Open quantum phase transitions - universality and underlying mechanism

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Motivation

✦ Equilibrium phase transitions (continuous):

- *Universal* - few critical exponents determine the physics (microscopic details irrelevant).
- *Spontaneous symmetry breaking* - in one phase the state does not possess the same symmetry as the full Hamiltonian.
- *Excitations* - energy gap closes at the critical point. Excitations in 'symmetry broken phase' either gapped (*Higgs*) or continuous (*Goldstone*).
- *Mermin-Wagner theorem* - the type of symmetry + the dimensionality determine which type of transition that is allowed.

✦ (Especially) cold atom experiments. Drive them and engineer desired coupling to reservoir $\longrightarrow$ non-trivial steady states.

✦ How do the above general results translate to these new phase transitions?

Outlook

1. (Equilibrium) Quantum phase transitions
   - Symmetry breaking.
   - Scale invariance and universality.
2. Open quantum systems
   - Composite systems, density operators.
   - Lindblad master equation.
3. Open quantum phase transitions
4. A new type of phase transition
5. Prospects

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Phase transitions

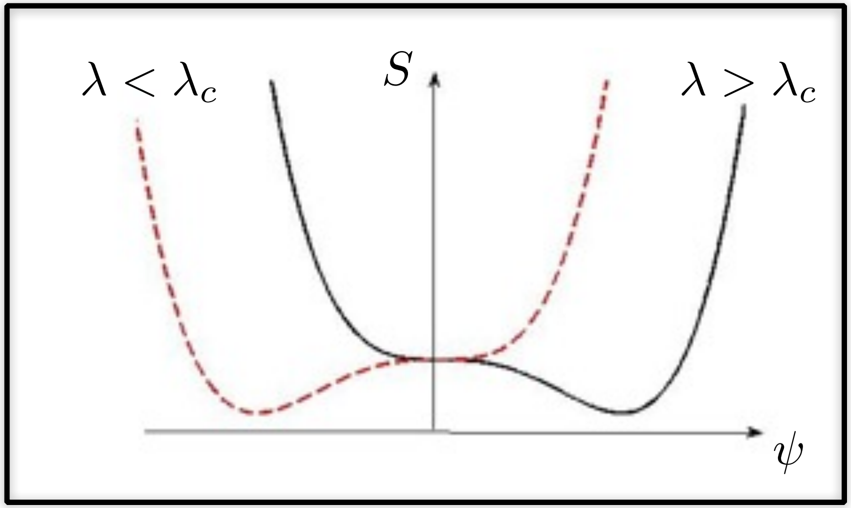
Phase transition

- Action $S[\bar{\psi},\psi]$, giving partition function

$$\mathcal{Z} = \int D(\bar{\psi},\psi)\, e^{-S[\bar{\psi},\psi]}$$

Mean-field solution: minimizing the action for *order parameter* ψ*.*

- PT $\longrightarrow$ change in order parameter



Sudden jump - first order PT.

Phase transitions

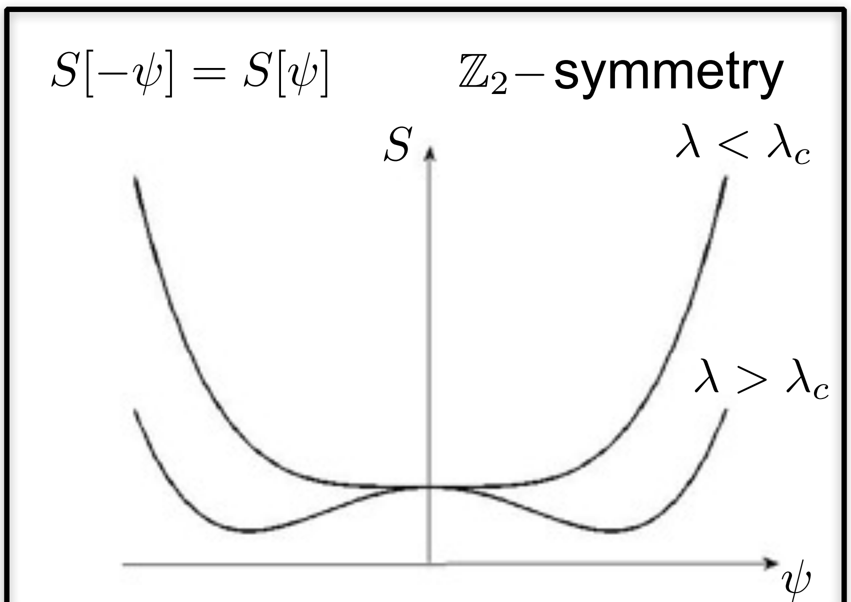
Phase transition

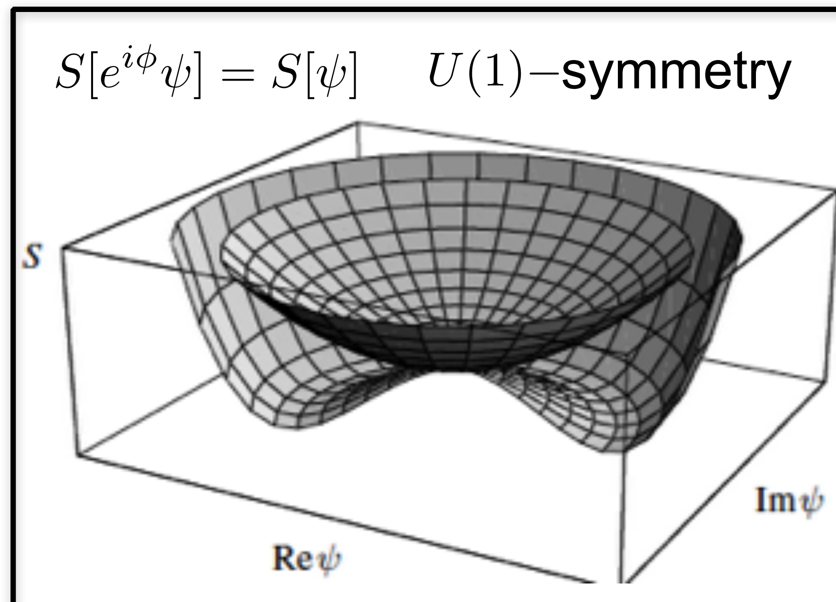
✦ Action $S[\bar{\psi}, \psi]$, giving partition function

$$\mathcal{Z} = \int D(\bar{\psi}, \psi)\, e^{-S[\bar{\psi}, \psi]}$$

Mean-field solution: minimizing the action for *order parameter* ψ*.*

✦ Symmetries $S[g\bar{\psi}, g\psi] = S[\bar{\psi}, \psi]$.





