<u>QFT-I problem set 4</u>

(Fawad Hassan)

Deadline: May 27, 2007

1. The Lagrangian density for the complex free scalar field $\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi$ is invariant under global phase transformations $\Phi \to \widetilde{\Phi} = e^{i\theta}\Phi$ and $\Phi^\dagger \to \widetilde{\Phi}^\dagger = e^{-i\theta}\Phi^\dagger$, but it is not invariant if $\theta = \theta(x)$ is a space-time dependent parameter. Construct the locally gauge invariant Lagrangian, and verify its invariance under the local $U(1)$ gauge transformation. Write down the interaction Lagrangian describing the coupling of the scalar field to the photon field. Draw the interaction vertices and give the corresponding Feynmen rules.

2. Consider the elastic electron-neutrino scattering processes, $\nu_\mu + e^- \to \nu_\mu + e^-$ and $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$. Derive the following expressions for the total cross sections,

$$\sigma_T(\nu_\mu e) = \frac{G^2 s}{3\pi}(g_V^2 + g_V g_A + g_A^2)\,, \qquad \sigma_T(\bar{\nu}_\mu e) = \frac{G^2 s}{3\pi}(g_V^2 - g_V g_A + g_A^2)$$

3. Starting from the generalized Higgs-neutrino coupling term (see Mandl and Shaw for details on the conventions),

$$-G_{l'l}\bar{\Psi}^L_{l'}(x)\psi^R_{\nu_l}(x)\tilde{\Phi}(x) - G^*_{l'l}\tilde{\Phi}^\dagger(x)\bar{\psi}^R_{\nu_l}(x)\Psi^L_{l'}(x) \qquad (A)$$

show that in the unitary gauge, it reduces to

$$-\frac{1}{\sqrt{2}}\sum_j \lambda_j \bar{\psi}_j(x)\psi_j(x)[v + \sigma(x)]$$

where $\psi_j = \sum_l U_{jl}\psi_{\nu_l}$ and U is the unitary matrix that diagonalizes the Hermitian coupling matrix G, *i.e.*, $(UGU^\dagger)_{ij} = \lambda_i \delta_{ij}$. Hence, show that the coupling term (A) leads to eigenstate neutrinos ν_j associated with the fields $\psi_j(x)$ with masses

$$m_j = \lambda_j v/\sqrt{2}$$

Draw the Higgs-neutrino (ν_j) interaction vertex and show that it comes with vertex factor $(-i/v)m_j$.