Reading instructions

List of which literature (Goldstein's book *Classical Mechanics* and Ingemar Bengtsson's lecture notes [1]) that you're supposed to study. Chapters and sections not mentioned below are not part of the course.

1 Chapter 1 – *Survey of the Elementary Principles*

1.1 Section 1.1 – *Mechanics of a particle*

Hopefully most of this section is familiar to you. However, back when you studied Newtonin mechanics you did not have the same mathematical machinery, and some results here might be a bit more abstractly derived. The same should apply to the three conservation laws, i.e. that you have met them before. They say that if there are no forces acting on the particle then momentum and angular momentum must be preserved. Furthermore, if the force is conservative (important to know what a conservative force is!), then the particle's energy is also preserved.

1.2 Section 1.2 – *Mechanics of a system of particles*

Center-of-mass and relative variables are important for later when we discuss motion in central potentials and the inertia tensor.

1.3 Section 1.3 – *Constraints*

This is essential for analytical mechanics, and thereby important that you get the ideas. A constraint could be if a particle is in a box, then it is

restricted to a certain part of space. Or a particle in the atmosphere of the earth. Constraints replaces some forces, for example the walls of the box is nothing but a very strong potentials. So one can talk about *forces of constraints* (however, they are not normally specified), instead you introduce *generalized coordinates* q_i that obey the constraints. Another concept that appear throughout the course. The type of constraints is also to get familiar with; *holonomic*, *non-holonomic*, and *rheonomous* vs. *scleronomous*.

1.4 Section 1.4 – *D'Alembert's principle and Lagrange's equations*

Now it really gets started! D'Alembert's principle is good to have seen, even though it is the Lagrange's equations that will be returning method in this course. The derivations of d'Alembert's prionciple and Lagrange's equations are good exercises, and as you'll see further they are typical for how derivations in this course. Thus, it is recommended that you through them step-by-step. Learn also the difference between a full derivative and a partial derivative, *e.g.* $\frac{d}{dt}$ vs. $\frac{\partial}{\partial t}$. The *Lagrangian* is something that we come back to over and over. It is actually something you most likely meet also in other courses.

1.5 Section 1.5 – *Velocity dependent potentials and the dissipation function*

We will actually not talk too much about velocity dependent potential in the course, but the example of a charge particle in. magnetic field is so important such that it must be included in a course like this. The last part about the *Rayleigh's dissipation function* is not central.

1.6 Section 1.5 – *Simple applications of the Lagrange formalism*

I think that one has to work through numerous examples and problems in order to get acquainted with generalized coordinates and the Lagrange formalism. In this section they give the most simple ones. Goldstein returns every now and then to his expansion (1.73) of the kinetic energy. It is nothing I use later in the course though. If you feel uncomfortable with different

coordinates, Cartesian, polar, cylindrical and spherical, I recommend that you spend a few minutes to repeat that.

2 Chapter 2 – *Variational Principles and the Lagrange's Equations*

2.1 Section 2.1 – *Hamilton's principle*

This is one of the central topics in analytical mechanics. An *action integral* is something that is central in a whole set of different areas of physics (quantum mechanics, optics, quantum field theory). Actually, the idea of using variations of an action to derive the equations of motion is something you will meet in later courses if you follow the theory program.

2.2 Section 2.2 – *Some techniques of the calculus of variations*

The mathematics we need for the next section. Even though it is mathematics I think it is important for fully appreciating Hamilton's principle. When applied to quantum mechanics (not in this course) it also give a nice alternative approach (path integrals), and provides novel insight into the physics. Maybe some of you, especially those taking any course on *quantum field theory*, this is good training in functional derivatives. If you need more motivation, it is also fun mathematics.

2.3 Section 2.3 – *Derivation of Lagrange's equations from Hamilton's principle*

Another very typical and crucial derivation.

2.4 Section 2.4 – *Extending Hamilton's principle to systems with constraints*

Now the abstraction level is raised one step. We will not use this much later in the course, but I will mention it in the lecture. It works with *Lagrange*

multipliers which is something useful to know as a physicist. Focus on the derivation up till equation (2.23).

