QFT-I problem set 2

(Fawad Hassan)

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1. Using the expansion of the Dirac field $\psi(x)$ in terms of creation and annihilation operators, compute the fermion propagator $\langle 0|T\left(\psi(x)\bar{\psi}(x')\right)|0\rangle$ and show that the answer can be expressed as $iS_F(x-x')$ with

$$S_F(x) = \frac{\hbar}{(2\pi\hbar)^4}\int d^3p \int_{C_F} dp^0 e^{-ipx/\hbar}\frac{\not{p}+mc}{p^2-m^2c^2}$$

   where C_F is the Feynman contour in the complex p^0 plane.

2. Starting with the Lorentz transformation of the Dirac field ψ show that $\bar{\psi}\psi$ is a Lorentz scalar while $\bar{\psi}\gamma^\mu\psi$ is a Lorentz vector. Express both in terms of $\psi_L = \frac{1}{2}(1-\gamma_5)\psi$ and $\psi_R = \frac{1}{2}(1+\gamma_5)\psi$.

3. Problem 4.5 (page 80) from Mandl and Shaw

4. In the Dirac theory, the projection of the spin operator along the particle momentum is given by (in units where $\hbar = 1$)

$$S_p = \frac{1}{2}\int d^3x\, N(\psi^\dagger\sigma_p\psi)$$

   Show that in terms of lowering and raining operators,

$$S_p = \frac{1}{2}\sum_{r,\vec{p}}(-1)^{r+1}\left(d_r^\dagger(\vec{p})d_r(\vec{p}) + c_r^\dagger(\vec{p})c_r(\vec{p})\right)$$

5. Consider the photon field A_μ in the Lorenz gauge with polarization tensor denoted by $\epsilon(\vec{k})$. Explain why $\epsilon_r^\mu(\vec{k}) \to \epsilon_r^\mu(\vec{k}) + \lambda_r k^\mu$, corresponds to a gauge transformation of the photon field. Show that the complete amplitude $\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b$ for the Compton scattering process $e^-(p,r) + \gamma(k,s) \to e^-(p',r') + \gamma(k',s')$ is invariant under the gauge transformations $\epsilon(\vec{k}) \to \epsilon(\vec{k}) + \lambda k$, $\epsilon'(\vec{k}') \to \epsilon'(\vec{k}') + \lambda' k'$.