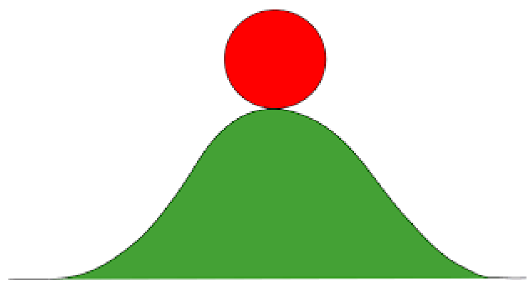
The Kepler problem with negative mass

Analyzing the consequences of the sign of mass m in a simple system of two interacting particles/masses.



1 Problem

A particle with position $\mathbf{x}_1(t)$ and mass $m_1 > 0$ and a particle with position $\mathbf{x}_2(t)$ and mass $m_2 < 0$ are moving in three dimensions and interacting according to Newton's law of gravity.

1. Set up the Lagrangian for this system.[1] Discuss the conserved quantities, *i.e.* energy, momentum, and angular momentum. Furthermore, analyze the circular orbits, and especially in the case when the reduced mass $\mu \equiv m_1 m_2/(m_1 + m_2) < 0$.

2. Make a Legendre transformation and derive the Hamiltonian of this system. Discuss what happens if $m_2 < 0$. Consider the case $\mu = 0$ by solving the Hamilton's equations. Assume that the particles start from rest, $\dot{\mathbf{x}}_1(0) = \dot{\mathbf{x}}_2(0) = 0$.

[1] *Hint*: You may find chapter 3 of Goldstein et al. helpful.

3. Compare the above result with Coulomb's law of classical electromagnetism for two particles of opposite electric charge. The Hamiltonian is again unbounded from below. Why is this less problematic than the case with two masses of opposite sign?

4. Considering the fact that Newton's law of gravity and Coulomb's law take the same mathematical form, why does the gravitational case behave so differently as compared to the electromagnetic case?