Electrodynamics-I, Problem set 1

Due date: Wednesday, Sept. 19, 2012

Please explain your reasoning clearly and show the steps in your calculations

1. (a) For integer n, show that

$$\frac{1}{a}\int_0^a e^{2i\pi nx/a}dx = \delta_{n,0}$$

   (b) Let $f(x)$ be a function defined over a finite interval $0 \le x \le a$. