Final Examination Paper for Electrodynamics-I

Allowed help material: *Physics and Mathematics handbooks*
Date: *Friday, April 02, 2004*, Time: *09:00 - 15:00*
Number of questions: 6

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1. Consider a continuous charge distribution $\rho(x)$ giving rise to an electric field $\vec{E}(x)$. Show that the electrostatic energy density of the system is given by
$$w = \frac{1}{8\pi}|\vec{E}|^2$$
In what major respect does the energy of a continuous charge distribution differ from that of a discrete charge distribution? Comment qualitatively.

2. (a) Consider a potential problem which is invariant under rotations around the z-axis (that is, with azimuthal symmetry). Assume that we are given an expression for the potential along a line that makes an angle θ_0 with the z-axis as,
$$\Phi(r, \theta_0) = \sum_{l=0}^{\infty} \frac{A_l(\theta_0)}{r^{l+1}}$$
Write down the solution for arbitrary angle θ in terms of the coefficients $A_l(\theta_0)$, using the uniqueness of such expansions.
(b) Show that, up to a constant, the solutions of the Poisson equation, $\nabla^2\phi = -4\pi\rho$, are uniquely specified by *Neumann* and *Dirichlet* boundary conditions. (Hint: You need *Green's first identity*, $\int_V (\phi\nabla^2\psi + \vec{\nabla}\phi \cdot \vec{\nabla}\psi) d^3x = \oint_S \phi \frac{\partial\psi}{\partial n} dS$).

3. (a) For two closely separated charges $+q$ and $-q$, write down the potential at a far away point in terms of the dipole moment $\vec{p}$ of the system.
(b) Consider two colinear, but oppositely oriented dipoles $\vec{p}$ and $-\vec{p}$. Evaluate the dipole and quadrupole moments of this system.

4. Consider a straight conducting wire (of infinite length) through which a total electric current I flows. Using Maxwell's equations show that the magnetic field $|\vec{B}|$ at a distance $\vec{r}$ from the wire is given by
$$|\vec{B}(r)| = \frac{2I}{c\,r}$$

5. (a) Prove the *Poynting theorem*,
$$\vec{J} \cdot \vec{E} + \frac{1}{8\pi}\frac{\partial}{\partial t}\left(\epsilon\vec{E} \cdot \vec{E} + \frac{1}{\mu}\vec{B} \cdot \vec{B}\right) + \frac{c}{4\pi}\nabla \cdot (\vec{E} \times \vec{H}) = 0$$
(Hint: You will need the vector identity $\nabla \cdot (\vec{P} \times \vec{Q}) = (\nabla \times \vec{P}) \cdot \vec{Q} - \vec{P} \cdot (\nabla \times \vec{Q})$)

(b) Give the physical interpretation of each term in this equation. What is the significance of the *Poynting theorem*?

6. Consider two frames S and $\widetilde{S}$ related by a Lorentz transformation matrix L so that $\widetilde{X}^\mu = L^\mu{}_\nu X^\nu$. Remember that $\mu = 0, 1, 2, 3$ and $X^0 = ct$.

   (a) Argue that the two postulates of Special Relativity (that the speed of light is a constant c and the laws of physics have the same form in all inertial reference frames) imply that

   $$(X^0)^2 - (X^1)^2 + (X^2)^2 + (X^3)^2 = (\widetilde{X}^0)^2 - (\widetilde{X}^1)^2 + (\widetilde{X}^2)^2 + (\widetilde{X}^3)^2$$

   (b) Find the restriction on the matrix L that follows from the above property of Lorentz transformations. Using this, express L^{-1} in terms of L^T.