Phase transitions
2nd order phase transition

✦ *Continuous* phase transition.

✦ Exists a symmetry $[\hat{U}, \hat{H}] = 0$.

✦ Quantum mechanics: $\hat{U}$ and $\hat{H}$ common eigenbasis - energy eigenstates well defined symmetry.

✦ Thermodynamic limit: *spontaneous symmetry breaking* - ground state $|\psi_0(\lambda)\rangle$ does not have a defined symmetry!

✦ "Potential barrier" becomes infinite $\longrightarrow$ degeneracy.

✦ If $\hat{U}$ continuous $\longrightarrow$ *Goldstone* (gapless) *excitations.*

$\hat{U}$ *discontinuous* $\longrightarrow$ *Higgs* (gapped) *excitations.*

Phase transitions

2nd order phase transition

- ✦ Ground state energy $E_0(\lambda)$ continuous, λ system parameter.
- ✦ Derivatives $\partial_\lambda^n E_0(\lambda)$ can be discontinuous for some *critical coupling* λ_c.
- ✦ When and why?
  - *Thermodynamic limit* - system 'size' infinite.
  - Competing terms supporting different properties

$$\hat{H} = \hat{H}_1 + \lambda \hat{H}_2, \qquad \left[\hat{H}_1, \hat{H}_2\right] \neq 0$$

- ✦ Roughly, $\lambda < 1$ ground state properties from $\hat{H}_1$, $\lambda > 1$ from $\hat{H}_2$.
  - At $\lambda = \lambda_c \equiv 1$ spectrum gapless.
  - $|\psi_0(\lambda)\rangle$ and ψ 'non-analytic' at λ_c.
  - The PT driven by quantum fluctuations at $T = 0$ (classical PT, thermal fluctuations).

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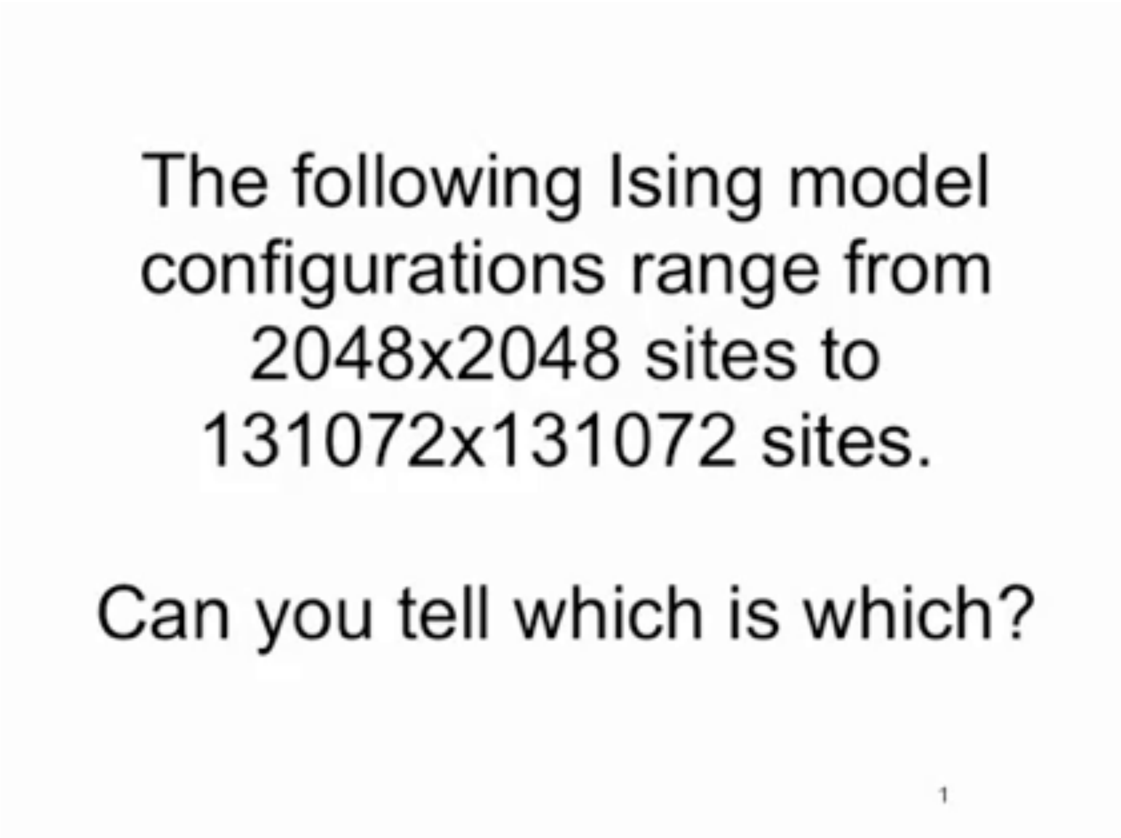
Phase transitions

Phase transition (2nd) - scale invariance

At the critical point λ_c



$$H_{class} = \sum_{\langle ij \rangle} s_i s_j - h \sum_i s_i$$

System the same independent of “zooming” - length scale diverges!!

Phase transitions

Phase transition (2nd) - universality

- ✦ Systematic method (*renormalization group*) - eliminates short length scales (‘high energies’).

- ✦ Effective low energy model - microscopic details irrelevant.

- ✦ Models belong to different ‘classes’ - universality (depend on macroscopic properties like symmetries).

