Chaos in the double pendulum

Study of chaos in a simple system. Combination of Lagrangian and Hamiltonian mechanics with numerical analysis.

1 Problem

The double pendulum is one of the, if not the, simplest model exhibiting chaotic behaviour. As you know from the exercises (see also the figure), there are two degrees-of-freedom. The only conserved quantity is the energy, so we have more degrees-of-freedom than preserved quantities as is needed for chaotic motion.

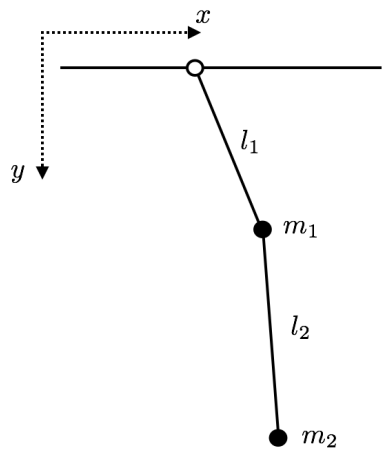

Figure 1: Sketch of the double pendulum problem. The lengths of the two arms are l_1 and l_2, and the point masses m_1 and m_2.

This assignment is different from the others as it requires numerical simulations of the corresponding equations-of-motion. The idea is that you should

identify the chaotic motion. Try to discuss the type evolution in terms of the system energy and parameters (l_1, l_2, m_1, $1m_2$). For example, where do we see pronounced chaotic behaviour and when is the motion regular? To characterize the dynamics it is recommended that you look at Poincar sections, or Lyapunov exponents. In order to simulate the evolution, you first need to derive the corresponding equations-of-motion, either in the Lagrange or Hamilton formalism (or both). One relevant reference is given below.

References

[1] https://hal.archives-ouvertes.fr/hal-01389907/document