2.5 Section 2.6 – *Conservation theorems and symmetry properties*

Needless to say, symmetries is much of physics. How it connects to conservation laws is due to Emmy Noether (why didn't she get the Nobel prize?). Here the *canonical/conjugate momentum* is introduced for the first time, you better remember what it is. Note that it does not need to be equal to the physical momentum. Since the whole formalism is general, the generalized coordinates could be something else than spatial coordinates, and then also the corresponding conjugate momentum will not be a momentum. Remember also what is meant by *cyclic coordinates* since it is something we come back to frequently. My lecture is somewhat different from this section, but it does not hurt to get both views.

2.6 Section 2.7 – *Energy function and the conservation of energy*

We will talk more about this when we discuss the Hamilton formalism. Read up to equation (2.58).

3 Chapter 8 – *The Hamilton Equation of Motion*

3.1 Section 8.1 – *Legendre transforms and Hamilton's equations of motion*

Important section. The Legendre transform is an abstract tool that works. It is difficult to get a picture of what is going on. The parts about thermodynamics not included in this course, but read it anyway (only one page). I do not introduce the matrix formulation like Goldstein does on page 339. You can do it as well if you want, or do the calculations by brute force by writing out every term (like what I do). The example of a charged particle in an electromagnetic field is essential in physics! The last page about symplectic notation is not part of the course (but for one of the assignments).

3.2 Section 8.2 – *Cyclic coordinates and conservation theorems*

Here you only need to note that for a cyclic coordinate, its conjugate momentum is constant according to Hamilton's equations (first equation on page 344). You don't need to go into further details after this equation.

4 Chapter 9 – *Canonical Transformations*

4.1 Section 9.1 – *The equations of canonical transformations*

This is an important section, and most likely a bit hard to digest. In other words, pay some extra time to it. The four basic transformations are presented in Table 9.1, and note how the change of variables are performed by a Legendre transformation.

4.2 Section 9.2 – *Examples of canonical transformations*

Following the rules from the previous section, this section is hopefully rather straightforward. It is not mentioned, but note how the *Jacobian matrix* on page 376. You met the Jacobian matrix in mathematics when you, for example, changed between Cartesian and spherical coordinates.

4.3 Section 9.3 – *The harmonic oscillator*

This section gives the first non-trivial example of how to solve a problem using canonical transformation. It is not the simplest way for solving the harmonic oscillator, but an instructive example for how to apply the idea of canonical transformation. So make sure you understand the steps in the derivation.

4.4 Section 9.5 – *Poisson brackets and other canonical Invariants*

Goldstein rapidly moves into the matrix representation of Poisson brackets. I chose to not take that path. It clearly has its advantages, once you are used to it the derivations become short. However, it adds a level of abstraction

that I don't think we need. So instead I write out the derivations step-by-step. Nevertheless, the first four pages of Section **9.5** are important. As I said, in the lecture I do the same but without the matrix formalism. You can skip the discussion about Lagrange brackets and the last pages of the section. You're of course also welcome to read pages 97-98 in Ingemar's note [1] on the same topic.

4.5 Section 9.6 – *Equations of motion, ...*

The first part up to half of page 398 is included. The remaining part of the section is interesting, for example it makes the connection to quantum mechanics even more striking, but it is only for the interested reader.

4.6 Section 9.9 – *Liouville's theorem*

The whole section is part of the course. However, my derivation of the theorem on the lecture is slightly different from that presented in Goldstein. I derive a continuity equation and from that I can prove the theorem. Goldstein uses a result (*Poincaré's integral invariant*) from part of a section that is not included in the course. With this result Liouville's theorem follows easily. If you buy the result of Poincaré's integral invariant, namely that a volume element $\Delta V = \Delta q \Delta p$ is invariant under time-evolution, then you should be able to grasp Goldstein's proof (which is also intuitively nice).

5 Ingemar notes – *Phase space*

Goldstein lacks any deeper discussion about phase space, it is mentioned as if the reader is familiar with it already. However, in Ingemar's notes [1] there is a good description about phase space, Section 1.4. It is important that you feel comfortable with the phase space, and that you understand the different types of *fixed points*. Fixed points is something returning in physics, we will most likely discuss them further in terms of chaos, and if you read about the *renormalization group* later during your studies you'll find that they are extremely important there as well.

6 Ingemar notes – *Connection to Quantum Mechanics*

Goldstein does not discuss this more than mentioning it at a few occasions, for example he sells the Hamilton formalism partly as bridging classical and quantum mechanics (QM). However, I don't see why he could not add one section about it.

