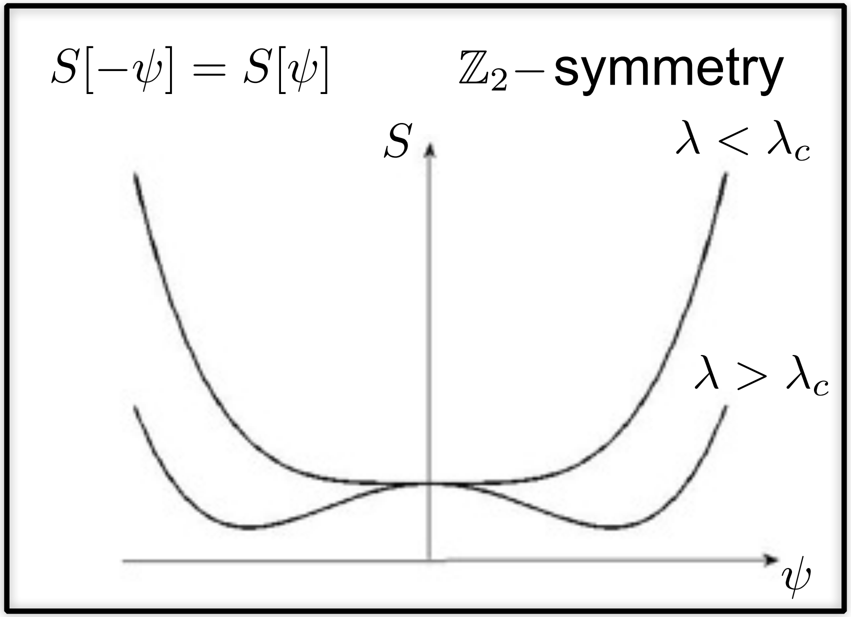
Phase transition

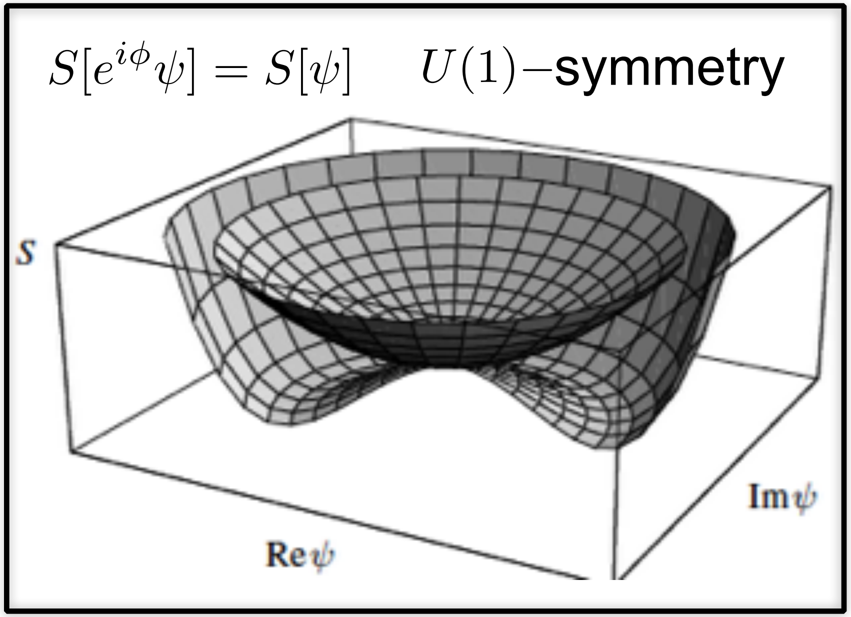
✦ Action $S[\bar{\psi}, \psi]$, giving partition function

$$\mathcal{Z} = \int D(\bar{\psi}, \psi)\, e^{-S[\bar{\psi}, \psi]}$$

Mean-field solution: minimizing the action for *order parameter* ψ.

✦ Symmetries $S[g\bar{\psi}, g\psi] = S[\bar{\psi}, \psi]$.





Phase transitions
2nd order phase transition

- *Continuous* phase transition.
- Exists a symmetry $[\hat{U}, \hat{H}] = 0$.
- Quantum mechanics: $\hat{U}$ and $\hat{H}$ common eigenbasis - energy eigenstates well defined symmetry.
- Thermodynamic limit: *spontaneous symmetry breaking* - ground state $|\psi_0(\lambda)\rangle$ does not have a defined symmetry!
- "Potential barrier" becomes infinite $\rightarrow$ degeneracy.
- If $\hat{U}$ continuous $\rightarrow$ *Goldstone* (gapless) *excitations.*
  $\hat{U}$ *discontinuous* $\rightarrow$ *Higgs* (gapped) *excitations.*

N. Goldenfeld, *Lectures on Phase Transitions…*

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Phase transitions

Phase transition (2nd) - scale invariance

At the critical point λ_c

The following Ising model configurations range from 2048x2048 sites to 131072x131072 sites.

Can you tell which is which?

$$H_{class} = \sum_{\langle ij \rangle} s_i s_j - h \sum_i s_i$$

System the same independent of "zooming" - length scale diverges!!

N. Goldenfeld, *Lectures on Phase Transitions…*

Phase transitions

Phase transition (2nd) - universality

✦ Systematic method (*renormalization group*) - eliminates short length scales ('high energies').

✦ Effective low energy model - microscopic details irrelevant.

✦ Models belong to different 'classes' - universality (depend on macroscopic properties like symmetries).

✦ Critical regime:

$$\xi \propto |\lambda - \lambda_c|^{-\nu}, \quad \text{Characteristic length}$$

$$\tau \propto |\lambda - \lambda_c|^{-\delta}, \quad \text{Characteristic time}$$

$$\Delta E \propto |\lambda - \lambda_c|^{z\nu}, \quad \text{Gap closing}$$

$$\dots$$

Critical exponents $\nu, \delta, z, \ldots$, different *universality classes*.

N. Goldenfeld, *Lectures on Phase Transitions…*

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Phase transitions in open quantum systems

- Quantum PT's, non-analyticity in ground state $|\psi_0(\lambda)\rangle$ at critical coupling λ_c.
- Transition due to quantum fluctuations.
- Open quantum system

$$\partial_t \hat{\rho}(t) = i\left[\hat{\rho}(t), \hat{H}_S\right] + \hat{\mathcal{L}}\left[\hat{\rho}(t)\right],$$

$$\hat{\mathcal{L}}\left[\hat{\rho}(t)\right] = \sum_i g_i \left(2\hat{A}_i \hat{\rho}(t) \hat{A}_i^\dagger - \hat{A}_i^\dagger \hat{A}_i \hat{\rho}(t) - \hat{\rho}(t) \hat{A}_i^\dagger \hat{A}_i\right)$$

- **No** ground state, **no** energy spectrum!!

What do we mean by a PT here, what is the relevant state?

C.-E. Bardyn *et al.*, NJP **15** (2013)

Phase transitions in open quantum systems

- One (obvious) physically relevant state is the steady state

$$\partial_t \hat{\rho}_{ss}(t) = 0 \quad \to \quad i\left[\hat{\rho}_{ss}, \hat{H}_S\right] + \hat{\mathcal{L}}[\hat{\rho}_{ss}] = 0$$

- May be non-equilibrium, but time-independent.

- Closed system, all energy eigenstates also steady states.

- If:
  1. $[\hat{A}_i, \hat{H}_S] = 0, \ \forall \hat{A}_i$, energy eigenstates also steady states.
  2. $\hat{H}_S$ critical and $\hat{\mathcal{L}}\left[|\psi_0(\lambda < \lambda_c)\rangle\langle\psi_0(\lambda < \lambda_c)|\right] = 0$, the environment is expected to support the symmetric phase.

- If neither of the two $\longrightarrow$ new physics???

- Phase transition if $\hat{\rho}_{ss}$ non-analytic for some λ_c .

