Disorder and symmetry classes in cold atom systems

Jonas Larson

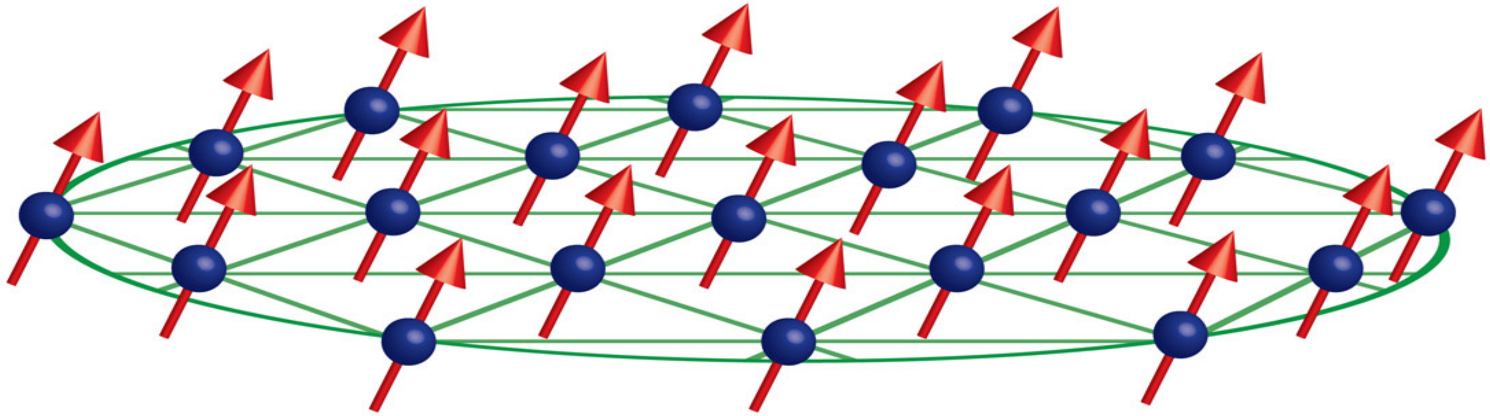
with Fernanda Pinheiro

Stockholm University and Universität zu Köln

Lund 11/5-2015

Motivation

- Optical lattices + control → quantum simulators.



- Hubbard models, spin models (magnetism), topological models (quantum computing), new models,…
- Controlled disorder. Study paradigm problems from cond-mat, fundamental questions about dynamics,…

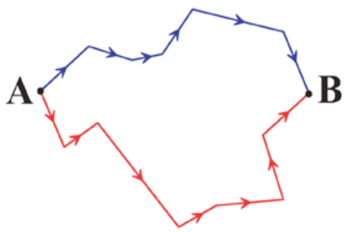
Outline

1. Anderson localization: idea.
2. Symmetry classes.
3. Anderson localization: realization with cold atoms.
4. Beyond current experiments.
5. Order vs. disorder.
6. And then…

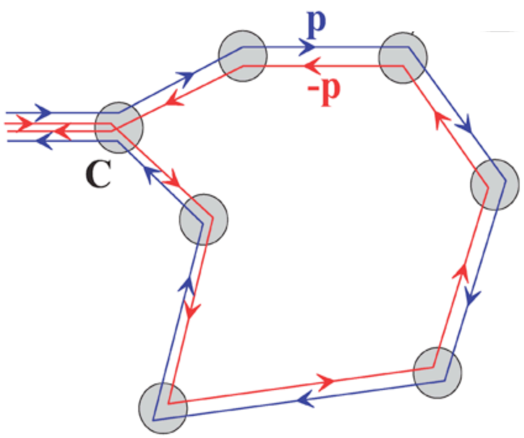
Anderson localization: idea

Weak localization

- Imagine a lattice model with onsite disorder.
- Random phase contributions.
- Going from **A** to **B**, sum all paths coherently.



- Amplitude to ”stay”, **C** to **C**. Closed loops, clockwise and anti-clockwise.
- Time-reversal symmetry; constructive interference if $\mathcal{T}^2 = +1$, destructive interference if $\mathcal{T}^2 = -1$.



Strong localization

- Anderson 1958: sufficently strong disorder may **fully** prohibit conduction.
- Destructive interference – quantum effect.
- Single-particle case – no interaction.