- ✦ Critical regime:

$$\xi \propto |\lambda - \lambda_c|^{-\nu}, \quad \text{Characteristic length}$$

$$\tau \propto |\lambda - \lambda_c|^{-\delta}, \quad \text{Characteristic time}$$

$$\Delta E \propto |\lambda - \lambda_c|^{z\nu}, \quad \text{Gap closing}$$

$$\ldots$$

Critical exponents ν, δ, z, ... , different *universality classes*.

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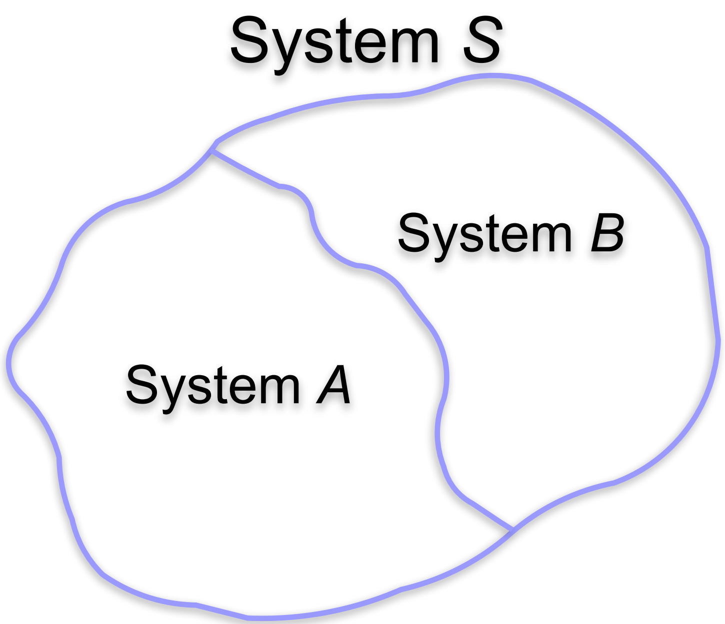
Open quantum systems
Density operator



$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$$|\psi\rangle = \sum_{ij} c_{ij} |\psi_i^A\rangle \otimes |\psi_j^B\rangle$$

✦ Only access to *A*, local observables $\hat{O}_A$.

✦ What is the state of subsystem *A*?

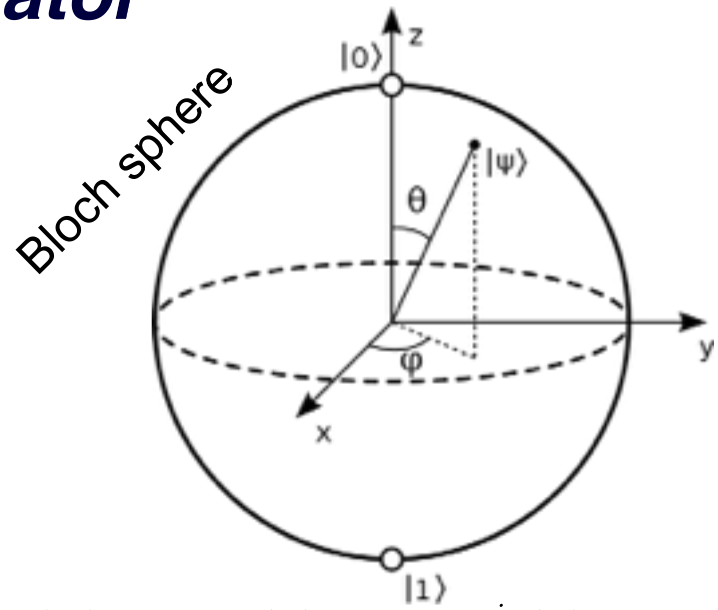
Open quantum systems
Density operator

$$\begin{aligned}
\langle \hat{\mathcal{O}}_A \rangle &= \textstyle\sum_{ijnm} c^*_{ij} c_{nm} \langle \phi^B_i | \langle \phi^A_j | \hat{\mathcal{O}}_A | \phi^A_n \rangle | \phi^B_m \rangle \\
&= \textstyle\sum_{ijn} c^*_{ij} c_{ni} \langle \phi^A_j | \hat{\mathcal{O}}_A | \phi^A_n \rangle \\
&= \textstyle\sum_l \sum_{ijn} c^*_{ij} c_{ni} \langle \phi^A_j | \hat{\mathcal{O}}_A | l \rangle \langle l | \phi^A_n \rangle \\
&= \textstyle\sum_l \sum_{ijn} c^*_{ij} c_{ni} \langle l | \phi^A_n \rangle \langle \phi^A_j | \hat{\mathcal{O}}_A | l \rangle \\
&= \textstyle\sum_l \langle l | \hat{\rho}_A \hat{\mathcal{O}}_A | l \rangle = \mathrm{Tr}_A \left[\hat{\rho}_A \hat{\mathcal{O}}_A \right]
\end{aligned}$$

Reduced density operator $\hat{\rho}_A = \sum_{ijn} c^*_{ij} c_{ni} |\phi^A_n\rangle\langle\phi^A_j| = \mathrm{Tr}_B\left[\hat{\rho}\right]$, with $\hat{\rho} = |\psi\rangle\langle\psi|$.

State of system *S*: $\hat{\rho}$, state of subsystem *A*: $\hat{\rho}_A$. In general $\hat{\rho} \neq \hat{\rho}_A \otimes \hat{\rho}_B$.

Open quantum systems
Density operator



- In general $\hat{\rho} \neq |\psi\rangle\langle\psi|$.

- Physical:

$$\mathrm{Tr}\,[\hat{\rho}] = 1,$$
$$\hat{\rho}^\dagger = \hat{\rho},$$
$$||\hat{\rho}|| \geq 0$$

- If $\hat{\rho} = |\psi\rangle\langle\psi|$ (*pure state*), two-level state $|\psi\rangle = \cos\theta|0\rangle + \sin\theta e^{i\varphi}|1\rangle$.

