ARQFT problem set 1

Due date: Friday, Dec 14, 2012

1. (a) Starting from the expression for the conserved charge as given by Noether's theorm, find the expression for the $U(1)$ charges of the complex scalar field and the Dirac field. Express the results in terms of creation and annihilation operators for the corresponding fields.

   (b) Starting from the expression for the Noether charge, express the Hamiltonian H, momentum P_i and angular momentum M_{ij} operators in terms of the fields for (a) real scalar fields and (b) Dirac fields.

   Express the conserved quantities H, P_i in terms of the creation and annihilation operators for the corresponding fields.

   (c) In the Dirac theory, the projection of the spin operator along the particle momentum is given by (in units where $\hbar = 1$)

   $$S_p = \frac{1}{2}\int d^3x\, N(\psi^\dagger \sigma_p \psi)$$

   (See Mandl and Shaw for notations). Show that in terms of lowering and raining operators,

   $$S_p = \frac{1}{2}\sum_{r,\vec{p}}(-1)^{r+1}\left(d_r^\dagger(\vec{p})d_r(\vec{p}) + c_r^\dagger(\vec{p})c_r(\vec{p})\right)$$

2. Using the expansion of the Dirac field $\psi(x)$ in terms of creation and annihilation operators, compute the fermion propagator $\langle 0|T\left(\psi(x)\bar{\psi}(x')\right)|0\rangle$.

   Show that the answer can be expressed in terms of $S_F(x-x')$ given by the contour integral

   $$S_F(x) = \frac{\hbar}{(2\pi\hbar)^4}\int d^3p \int_{C_F} dp^0 e^{-ipx/\hbar}\frac{\not{p} + mc}{p^2 - m^2c^2}$$

   where C_F is the Feynman contour in the complex p^0 plane.