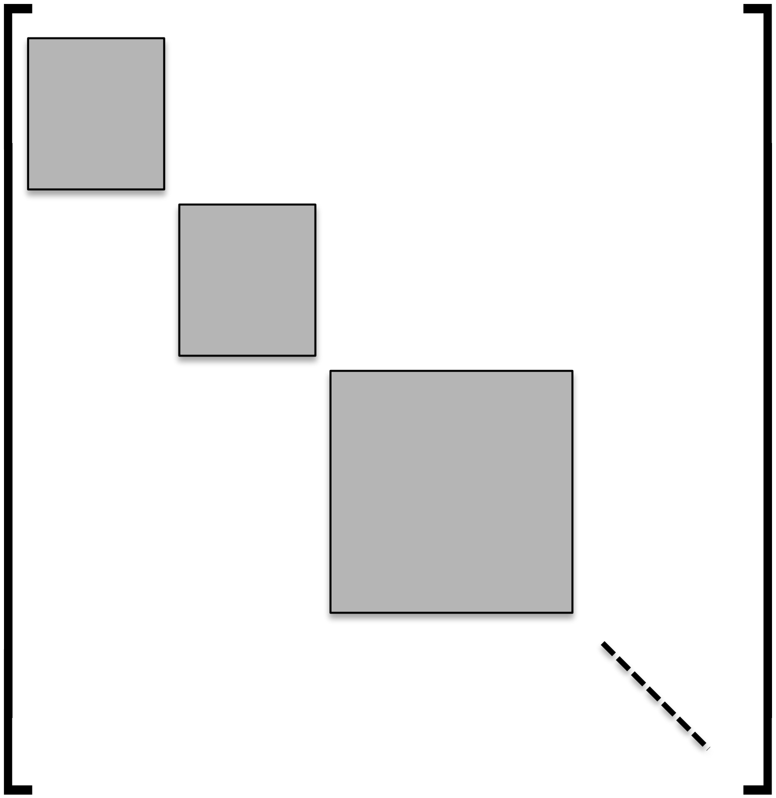
Theory:

- 1D, **all** eigenstates are localized.
- 2D, **all** eigenstates are localized, or…
- 3D, a *mobility edge:* some energy E_c separates localized from delocalized eigenstates.

Symmetry classes

Symmetry classes

- Symmetry: unitary $[\hat{U}, \hat{H}] = 0$.
- Hamiltonian on block form, each block an ’irreducible’ Hamiltonan.
- Should look for ’different’ symmetries.



Rye *et al.*, NJP **12**, 065010 (2009)

Symmetry classes

- *Time-reversal symmetry* $\mathcal{T}$:

$$\hat{U}_{\mathcal{T}}^{\dagger} \hat{H}^{*} \hat{U}_{\mathcal{T}} = +\hat{H}$$

- *Particle-hole symmetry* $\mathcal{C}$:

$$\hat{U}_{\mathcal{C}}^{\dagger} \hat{H}^{*} \hat{U}_{\mathcal{C}} = -\hat{H}$$

- Furthermore we may have

$$\mathcal{T}^2 = \pm 1, \qquad \mathcal{C}^2 = \pm 1$$

- Finally $\mathcal{T} \cdot \mathcal{C}$ is non-trivial and called *chiral symmetry* $\mathcal{S}$:

$$\left[\hat{U}_{\mathcal{S}}, \hat{H}\right]_{+} = 0$$

Rye *et al.*, NJP **12**, 065010 (2009)

Symmetry classes

- In this work:

Label	$\mathcal{T}$	$\mathcal{C}$	$\mathcal{S}$
A (unitary)	0	0	0
AI (orthogonal)	+1	0	0
AII (symplectic)	-1	0	0
AIII (chiral unitary)	0	0	1
BDI (chiral orthogonal)	+1	+1	1
CII (chiral symplectic)	-1	-1	1
D (BdG)	0	+1	0
C (BdG)	0	-1	0
DIII (BdG)	-1	+1	1
CI (BdG)	+1	-1	1

Pinheiro & Larson, arXiv:1503.07777

Symmetry classes and localization

- Focus on A, AI, AIII, and BDI (2D).

    1) Class A - *unitary*. All states localized, potentially topological.
    2) Class AI – *Wigner-Dyson orthogonal*. All states localized, topologically trivial.
    3) Class AIII – *chiral unitary*. State at $E=0$ may be extended, other states localized, topologically trivial.
    4) Class BDI – *chiral orthogonal*. Same as for AIII.