- *Bloch vector* $\bar{R} = (x, y, z)$, $\alpha = \mathrm{Tr}\,[\hat{\sigma}_\alpha \hat{\rho}]$, with $\hat{\sigma}_\alpha$ *Pauli matrices.*

  Pure state $|\bar{R}| = 1$, in general $|\bar{R}| < 1$.

Majority of states not pure!

- Random multi-qubit state $\hat{\rho} \;\longrightarrow$ reduced single qubit state $\hat{\rho}_1 \approx \mathbb{I}/2$.

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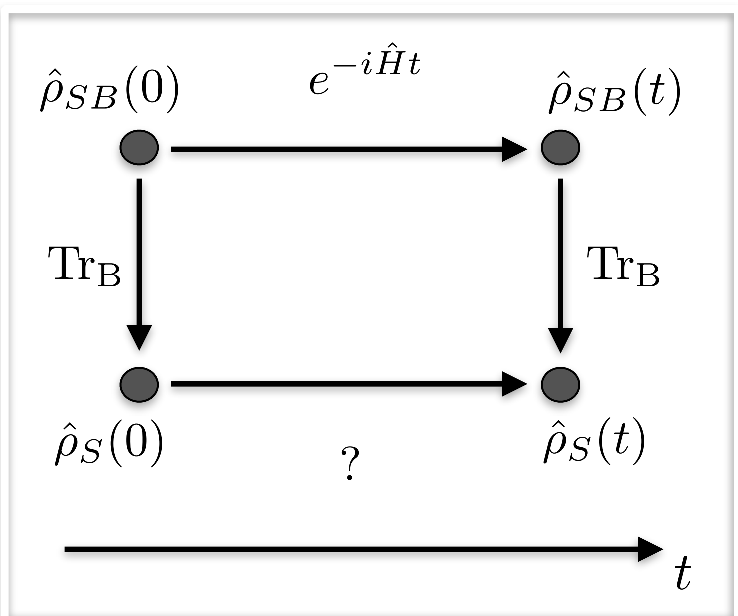
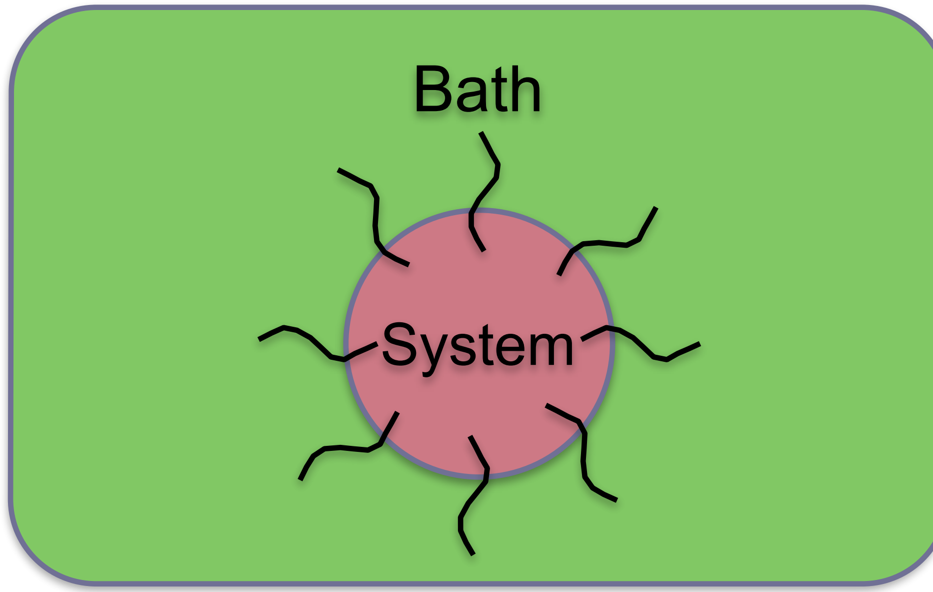
Open quantum systems
Density operator

Combined system

$$\hat{H} = \hat{H}_S + \hat{H}_B + \hat{H}_{SB}$$





Generator “?”, operators defined on $\mathcal{H}_S$. Generally **not** possible!!

Open quantum systems
Density operator

✦ Weak coupling, “big” bath

1. Bath time-scale short: no memory, *Markovian.*
2. System negligible influence on bath, *Born.*
3. *Rotating wave approximation.*

$$\partial_t \hat{\rho}(t) = i\left[\hat{\rho}(t), \hat{H}_S\right] + \hat{\mathcal{L}}\left[\hat{\rho}(t)\right],$$

$$\hat{\mathcal{L}}\left[\hat{\rho}(t)\right] = \sum_i g_i \left(2\hat{A}_i\hat{\rho}(t)\hat{A}_i^\dagger - \hat{A}_i^\dagger\hat{A}_i\hat{\rho}(t) - \hat{\rho}(t)\hat{A}_i^\dagger\hat{A}_i\right)$$

Lindblad master equation

$\hat{A}_i$ *Lindblad jump operators.* Non-unitary evolution, $\hat{\rho}(t)$ not generally pure.

Phase transitions in open quantum systems

- Quantum PT's, non-analyticity in ground state $|\psi_0(\lambda)\rangle$ at critical coupling λ_c.

- Transition due to quantum fluctuations.

- Open quantum system

$$\partial_t \hat{\rho}(t) = i\left[\hat{\rho}(t), \hat{H}_S\right] + \hat{\mathcal{L}}\left[\hat{\rho}(t)\right],$$

$$\hat{\mathcal{L}}\left[\hat{\rho}(t)\right] = \sum_i g_i \left(2\hat{A}_i\hat{\rho}(t)\hat{A}_i^\dagger - \hat{A}_i^\dagger\hat{A}_i\hat{\rho}(t) - \hat{\rho}(t)\hat{A}_i^\dagger\hat{A}_i\right)$$

- **No** ground state, **no** energy spectrum!!

What do we mean by a PT here, what is the relevant state?

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Phase transitions in open quantum systems

- One (obvious) physically relevant state is the steady state

$$\partial_t \hat{\rho}_{ss}(t) = 0 \quad \rightarrow \quad i\left[\hat{\rho}_{ss}, \hat{H}_S\right] + \hat{\mathcal{L}}[\hat{\rho}_{ss}] = 0$$

- May be non-equilibrium, but time-independent.