C.-E. Bardyn *et al.*, NJP **15** (2013)

Outlook

1. (Equilibrium) Quantum phase transitions
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2. Open quantum systems
   - Composite systems, density operators.
   - Lindblad master equation.
3. Open quantum phase transitions
4. **A new type of phase transition**
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New type of phase transition
Model

✦ Simplest (non-trivial) scenario

- $\partial_t \hat{\rho} = i[\hat{\rho}, \hat{H}_s]$ trivial
- $\partial_t \hat{\rho} = \kappa(2\hat{A}\hat{\rho}\hat{A}^\dagger - \hat{A}^\dagger \hat{A}\hat{\rho} - \hat{\rho}\hat{A}^\dagger \hat{A})$ trivial
- $\partial_t \hat{\rho} = i[\hat{\rho}, \hat{H}_s] + \kappa(2\hat{A}\hat{\rho}\hat{A}^\dagger - \hat{A}^\dagger \hat{A}\hat{\rho} - \hat{\rho}\hat{A}^\dagger \hat{A})$ critical

✦ One such model (which is also solvable analytically!)

$[\hat{S}_i, \hat{S}_j] = i\varepsilon_{ijk}\hat{S}_k$ Large spin-*S* (preserved)

$\hat{H}_S = \omega \hat{S}_x$ Coherent drive

$\hat{A} = \hat{S}_-$ Spontaneous decay to state $|S, -S\rangle$

D. F. Walls *et al.*, Prog. Theo. Phys. **64** (1978)

New type of phase transition
Model

- Equation to solve

$$\partial \hat{\rho} = i[\hat{\rho}, \omega \hat{S}_x] + \kappa(2\hat{S}_- \hat{\rho} \hat{S}_+ - \hat{S}_+ \hat{S}_- \hat{\rho} - \hat{\rho} \hat{S}_+ \hat{S}_-)$$

- Limiting cases

$$\frac{\kappa}{\omega} = 0 \quad \rightarrow \quad |S, -S\rangle_x$$ Hamiltonian dominates

$$\frac{\omega}{\kappa} = 0 \quad \rightarrow \quad |S, -S\rangle_z$$ Lindbladian dominates

- Phase transition somewhere between?

New type of phase transition
Mean-field solution

- Mean-field (normal ordering): $S_\alpha = \langle \hat{S}_\alpha \rangle, \quad \langle \hat{S}_\alpha \hat{S}_\beta \rangle = \langle \hat{S}_\alpha \rangle \langle \hat{S}_\beta \rangle$.