- Renormalization group calculation gives localization length

$$\lambda(E) \propto e^{g^{-1}\sqrt{\log(\Delta/|E|)}}$$

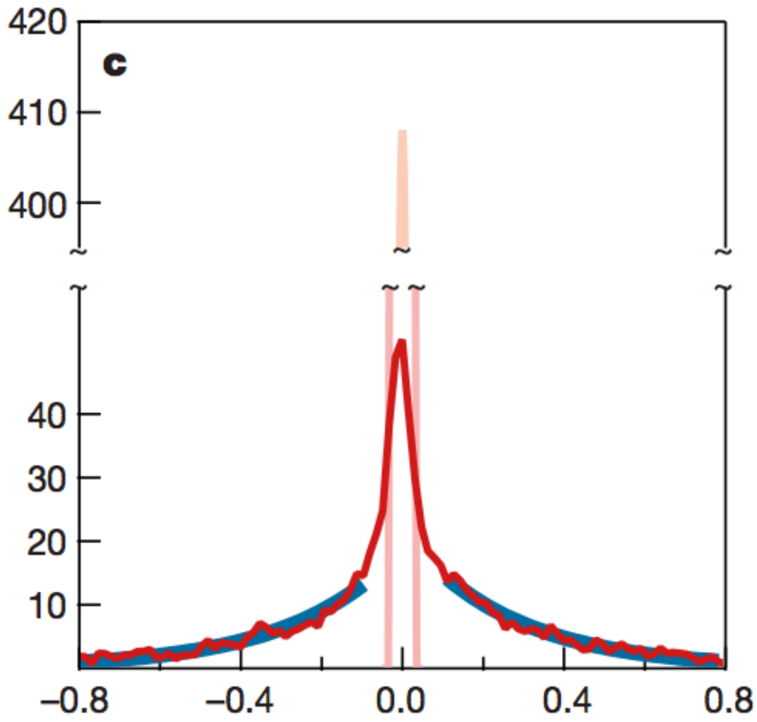
Here, Δ is a band "width" of clean system, and g^{-1} is proportional to the conductance.

Evers & Mirlin, PRM (2008), König *et al.*, PRB (2012).

Anderson localization: realization with cold atoms

Anderson in the atomic lab

1. 2008: 1D, localization and localization length (Nature **453**; 891 & 895).
2. 2009: 3D, suppression of superfluidity (PRL **102**, 055301).
3. 2010: 3D, *Bose glass* (Nature Phys. **6**, 677).
4. 2011/2012: 3D, presence of a mobility edge (Science **334**, 66; Nature Phys. **8**, 398).

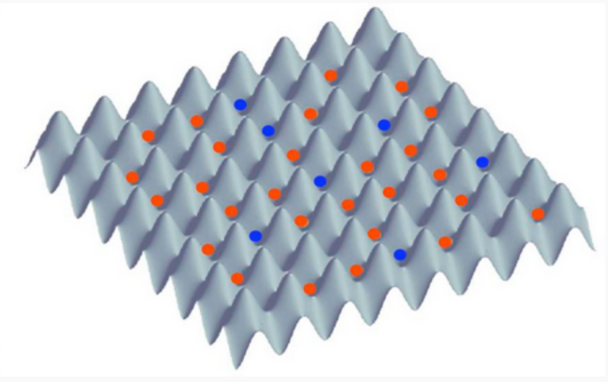


- All experiments use a standard optical potential $\longrightarrow$ class AI.

Beyond current experiments

Model Hamiltonian

- 2D isotropic square optical lattice + two atomic species.



- Tunneling t, chemical potentials μ_a and μ_b, and random (onsite) coupling between the species $h_{\mathbf{i}}e^{i\varphi_{\mathbf{i}}}$.

$$\hat{H} = -t\sum_{\langle\mathbf{ij}\rangle}\left(\hat{a}_{\mathbf{i}}^{\dagger}\hat{a}_{\mathbf{j}} + \hat{b}_{\mathbf{i}}^{\dagger}\hat{b}_{\mathbf{j}}\right) + \sum_{\mathbf{i}}\left(\mu_a\hat{n}_a + \mu_b\hat{n}_b\right) + \sum_{\mathbf{i}}h_{\mathbf{i}}\left(e^{i\varphi_{\mathbf{i}}}\hat{a}_{\mathbf{i}}^{\dagger}\hat{b}_{\mathbf{i}} + h.c.\right)$$

- Aternative: Bilayer single species model with random hopping between layers.