- Closed system, all energy eigenstates also steady states.

- If:
  1. $[\hat{A}_i, \hat{H}_S] = 0, \ \forall \hat{A}_i$, energy eigenstates also steady states.
  2. $\hat{H}_S$ critical and $\hat{\mathcal{L}}\left[|\psi_0(\lambda < \lambda_c)\rangle\langle\psi_0(\lambda < \lambda_c)|\right] = 0$, the environment is expected to support the symmetric phase.

- If neither of the two $\longrightarrow$ new physics???

- Phase transition if $\hat{\rho}_{ss}$ non-analytic for some λ_c .

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New type of phase transition
Model

- Simplest (non-trivial) scenario
  - $\partial_t \hat{\rho} = i[\hat{\rho}, \hat{H}_s]$ trivial
  - $\partial_t \hat{\rho} = \kappa(2\hat{A}\hat{\rho}\hat{A}^\dagger - \hat{A}^\dagger \hat{A}\hat{\rho} - \hat{\rho}\hat{A}^\dagger \hat{A})$ trivial
  - $\partial_t \hat{\rho} = i[\hat{\rho}, \hat{H}_s] + \kappa(2\hat{A}\hat{\rho}\hat{A}^\dagger - \hat{A}^\dagger \hat{A}\hat{\rho} - \hat{\rho}\hat{A}^\dagger \hat{A})$ critical
- One such model (which is also solvable analytically!)

$[\hat{S}_i, \hat{S}_j] = i\varepsilon_{ijk}\hat{S}_k$ Large spin-*S* (preserved)

$\hat{H}_S = \omega \hat{S}_x$ Coherent drive

$\hat{A} = \hat{S}_-$ Spontaneous decay to state $|S, -S\rangle$

New type of phase transition
Model

- Equation to solve

$$\partial\hat{\rho} = i[\hat{\rho}, \omega\hat{S}_x] + \kappa(2\hat{S}_-\hat{\rho}\hat{S}_+ - \hat{S}_+\hat{S}_-\hat{\rho} - \hat{\rho}\hat{S}_+\hat{S}_-)$$

- Limiting cases

$$\frac{\kappa}{\omega} = 0 \quad \rightarrow \quad |S, -S\rangle_x$$

Hamiltonian dominates

$$\frac{\omega}{\kappa} = 0 \quad \rightarrow \quad |S, -S\rangle_z$$

Lindbladian dominates

- Phase transition somewhere between?

New type of phase transition
Mean-field solution

✦ Mean-field (normal ordering): $S_\alpha = \langle \hat{S}_\alpha \rangle, \quad \langle \hat{S}_\alpha \hat{S}_\beta \rangle = \langle \hat{S}_\alpha \rangle \langle \hat{S}_\beta \rangle$.

$$\dot{S}_x = 2\frac{\kappa}{S} S_x S_z,$$

$$\dot{S}_y = S_z \left(2\frac{\kappa}{S} S_y - \omega \right),$$

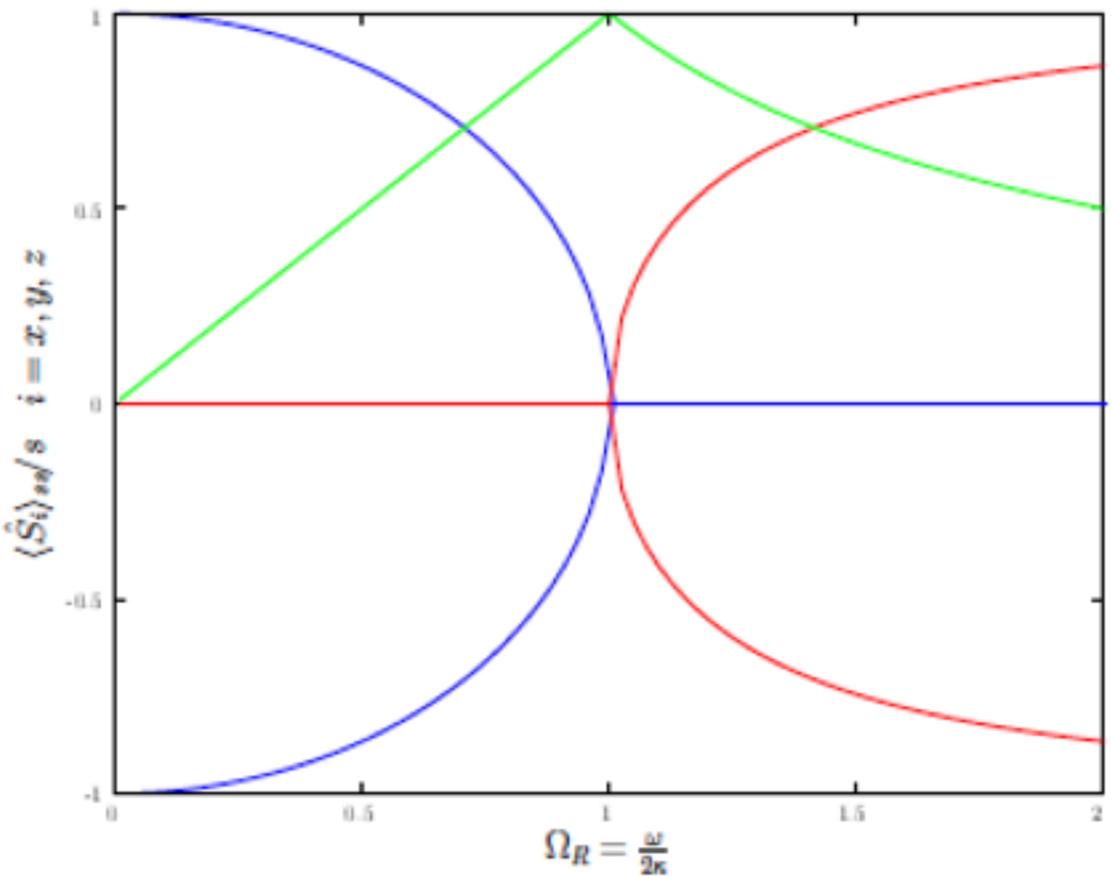
$$\dot{S}_z = \omega S_y - 2\frac{\kappa}{S} \left(S_x^2 + S_y^2 \right)$$

✦ Conserved spin: $S^2 = S_x^2 + S_y^2 + S_z^2$.