$$\dot{S}_x = 2\frac{\kappa}{S} S_x S_z ,$$

$$\dot{S}_y = S_z \left(2\frac{\kappa}{S} S_y - \omega \right),$$

$$\dot{S}_z = \omega S_y - 2\frac{\kappa}{S}\left(S_x^2 + S_y^2\right)$$

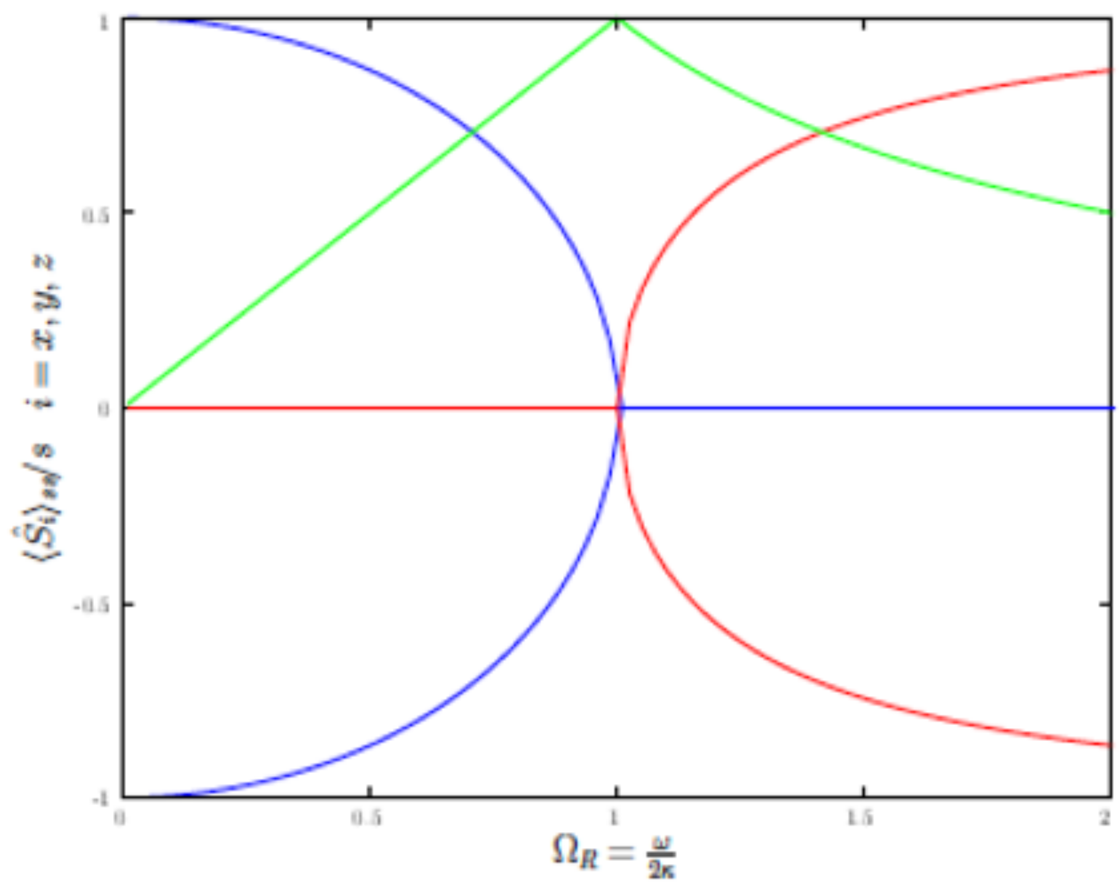
- Conserved spin: $S^2 = S_x^2 + S_y^2 + S_z^2$.

- Steady state solutions

$$(S_x, S_y, S_z)/S = \left(\pm\sqrt{1 - \frac{4\kappa^2}{\omega^2}}, \frac{2\kappa}{\omega}, 0 \right), \qquad (S_x, S_y, S_z)/S = \left(0 \frac{\omega}{2\kappa}, \pm\sqrt{1 - \frac{\omega^2}{4\kappa^2}} \right)$$

New type of phase transition

Mean-field solution



Classical steady state solutions: *x* (red), *y* (green), *z* (blue).

Red curve, *Hopf bifurcation*

Blue curve, *Strange bifurcation*…

New type of phase transition
Universality

- ✦ Universality - critical exponents.
  - Mean-field. $\lambda = \omega/\kappa \longrightarrow$ "Magnetization" $S_z \propto |\lambda_c - \lambda|^{1/2} \longrightarrow \nu = 1/2$.
  - Quantum. $\nu = 1/2$.
- ✦ Seems to be universal!
- ✦ No length scale! Critical slowing down??

New type of phase transition
Universality

✦ Quantum treatment. Analytical solution

$$\hat{\rho}_{ss} = \frac{1}{D} \sum_{n,m=0}^{2S} (g^*)^{-m} g^{-n} \hat{S}_-^m \hat{S}_+^n, \qquad g = i\omega S/\kappa$$

Expectations $\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}[\hat{\mathcal{O}} \hat{\rho}_{ss}]$. Truncate for some *S*.

✦ Critical exponent the same as for mean-field.

✦ Fluctuations in $\hat{S}_x$: $\langle \hat{S}_x^2 \rangle \propto |\lambda_c - \lambda|^{1/2}$.

New type of phase transition
Symmetries

✦ Symmetries.

