Electrodynamics-I, Problem set 3

Due date: Wednesday, Oct 31, 2012

Please explain your reasoning clearly and show the steps in your calculations

1. Consider two parallel conducting wires of infinite length placed a distance d apart and carrying currents I_1 and I_2, respectively. Calculate the force per unit length between the wires. Indicate when the force is attractive and when repulsive.

2. Consider the current density $\vec{J}(\vec{x}) = \sum_i q_i \vec{v}(\vec{x})\delta(\vec{x} - \vec{x}_i)$ corresponding to discrete charges q_i moving with velocities $\vec{v}_i$. Evaluate the magnetic moment $\vec{m}$ of the system in terms of its angular momentum. What is the potential energy of the system when placed in a magnetic field $\vec{B}$?

3. Write down monocromatic plane-wave solutions for the vector and scalar potentials $\vec{A}$ and Φ in the Lorenz gauge and in the absence of sources. What is the implication of the Lorenz gauge condition? From this derive the expressions for the electric field $\vec{E}$ and the magnetic field $\vec{B}$ and show that, along with the wave vector $\vec{k}$ they form a set of three perpendicular vectors.

4. (a) Consider a reference frame $\widetilde{S}$ moving away from a frame S with velocity v along the x^1-axis (there is no relative motion in the x^2 and x^3 directions). Show that the corresponding Lorentz transformations keep the space time interval invariant, i.e.,

$$(\widetilde{x}^0)^2 - (\widetilde{x}^1)^2 - (\widetilde{x}^2)^2 - (\widetilde{x}^3)^2 = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

   (b) For the above frames, derive the transformation laws for $\frac{\partial}{\partial x^0}$ and $\frac{\partial}{\partial x^1}$. Using these show that if J^μ is a contravariant 4-vector, then $\partial_\mu J^\mu$ is Lorentz invariant, i.e.,

$$\sum_{\mu=0}^{3} \frac{\partial \widetilde{J}^\mu}{\partial \widetilde{x}^\mu} = \sum_{\mu=0}^{3} \frac{\partial J^\mu}{\partial x^\mu}$$