✦ Steady state solutions

$$(S_x, S_y, S_z)/S = \left(\pm\sqrt{1 - \frac{4\kappa^2}{\omega^2}}, \frac{2\kappa}{\omega}, 0 \right), \qquad (S_x, S_y, S_z)/S = \left(0 \frac{\omega}{2\kappa}, \pm\sqrt{1 - \frac{\omega^2}{4\kappa^2}} \right)$$

New type of phase transition
Mean-field solution



Classical steady state solutions: *x* (red), *y* (green), *z* (blue).

Red curve, *Hopf bifurcation*

Blue curve, *Strange bifurcation*…

New type of phase transition
Universality

✦ Universality - critical exponents.

- Mean-field. $\lambda = \omega/\kappa \longrightarrow$ "Magnetization" $S_z \propto |\lambda_c - \lambda|^{1/2} \longrightarrow \nu = 1/2$.
- Quantum. $\nu = 1/2$

✦ Seems to be universal!

✦ No length scale! Critical slowing down??

New type of phase transition

Universality

✦ Quantum treatment. Analytical solution

$$\hat{\rho}_{ss} = \frac{1}{D} \sum_{n,m=0}^{2S} (g^*)^{-m} g^{-n} \hat{S}_-^m \hat{S}_+^n, \qquad g = i\omega S/\kappa$$

✦ Expectations $\langle \hat{\mathcal{O}} \rangle = \mathrm{Tr}[\hat{\mathcal{O}} \hat{\rho}_{ss}]$. Truncate for some *S*.

✦ Critical exponent the same as for mean-field.

✦ Fluctuations in $\hat{S}_x$: $\langle \hat{S}_x^2 \rangle \propto |\lambda_c - \lambda|^{1/2}$.

New type of phase transition

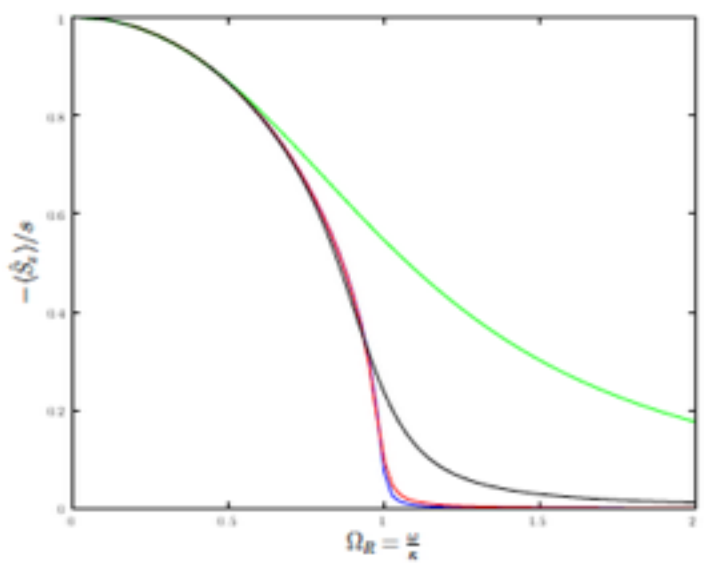
Universality

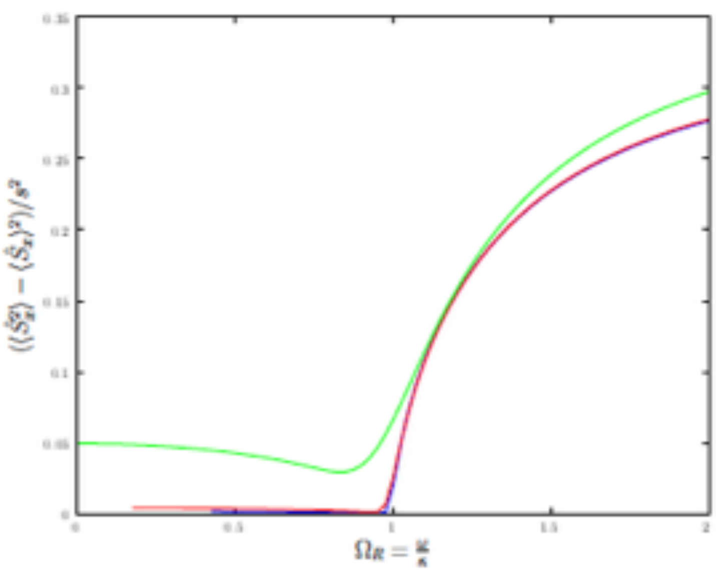
- $\langle \hat{S}_z \rangle / S \to 0, \quad \omega/\kappa \to \infty$. Hamiltonian part dominates.

- $\langle \hat{S}_z \rangle / S \to 1, \quad \omega/\kappa \to 0$. Pure state, fluctuations $\langle \hat{S}_x^2 \rangle / S^2 \to 0,$

- In general, quantum fluctuations relative system size $\mathcal{O}(S^{-1})$.

- But $\langle \hat{S}_x^2 \rangle / S^2 \to 1/3, \quad \omega/\kappa \to \infty$! Reservoir-induced-fluctuations.





System size
green
…
blue
($S = 200$)

New type of phase transition
Symmetries

✦ Symmetries.

- *Closed case*. $[\hat{A},\hat{H}]=0 \leftrightarrow \hat{H}=\hat{U}\hat{H}\hat{U}^{-1} \quad \longrightarrow \quad \hat{A}$ conserved.
- *Open case*. $\partial\hat{\rho}=\mathcal{L}_{\hat{L}}[\hat{\rho}]=\mathcal{L}_{\hat{U}\hat{L}\hat{U}^{-1}}[\hat{\rho}] \quad \longrightarrow$ generally no conserved quantity.

