

Electrodynamics-I, Problem set 2

Due date: Thursday, Oct. 15, 2011

Please explain your reasoning clearly and show the steps in your calculations

1. Show that:

- (a) The value of the electric field \vec{E} inside a conductor vanishes when charges within it are in equilibrium (*i.e.*, there are no currents).
- (b) Any excess charge placed on a conductor must lie entirely on its surface.
- (c) A closed hollow conductor shields its interior from fields due to charges outside, but does not shield its exterior from the fields due to charges placed inside it.
- (d) The electric field at the surface of a conductor is normal to the surface and its tangential component along the surface vanishes. The normal component has a magnitude $4\pi\sigma$, where σ is the surface charge density.

2. The time average potential of a neutral hydrogen atom is given by

$$\Phi(r) = q \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2} \right)$$

where q is the magnitude of the electronic charge and α is a constant. Find the distribution of charge (both continuous and discrete) which will give this potential and interpret your result physically.

3. (a) Consider the multipole moments of a charge distribution $\rho(r, \theta, \phi)$,

$$q_{lm} = \int r^2 dr \int \sin \theta d\theta \int d\phi \rho(r, \theta, \phi) r^l Y_{lm}^*(\theta, \phi)$$

Show that for a *spherically symmetric* charge distribution, all moments beyond the monopole moment vanish (hence the higher moments quantify deviations from sphericity).

- (b) Write the components of a unit vector \hat{n} in spherical polar coordinates and then express them in terms of spherical harmonics (*Hint: Only $Y_{l=1,m}(\theta, \phi)$ will contribute*).
 - (c) Express the dipole moment \vec{p} in rectangular coordinate system in terms of the dipole moment $q_{1,m}$ in spherical coordinates.
4. (a) Consider two point charges $+q$ and $-q$ with a separation \vec{y} , which is small but *finite* (this system forms a non-ideal dipole). Compute the first non-vanishing multipole moment beyond the dipole moment.

- (b) Consider a system of two colinear but oppositely oriented dipoles \vec{p} and $-\vec{p}$ placed a distance \vec{y} apart. Find the dipole and quadrupole moments of this system.
- (c) Find the interaction energy between two non-coincident dipoles with moments \vec{p}_1 and \vec{p}_2 .