Pinheiro & Larson, arXiv:1503.07777

Model Hamiltonian

- Idea: control $\mu = \mu_a - \mu_b$ and $\varphi_{\mathbf{i}} = 0$ or $\varphi_{\mathbf{i}} \neq 0$. Different symmetry classes.

$h_{\mathbf{i}}$	μ	**Class**
Real-valued	Zero	BDI
Complex-valued	Zero	AIII
Real-valued	Non-zero	AI
Complex-valued	Non-zero	A

Pinheiro & Larson, arXiv:1503.07777

Onset of localization

- Divergent $\lambda(E=0)$ highly debated – hard to see numerically (too small systems) and never experimentally seen.

- Inverse partition ratio IPR

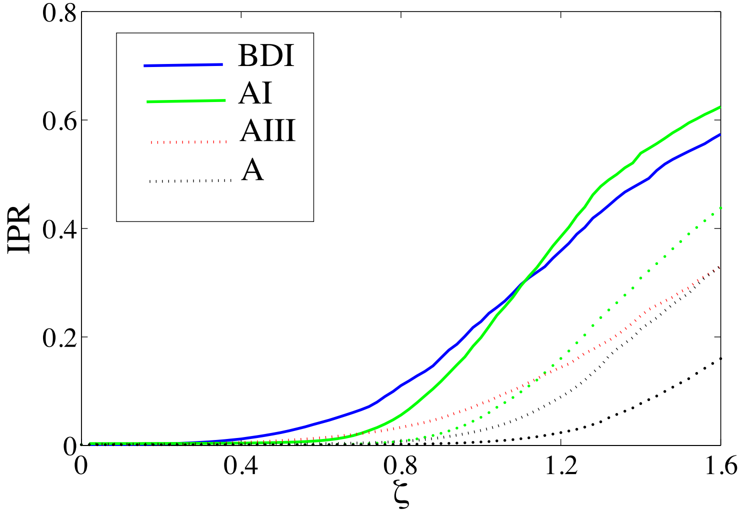
$$IPR(E) = \sum_{i,j} |\psi_E(i,j)|^4$$

$$IPR(E) = \begin{cases} \sim 1 & \text{localized} \\ N^{-1} & \text{extended} \end{cases}$$

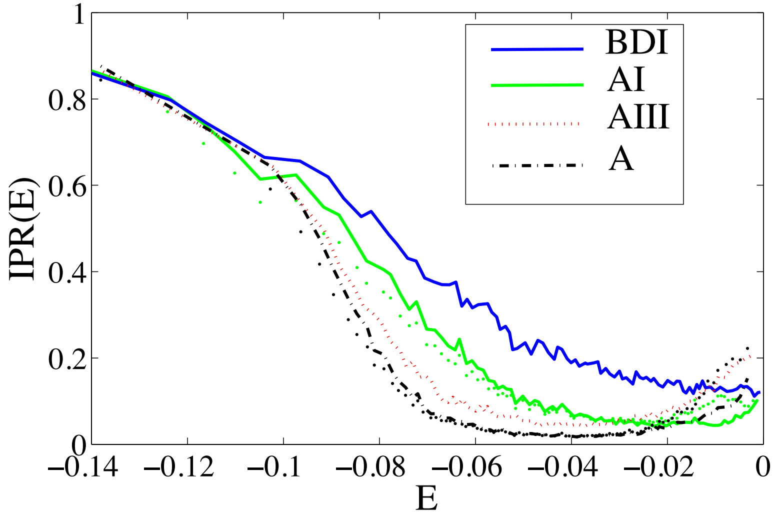
Pinheiro & Larson, arXiv:1503.07777

Onset of localization

Ground state IPR



Eigenstate IPR



The IPR for the different models and as a function of the coupling strength (disorder). Chiral models localize earlier. Dotted lines b species.

The IPR for the different models and energies. The flattening indicates that the localization length ~ system size (30x30).