✦ Guess $\hat{S}_x \to -\hat{S}_x,\ \hat{S}_y \to \hat{S}_y,\ \hat{S}_z \to -\hat{S}_z$

$$\partial\hat{\rho}=i[\hat{\rho},\omega\hat{S}_x]+\kappa\left(2\hat{S}_-\hat{\rho}\hat{S}_+-\hat{S}_+\hat{S}_-\hat{\rho}-\hat{\rho}\hat{S}_+\hat{S}_-\right)$$

$$\longrightarrow$$

$$\partial\hat{\rho}=-i[\hat{\rho},\omega\hat{S}_x]+\kappa\left(2\hat{S}_+\hat{\rho}\hat{S}_--\hat{S}_-\hat{S}_+\hat{\rho}-\hat{\rho}\hat{S}_-\hat{S}_+\right)$$

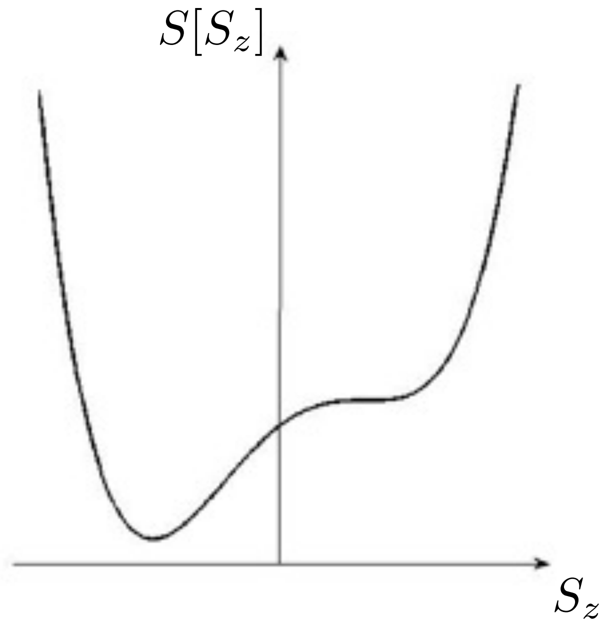
Dual symmetry! Flip spectrum + “step up” instead of “step down”.

New type of phase transition
Symmetries

✦ Mean-field.

- First bifurcation (S_z). **No** pitchfork, only one or two solutions. Stability: one stable, one unstable.

- Second bifurcation (S_x). Stability: the two solutions with purely imaginary eigenvalues $\longrightarrow$ Hopf. (Phase space a sphere so peculiar trajectories).



New type of phase transition

Second order phase transition with no symmetry breaking!!

✦ Corresponding Hamiltonian system (drive + 'internal' energy)

$$\hat{H} = \omega \hat{S}_x + \kappa \hat{S}_z$$

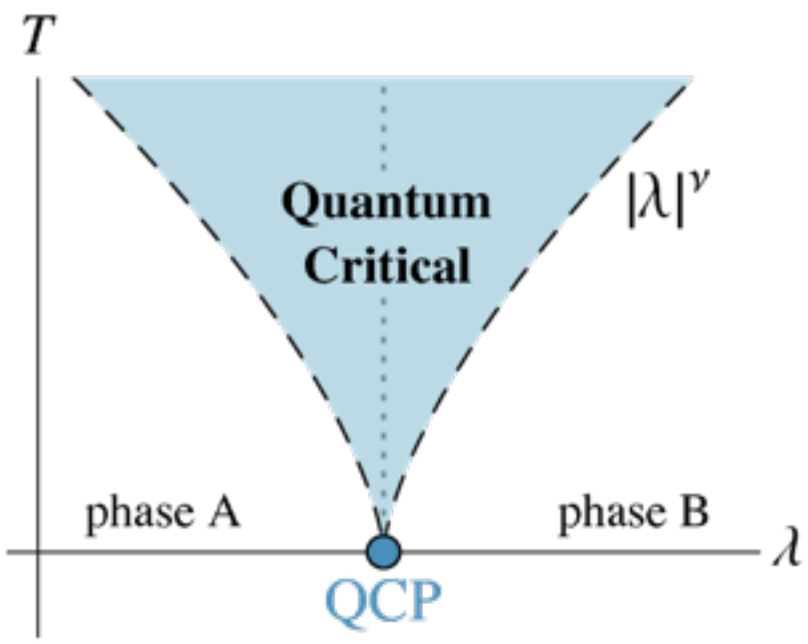
clearly not critical.

✦ Interplay between 'bath noise' and unitary evolution.

New type of phase transition
Quantum or classical

✦ Is it a quantum phase transition?

- Single degree of freedom. Quantum fluctuations vanishingly small in thermodynamic limit.
- Fluctuations due to coupling to reservoir.
- *Quantum critical region* - signatures of "quantum" survives finite temperatures in the vicinity of a critical point.