- *Closed case.* $[\hat{A}, \hat{H}] = 0 \leftrightarrow \hat{H} = \hat{U}\hat{H}\hat{U}^{-1} \longrightarrow$ $\hat{A}$ conserved.
- *Open case.* $\partial\hat{\rho} = \mathcal{L}_{\hat{L}}[\hat{\rho}] = \mathcal{L}_{\hat{U}\hat{L}\hat{U}^{-1}}[\hat{\rho}] \longrightarrow$ generally no conserved quantity.

✦ Guess $\hat{S}_x \to -\hat{S}_x,\ \hat{S}_y \to \hat{S}_y,\ \hat{S}_z \to -\hat{S}_z$

$$\partial\hat{\rho} = i[\hat{\rho}, \omega\hat{S}_x] + \kappa\left(2\hat{S}_-\hat{\rho}\hat{S}_+ - \hat{S}_+\hat{S}_-\hat{\rho} - \hat{\rho}\hat{S}_+\hat{S}_-\right)$$
$$\longrightarrow$$
$$\partial\hat{\rho} = -i[\hat{\rho}, \omega\hat{S}_x] + \kappa\left(2\hat{S}_+\hat{\rho}\hat{S}_- - \hat{S}_-\hat{S}_+\hat{\rho} - \hat{\rho}\hat{S}_-\hat{S}_+\right)$$

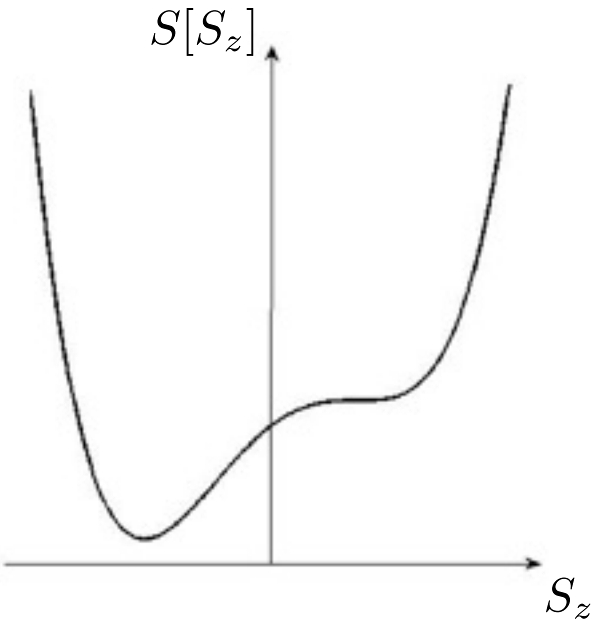
Dual symmetry! Flip spectrum + “step up” instead of “step down”.

New type of phase transition
Symmetries

✦ Mean-field.

- First bifurcation (S_z). **No** pitchfork, only one or two solutions. Stability: one stable, one unstable.

- Second bifurcation (S_x). Stability: the two solutions with purely imaginary eigenvalues $\longrightarrow$ Hopf. (Phase space a sphere so peculiar trajectories).



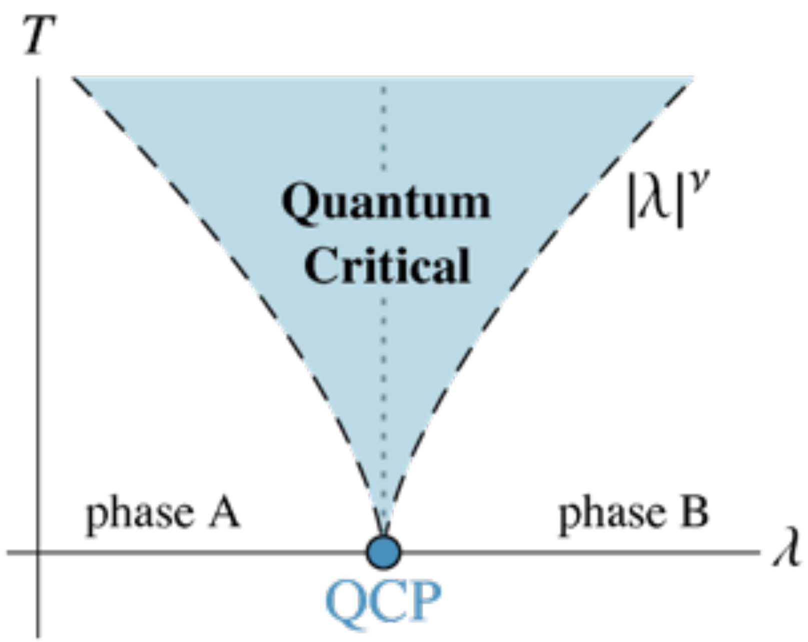
New type of phase transition

Second order phase transition with no symmetry breaking!!