Pinheiro & Larson, arXiv:1503.07777

Effective model for non-chiral cases

- Assume large μ, i.e. adiabatically elliminate (integrate out) a-species.
- Heisenberg equations of motion

$$\partial_t \hat{a}_{\mathbf{i}} = -ih_{\mathbf{i}}\hat{b}_{\mathbf{i}} - i\mu_a \hat{a}_{\mathbf{i}} + it \sum_{\mathbf{j}} \hat{a}_{\mathbf{j}},$$

$$\partial_t \hat{b}_{\mathbf{i}} = -ih_{\mathbf{i}}\hat{a}_{\mathbf{i}} - i\mu_b \hat{b}_{\mathbf{i}} + it \sum_{\mathbf{j}} \hat{b}_{\mathbf{j}}.$$

- Set $\partial_t \hat{a}_{\mathbf{i}} = 0$ giving $\hat{\mathbf{a}} = \mathbf{M}^{-1}\mathbf{h}\mathbf{b}$ with $\mathbf{h}$ diagonal and $\mathbf{M}$ tight-binding.
- Effective long range hopping for b-species

$$\left(\frac{t}{\mu_a}\right)^{-|\mathbf{i}-\mathbf{j}|}$$

- Not enough to kill localization!!

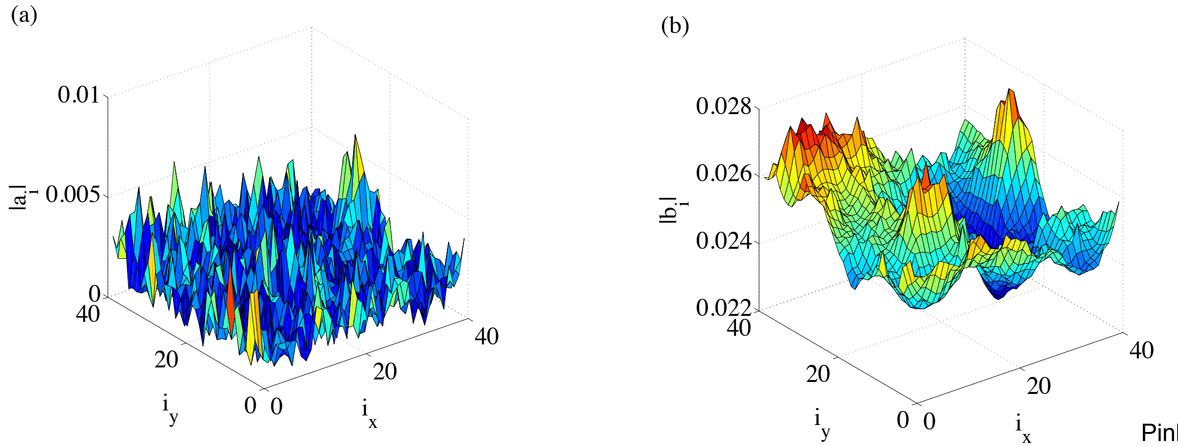
Pinheiro & Larson, arXiv:1503.07777

Effective model for non-chiral cases

$$\partial_t \hat{a}_{\mathbf{i}} = -ih_{\mathbf{i}}\hat{b}_{\mathbf{i}} - i\mu_a \hat{a}_{\mathbf{i}} + it\sum_{\mathbf{j}} \hat{a}_{\mathbf{j}},$$

$$\partial_t \hat{b}_{\mathbf{i}} = -ih_{\mathbf{i}}\hat{a}_{\mathbf{i}} - i\mu_b \hat{b}_{\mathbf{i}} + it\sum_{\mathbf{j}} \hat{b}_{\mathbf{j}}.$$

- Large population in one species gives large disorder for the other.
- If $\|\hat{b}_{\mathbf{i}}\| \gg \|\hat{a}_{\mathbf{i}}\|$, the ”small” species minimizes the disordered potential energy while the ”large” species minimizes the kinetic energy.