Ingemar discusses this partly in the section about Poisson brackets (section 8.2). This short part I recommend that you read. He then continues with a longer more mathematical discussion in section 8.5. This, however, is outside the scope of the course.

Below I summarize the important parts that you need to know.

When you learned about Poisson brackets I hope that you already saw numerous similarities with QM. If not you have to recall the Heisenberg picture of QM, in which states are static while operators evolve in time. The solution of the Schrödinger equation

$$i\partial_t|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \tag{1}$$

can be formally written as

$$|\psi(t)\rangle = \hat{U}(t,0)|\psi(0)\rangle, \qquad \hat{U}(t,0) = \exp\left(-i\hat{H}t/\hbar\right), \tag{2}$$

when assuming that the Hamiltonian $\hat{H}$ is time-independent (for a time-dependent Hamiltonian we have to modify the equations slightly, but everything works also for such cases). Expectations of some observable $\hat{A}$ (as an observable, the operator $\hat{A}$ is hermitian, i.e. $\hat{A} = \hat{A}^\dagger$) with respect to the state $|\psi(t)\rangle$ are

$$\langle\hat{A}\rangle = \langle\psi(t)|\hat{A}|\psi(t)\rangle = \langle\psi(0)|\hat{U}^\dagger(t)\hat{A}\hat{U}(t)|\psi(0)\rangle = \langle\psi(0)|\hat{A}(t)|\psi(0)\rangle,$$

where we have defined the time-dependent operator

$$\hat{A}(t) = \hat{U}^\dagger(t)\hat{A}\hat{U}(t). \tag{3}$$

If we take the time-derivative of this operator, and use the explicit form of the time-evolution operator $\hat{U}(t)$, one finds

$$\frac{d}{dt}\hat{A} = \frac{1}{i\hbar}[\hat{A},\hat{H}] \equiv \frac{1}{i\hbar}(\hat{A}\hat{H} - \hat{H}\hat{A}). \tag{4}$$

Solving this equation is equivalent to solving the Schrödinger equation (1). Knowing a state $|\psi(t)\rangle$ implies that we know as much as possible about the system, all the properties of the state are contained in the expectations $\langle\hat{A}\rangle_t = \langle\psi(t)|\hat{A}|\psi(t)\rangle$ for all possible operators $\hat{A}$ (not necessarily observables). However, knowing $\hat{A}(t)$ and the initial state $|\psi(0)\rangle$ we get the same information from $\langle\hat{A}\rangle_t = \langle\psi(0)|\hat{A}(t)|\psi(0)\rangle$. The former approach is the *Schrödinger picture*, while the latter is the *Heisenber picture*. If the operator $\hat{A}$ is explicitly time-dependent we modify the above equation-of-motion as

$$\frac{d}{dt}\hat{A} = \frac{1}{i\hbar}[\hat{A},\hat{H}] + \frac{\partial\hat{A}}{\partial t} \tag{5}$$

The above equation has the same structure as the one rendering time-evolution in classical mechanics (Goldstein, equation. (9.94)),

$$\frac{d}{dt}u = \{u,H\} + \frac{\partial u}{\partial t}. \tag{6}$$

By making the identification

$$\{..,..\} \leftrightarrow \frac{1}{i\hbar}[..,..] \tag{7}$$

the two equations (5) and (6) are formally identical. Thus, we swap between Poisson brackets and commutators, together with adding a factor $1/i\hbar$ where $\hbar$ is Planck's constant divided by 2π. Let's check this for the canonical conjugate variables q_i and p_j. Classically we have

$$\{q_i,p_j\} = \sum_l \left(\frac{\partial q_i}{\partial q_l}\frac{\partial p_i}{\partial p_l} - \frac{\partial q_i}{\partial p_l}\frac{\partial p_i}{\partial q_l}\right) = \frac{\partial q_i}{\partial q_i}\frac{\partial p_j}{\partial p_i} = \delta_{ij}. \tag{8}$$

The quantum commutator between the two operators $\hat{q}_i$ and $\hat{p}_j$ should therefor be $i\hbar\delta_{ij}$ according to (7);

$$\begin{aligned}[\hat{q}_i,\hat{p}_j] &= \hat{q}_i\left(-i\hbar\frac{\partial}{\partial\hat{q}_j}\right) - \left(-i\hbar\frac{\partial}{\partial\hat{q}_j}\right)\hat{q}_i \\ &= \hat{q}_i\left(-i\hbar\frac{\partial}{\partial\hat{q}_j}\right) - (-i\hbar\delta_{ij}) - \hat{q}_i\left(-i\hbar\frac{\partial}{\partial\hat{q}_j}\right) = i\hbar\delta_{ij}.\end{aligned} \tag{9}$$

You are welcome to derive the same 'correspondence' between the classical Poisson brackets $\{L_i,L_j\}$ and the quantum commutators $[\hat{L}_i,\hat{L}_j]$ for the angular momentum.