New type of phase transition
Quantum or classical

✦ Is it a quantum phase transition?

- Single degree of freedom. Quantum fluctuations vanishingly small in thermodynamic limit.
- Fluctuations due to coupling to reservoir.
- *Quantum critical region* - signatures of "quantum" survives finite temperatures in the vicinity of a critical point.



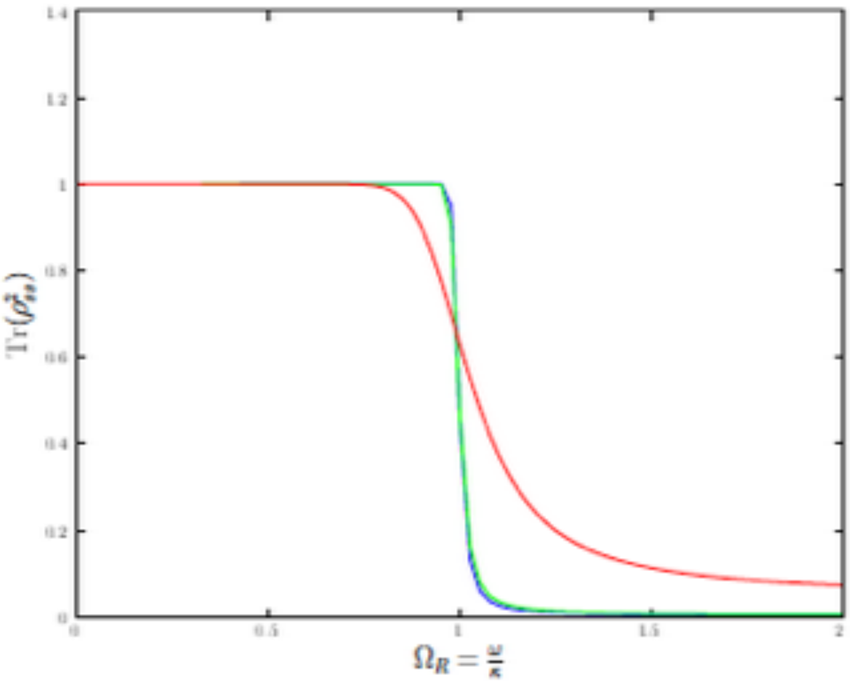
S. Sachdev, *Quantum Phase Transitions*

New type of phase transition

Quantum or classical

✦ "Quantum"

- Coherence: *Purity* $P \equiv \mathrm{Tr}[\hat{\rho}^2] > 0$ (thermodynamic limit).
- Entanglement: *Concurrence* $C(\hat{\rho})$ between two qubits. *Negativity* $N(\hat{\rho})$ between two equal halves.



Thermodynamic limit:

Reservoir dominated - pure state.

Hamiltonian dominated - maximally mixed.