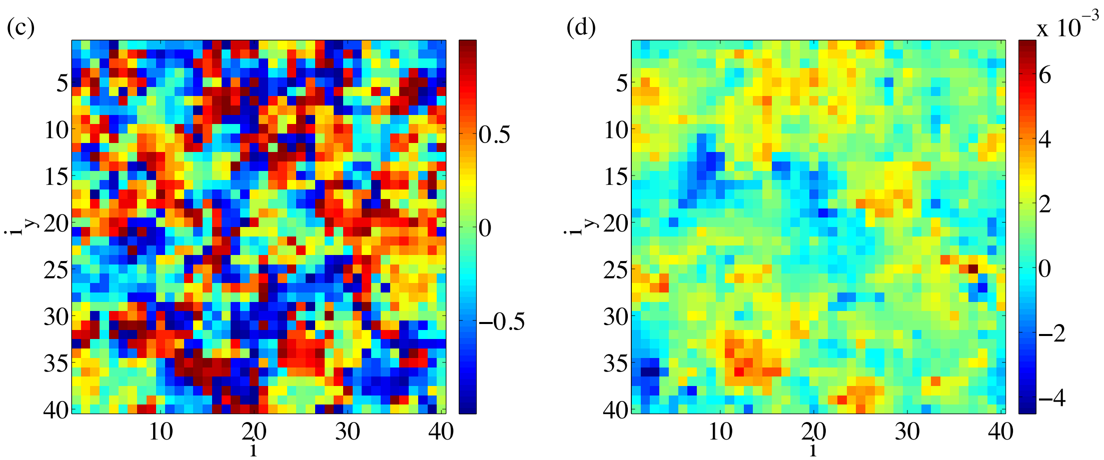
Pinheiro & Larson, arXiv:1503.07777

Effective model for non-chiral cases

$$\partial_t \hat{a}_{\mathbf{i}} = -ih_{\mathbf{i}}\hat{b}_{\mathbf{i}} - i\mu_a \hat{a}_{\mathbf{i}} + it\sum_{\mathbf{j}} \hat{a}_{\mathbf{j}},$$

$$\partial_t \hat{b}_{\mathbf{i}} = -ih_{\mathbf{i}}\hat{a}_{\mathbf{i}} - i\mu_b \hat{b}_{\mathbf{i}} + it\sum_{\mathbf{j}} \hat{b}_{\mathbf{j}}.$$

- Large population in one species gives large disorder for the other.
- If $\|\hat{b}_{\mathbf{i}}\| \gg \|\hat{a}_{\mathbf{i}}\|$, the ”small” species minimizes the disordered potential energy while the ”large” species minimizes the kinetic energy.



In the middle of the spectrum $\|\hat{b}_{\mathbf{i}}\| = \|\hat{a}_{\mathbf{i}}\|$ and for the most excited states $\|\hat{a}_{\mathbf{i}}\| \gg \|\hat{b}_{\mathbf{i}}\|$