Using the identification (7) is in fact a standard method for *quantizing* a classical system. If you know the Poisson brackets, for example the canonical conjugate variables, you postulate the same commutators (multiplied with $1/i\hbar$) for the corresponding quantum operators. This goes under the name the *correspondence principle*.

For those of you familiar with the density operator $\hat{\rho}$ of QM we find another nice correspondence. The density operator can formally be written as

$$\hat{\rho} = \sum_n p_n |\phi_n\rangle\langle\phi_n|, \qquad p_n \geq 0. \tag{10}$$

Here, p_n is the (classical) probability ($\sum_n p_n = 1$) for the system to be found in the state $|\phi_n\rangle$. The set of states $\{|\phi_n\rangle\}$ need not be orthonormal, but normalized. As for the state vector of $|\psi\rangle$ of quantum system, $\hat{\rho}$ provides the complete information about the system. The two formalisms are equivalent as long as we consider *pure* states $|\psi\rangle \Leftrightarrow \hat{\rho} = |\psi\rangle\langle\psi|$. However, the density formalism allows to describe a larger class of states, namely *mixed states* occurring when two or more of the probabilities p_n are non-zero. In such cases we may think of $\hat{\rho}$ as a classical ensemble of the quantum states $|\phi_n\rangle$ (note that this is not a quantum superposition of the same states!). In this way we can incorporate classical probabilities into QM. The evolution operator $\hat{U}(t)$ are as before (6), even when we work with density operators, but we have to act with it both from the left and the right since $\hat{\rho}$ is a matrix. A density operator thus evolves as

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^{-1}(t) = \sum_n p_n \hat{U}(0)|\phi_n(0)\rangle\langle\phi_n(0)|\hat{U}^{-1}(t) = \sum_n p_n |\phi_n(t)\rangle\langle\phi_n(t)|. \tag{11}$$

Differentiating this equation results in the Schrödinger equation for the density operator

$$\frac{d}{dt}\hat{\rho}(t) = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}(t)\right] + \frac{\partial}{\partial t}\hat{\rho}(t). \tag{12}$$

This is **not** a Heisenberg equation for the matrix/operator $\hat{\rho}$, but really the Schrödinger equation (we consider the Schrödinger picture where the state evolves in time). Even though it looks much like a Heisenberg equation there is a difference, there is a sign-difference in front of the first term (see Eq. (7)). This equation is also referred to as the *quantum Liouville equation*, to be compared to the classical Liouville equation

$$\frac{d}{dt}\rho(q,p,t) = \{\rho(q,p,t), H\} + \frac{\partial}{\partial t}\rho(q,p,t).$$

From the quantum Liouville equation (12) we can directly evaluate how expectation values of operators evolve in time from the identity $\frac{d}{dt}\langle\hat{A}\rangle = \frac{d}{dt}\mathrm{Tr}\left[\hat{A}\hat{\rho}\right] = \mathrm{Tr}\left[\hat{A}\frac{d}{dt}\hat{\rho}\right]$. By plugging in the expression (12) for $\frac{d}{dt}\hat{\rho}$ we arrive at the *Ehrenfest theorem*

$$\frac{d}{dt}\langle\hat{A}\rangle = \frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle + \frac{\partial}{\partial t}\langle\hat{A}\rangle.$$

This tells us that quantum expectation values obey the same equations of motion as their classical counterparts.

Classically $\rho(q,p,t)$ is a probability distribution of the canonical variables (q,p). Quantum mechanically, $\hat{q}$ and $\hat{p}$ are not independent variables due to the Heisenberg uncertainty relation. This causes problems in trying to construct a quantum counterpart of $\rho(q,p,t)$. However, there are ways to construct something similar to $\rho(q,p,t)$ also in QM. Even though one can prove that a direct analog is impossible. This is the topic of one of the assignment topics.