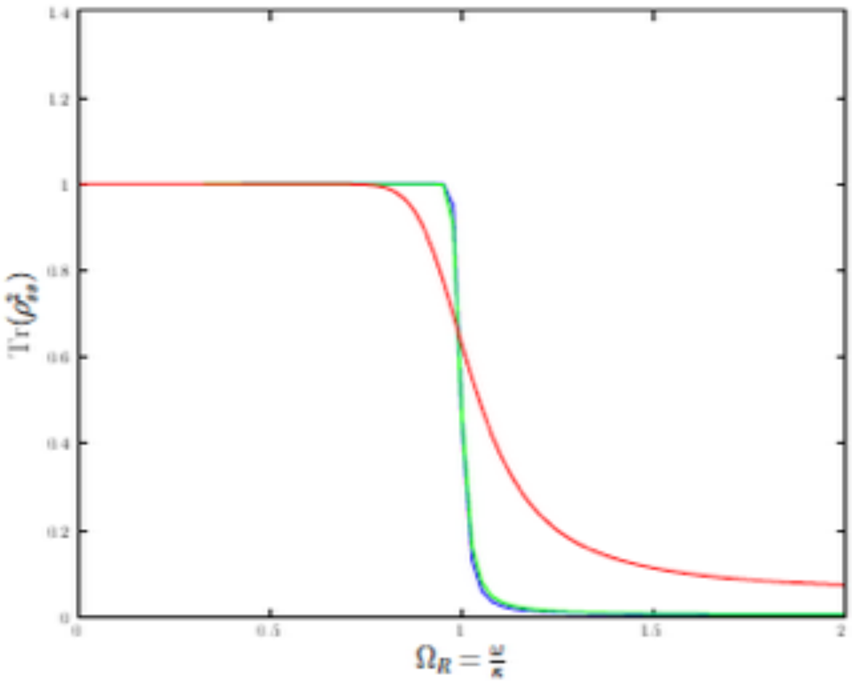


New type of phase transition

Quantum or classical

✦ "Quantum"

- Coherence: *Purity* $P \equiv \mathrm{Tr}[\hat{\rho}^2] > 0$ (thermodynamic limit).
- Entanglement: *Concurrence* $C(\hat{\rho})$ between two qubits. *Negativity* $N(\hat{\rho})$ between two equal halves.



Thermodynamic limit:

Reservoir dominated - pure state.

Hamiltonian dominated - maximally mixed.

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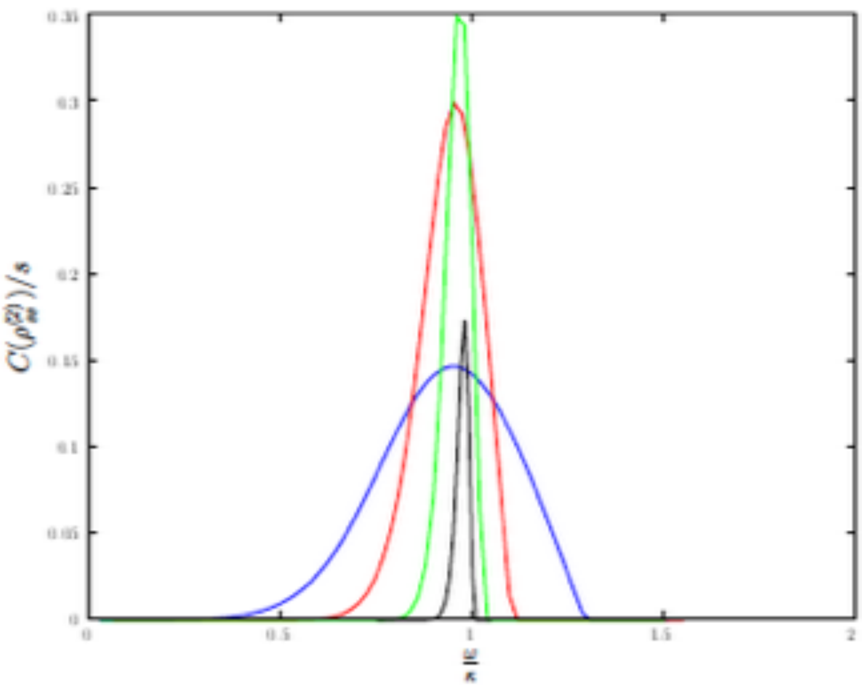

Thermodynamic limit:

No qubit-qubit entanglement outside the critical point.

New type of phase transition
Quantum or classical

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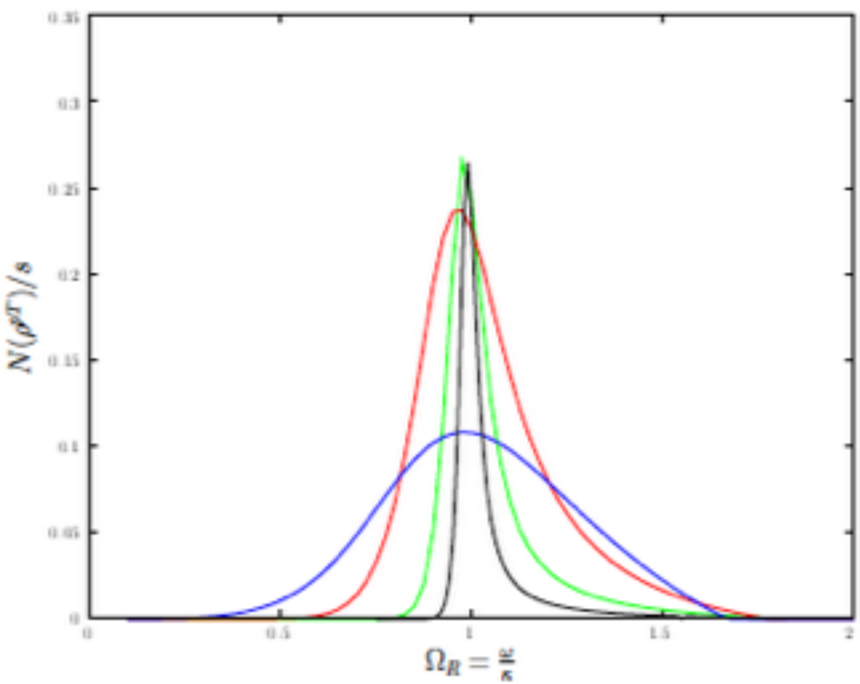

Thermodynamic limit:

No subsystem-subsystem entanglement outside the critical point.

Scales $N(\hat{\rho}) \sim S$ ('volume law').

'Critical exponents' infinite.

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Phase transitions in open quantum systems

✦ Found a model possessing an open PT, seemingly second order but lacks any symmetry breaking $\longrightarrow$ **New type**. Partly unexplored field.

Open questions

- Are these PT's universal? Scale invariance? New classes?
- Models driven more strongly by quantum fluctuations, length scale (lattice models).
- Gap closening of the *Liouvillian*, universal?
- Is there a *Mermin-Wagner theorem* for open QPT's?
- *Kibble-Zurek mechanism*?


Thanks!