Pinheiro & Larson, arXiv:1503.07777

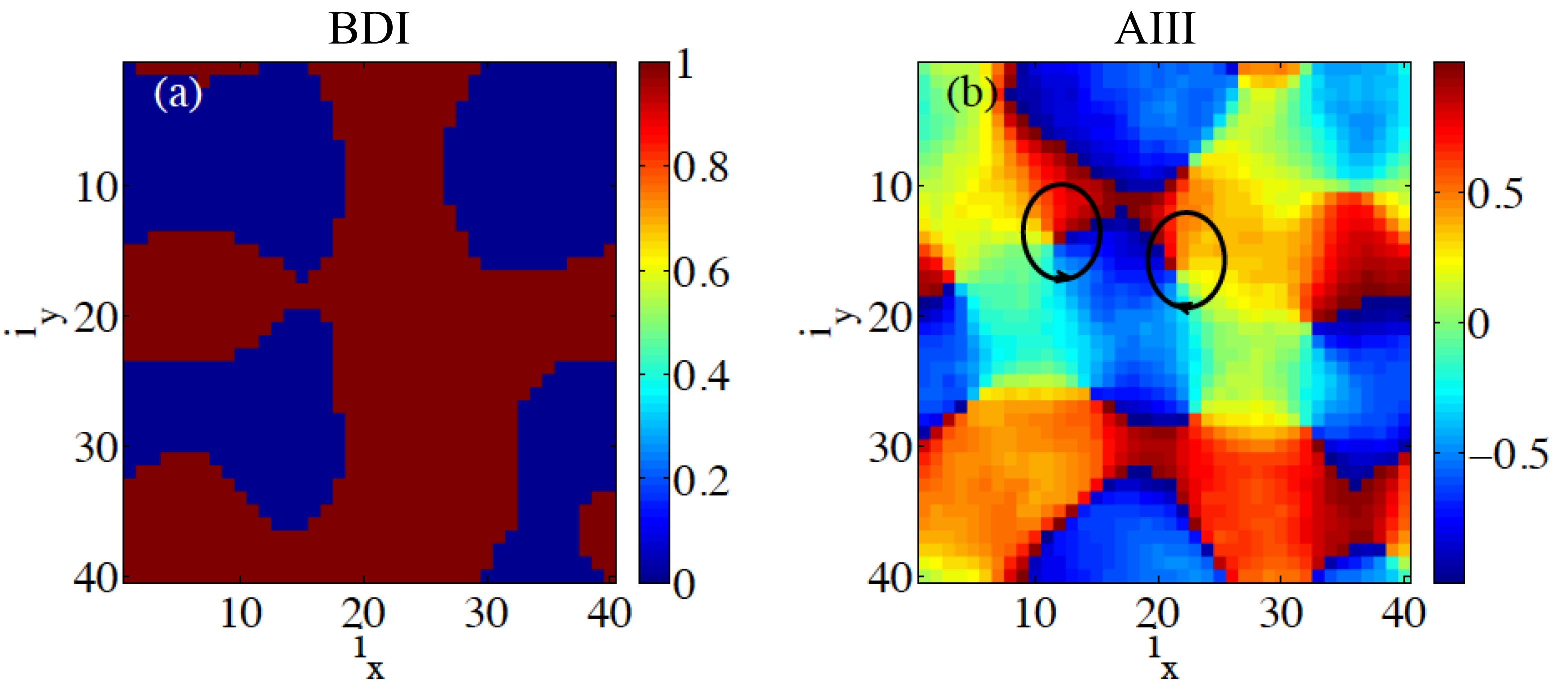
Characterizing excitations

$$\hat{H} = -t\sum_{\langle \mathbf{ij}\rangle}\left(\hat{a}_{\mathbf{i}}^{\dagger}\hat{a}_{\mathbf{j}} + \hat{b}_{\mathbf{i}}^{\dagger}\hat{b}_{\mathbf{j}}\right) + \sum_{\mathbf{i}}\left(\mu_a\hat{n}_a + \mu_b\hat{n}_b\right) + \sum_{\mathbf{i}} h_{\mathbf{i}}\left(e^{i\varphi_{\mathbf{i}}}\hat{a}_{\mathbf{i}}^{\dagger}\hat{b}_{\mathbf{i}} + h.c.\right)$$

- If $\varphi_{\mathbf{i}} = 0$, the last term favors a phase locking $\phi = 0,\ \pi$ between the onsite species.
- If $\varphi_{\mathbf{i}} \neq 0$, the the phase locking should compensate the random phase.
  1) First case (time-reversal symmetric, AI and BDI), excitations in terms of domain walls.
  2) Second case (A and AIII), excitations in terms of vortices.
- Long domain walls (many vortices) costly in terms of kinetic energy.

Pinheiro & Larson, arXiv:1503.07777

Characterizing excitations



Relative phase between the two species for eigenstate 20. Time-reversal model (left) → domain walls, complex Hamiltonian (right) → vortices.

Pinheiro & Larson, arXiv:1503.07777

Order vs disorder

“Random field induced order”

- *Mermin-Wagner theorem*: critical lower dimension were the system cannot order (too large fluctuations). Classical:

    1) Continuous symmetry, possible order for $D>2$.
    2) Discrete symmetry, possible order for $D>1$.

- Add a random field such that a continuous symmetry is broken down to a discrete one. Order not forbidden due to Mermin-Wagner - *Random-field induced order* (RFIO).

Abanin *et al.*, PRL (2007).

"Random field induced order"

- Niederberger *et al.* (PRL **100**, 030403 (2008)):

$$E = \int d\mathbf{r}[(\hbar^2/2m)|\nabla\psi_1|^2 + V(\mathbf{r})|\psi_1|^2 + (g_1/2)|\psi_1|^4 + (\hbar^2/2m)|\nabla\psi_2|^2 + V(\mathbf{r})|\psi_2|^2 + (g_2/2)|\psi_2|^4 + g_{12}|\psi_1|^2|\psi_2|^2 + (\hbar\Omega(\mathbf{r})/2)(\psi_1^*\psi_2 + \psi_2^*\psi_1)],$$

- The two BEC fields build up a $\pi/2$ phase locking due to RFIO.
- Do we see RFIO?
- No numerical evidence!
    - RFIO not well understood, even less now☺
    - Is interaction crucial?
    - Interacting random systems very different from "Anderson systems". Role of excitations?

And then…

Work in progress…

- Interaction:
    1) RFIO
    2) Excitations.
- Change of basis: phase of coupling appears on the tunneling terms – *synthetic gauge field*. *Hall physics*, *Hofstadter butterfly*,…
- Four species, can we get the symplectic class or the BdG classes?


Thanks!