Let us conclude by looking at another similarity between the two theories. If we assume that u is time-independent, then we had that if its Poisson bracket with the Hamiltonian H vanishes, i.e. $\{u,H\} = 0$, then u is an integral of motion – that is it is constant. The same holds, of course, in QM as well; if a time-independent operator $\hat{A}$ commutes with $\hat{H}$, then it is a constant of motion. Further, if $\hat{A}$ is hermitian we have that it defines a symmetry. More precisely, if $\hat{U}(\phi) = \exp(i\hat{A}\phi)$ for some parameter ϕ, then $[\hat{U}(\phi),\hat{H}] = 0$ and $\hat{U}(\phi)$ will be unitary. For example, for a potential $V(r)$ in spherical coordinates we have that $[\hat{L}_z,\hat{H}] = 0$, and we say that $\hat{L}_z$ is an integral/constant of motion. It also defines a symmetry, $\hat{U}(\phi) = \exp(i\hat{L}_z\phi)$ acting on the Hamiltonian $(\hat{U}(\phi)\hat{H}\hat{U}^{-1}(\phi))$ represents a rotation by an angle ϕ around the z-axis. We say that $\hat{L}_z$ is a *generator* of rotations around z. Likewise, the Hamiltonian is a generator of time-evolution, both in classical and QM. This is the quantum version of Noether's theorem. A caveat, the above holds true for continuous symmetries, not for discrete ones (e.g. parity $\hat{q}_i \leftrightarrow -\hat{q}_i$). In general, for classical systems Noether's theorem applies for continuous symmetries and not discrete ones.

7 Chapter 10 – *Hamilton-Jacobi Theory and the Action-Angle Variables*

7.1 10.1 – *The Hamilton-Jacobi equation...*

Essential section. It presents the ideas of the Hamilton-Jacobi theory, and it turns out good for understanding canonical transformations. It is important to get that we have not solved anything so far, only rewritten the equations-of-motions into a single non-linear first order partial differential equation for $S(\mathbf{q}; \alpha, t)$.

7.2 10.2 – *The harmonic oscillator problem...*

Good example to see the Hamilton-Jacobi 'in action'. You may focus on the first part, the 1D harmonic oscillator. The remaining two examples are not hard to follow, but the important ingredients are already in the first example (nevertheless, knowing what a *Lissajous curve* is might turn out valuable knowledge in a party or so).

7.3 10.3 – *...Hamilton's characteristic function*

With the *principle function* $S(\mathbf{q}, \mathbf{P}, t)$ we transformed the problem into one with $K \equiv 0$, *i.e.* the canonical $Q_i = \beta_i = \text{const.}$ and $P_i = \alpha_i = \text{const.}$ (I call this method 3). This approach is general and works for explicitly time-dependent Hamiltonians $H(\mathbf{q}, \mathbf{p}, t)$. Alternatively, if $\partial H/\partial t = 0$ a systematic approach is to transform the Hamiltonian into one with every Q_i cyclic (in my lecture I call this method 2). This is solved via the Hamilton-Jacobi equation ow taking the form

$$H\left(q_i, \frac{\partial W}{\partial q_i}\right) = \alpha_1,$$

i.e. first equation on page 441 in Goldstein. The procedure how to solve the problem follows similar steps as for case 1.

7.4 10.4 – *Separation of variables...*

In most solvable examples with more than one degree-of-freedom, the Hamilton-Jacobi method becomes helpful only when the action S can be separated in

the variables q_i. This is briefly discussed in this section.

7.5 10.5 – *Ignorable coordinates...*

To see the idea of variable separation in action, in this section I recommend that you look at the example of planar motion in a central force field that starts at the end of page 448 and continues for the whole page 449.

7.6 10.6 – *Action-angle variables in systems of one degree of freedom*

Important section. Also as a training in Hamilton-Jacobi theory.

7.7 10.7 – *Action-angle variables for completely separable systems*

The generalization to many variables is straightforward. Read up to equation (10.106). It is interesting that the method works for any separable system, even if the full system is not periodic. For the fully separable system, each canonical pair (q_i, p_i) is periodic with some frequency v_i. However, if the different v_i's are not commensurate the full system will not be periodic.

For the interested; if you look at trajectories in configuration space, like in two dimensions $(q_1(t), q_2(t))$ they will form Lissajous curves, for irrational v_1/v_2 the curves will never close.