New type of phase transition
Quantum or classical

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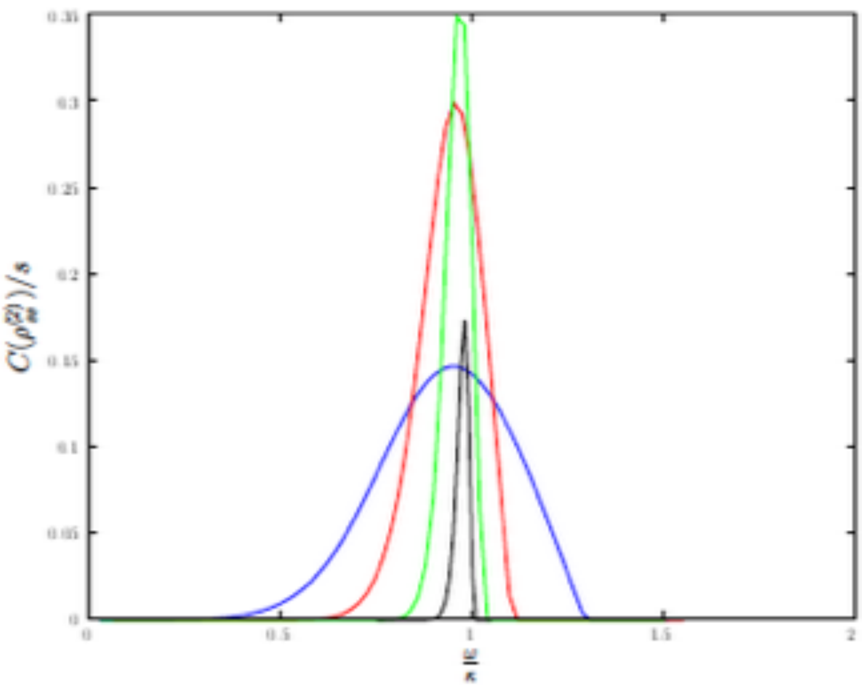

Thermodynamic limit:

No qubit-qubit entanglement outside the critical point.

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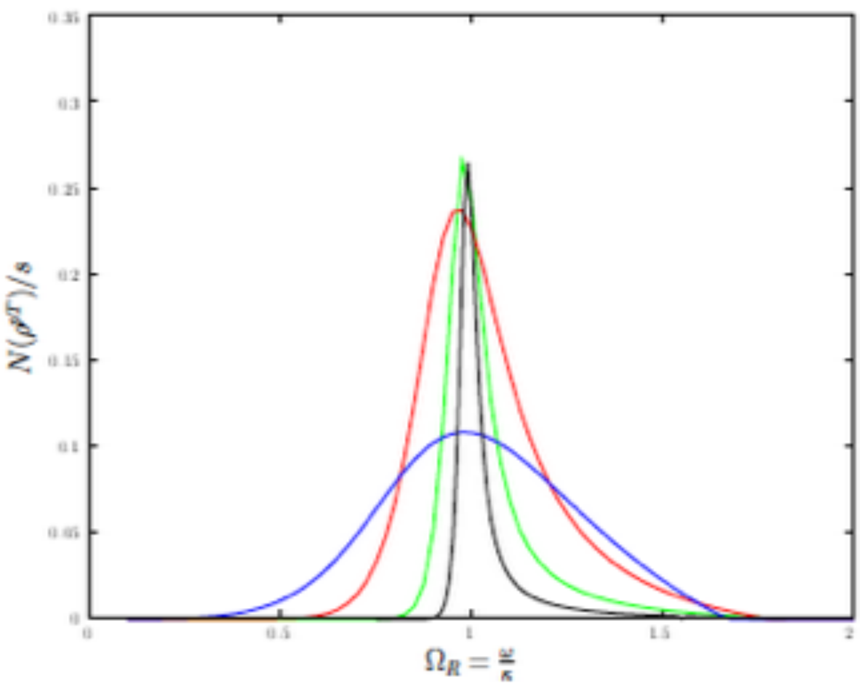

Thermodynamic limit:

No subsystem-subsystem entanglement outside the critical point.

Scales $N(\hat{\rho}) \sim S$ ('volume law').

'Critical exponents' infinite.

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Phase transitions in open quantum systems

✦ Found a model possessing an open PT, seemingly second order but lacks any symmetry breaking $\longrightarrow$ **New type**. Partly unexplored field.

Open questions

- Are these PT's universal? Scale invariance? New classes?
- Models driven more strongly by quantum fluctuations, length scale (lattice models).
- Gap closening of the *Liouvillian*, universal?
- Is there a *Mermin-Wagner theorem* for open QPT's?
- *Kibble-Zurek mechanism*?


Thanks!