8 Chapter 3 –*The Central Force Problem*

8.1 3.1 – *Reduction to the equivalent one-body problem*

This section is probably something you already studied in the first mechanics course and thereby familiar stuff. But nevertheless, it might be good to repeat concepts as center-of-mass variables, relative variables, and reduced masses.

8.2 3.2 – *The equations of motion and first integrals*

Important to understand how the conserved quantities help in solving the problem with a potential on the form $V(r)$ (r is the radial variable). Here

the Lagrange formalism is applied, which is a good exercise. Pay attention to the centrifugal term

$$\frac{1}{2}\frac{l^2}{mr^2}$$

that pops up in the Euler-Lagrange equation for the r-variable. You may connect this to what you learned about central forces in quantum mechanics.

9 Chapter 4 −*The Kinematics of Rigid Body Motion*

Most of this chapter is applied linear algebra, and hopefully you recognize much already.

9.1 4.1 −*The Independent coordinates...*

Probably stuff you remember from your first courses on classical mechanics.

9.2 4.2 −*Orthogonal transformations*

Read if you wish to refresh your knowledge about matrices that rotates vectors. You have seen this in your linear algebra courses. You may also recall quantum mechanics; unitary operators (operators leaving the scalar product invariant) is the generalization of orthogonal matrices. Orthogonal matrices are real and act as rotations in $\mathbb{R}^3$, while unitary matrices are in general complex and rotate vectors in Hilbert space $\mathcal{H}$.

9.3 4.3 −*Formal Properties...*

After quantum mechanics this is hopefully known for you: (1) matrices do not need to commute (for example rotations in $\mathbb{R}^3$), (2) an orthogonal matrix is diagonalizable and its spectrum is real, (3) the norms of the eigenvalues of an unitary matrix is 1, i.e. the determinant is ± 1. If this sounds familiar to you, no need to read this section.

9.4 4.4 –*The Euler angles*

The Euler angles is something you most likely will hear many times through your studies (maybe you have already). The Euler angles is a choice for parametrizing the rotation into three independent angles.

9.5 4.6 –*Euler's theorem ...*

Another section of linear algebra. It is shown that the norm of the eigenvalues of an orthogonal matrix must be unity. In quantum mechanics this is often motivated by the fact that the norm should be preserved. Using the mathematics of orthogonal matrices, it is shown that the motion of a rigid body with one fixed point is a rotation. This is, of course, an expected result, but nevertheless the chapter is useful for getting a better intuition for rotations and orthogonal matrices. A more general result is that any motion of a rigid body can be characterized by a translation and a rotation.

9.6 4.8 –*Infinitesimal rotations*

The derivation in this section is very similar to that of rotations in quantum mechanics (for you who took Sakurai), and one sees how the cross product naturally arises and also how the angular commutation relations derive. You might also recognize polar/axial vectors from electromagnetism. The section is good to have seen during your studies, but it is not central for this course.

9.7 4.9 –*Rate of change of a vector*

Some of the results of this section are needed when discussing the inertia tensor in chapter 5. Especially the result Eq. (4.86) will be needed later on. The tensor derivation of this result can be skipped.

10 Chapter 5 – *The Rigid Body Equations of Motion*

10.1 5.1 –*Angular momentum and ...*

An important section, not so much for understanding the ideas behind analytical mechanics. However, rotations and central motion is so important in

physics that it is good to pay some extra thinking about these things.

10.2 5.3 –*The inertia tensor and ...*

A good exercise in how the developed theory is applied is presented in this section. It ends with an example of how to derive the inertia tensor for a homogeneous cube.

10.3 5.4 –*The eigenvalues of the ...*

As for the previous section, an exercise that deepens the understanding for the inertia tensor. This idea of diagonalizing **I** is likely that you will meet in later courses. The moment of inertia can be written as a quadratic form, and this often also happens in quantum mechanical problems and whenever the Hamiltonian is written on quadratic form it is usually easy to diagonalize it.

11 Chapter 11 – *Classical Chaos*

Classical chaos could make up a course on its own, and if you ask me an interesting and fun course. Unfortunately, there will not be much time to go into this topic in any depth, but I'll try to mention some basic concepts and ideas.

11.1 11.1 –*Periodic motion*

References

[1] Ingemar's notes: http://3dhouse.se/ingemar/anmek1.pdf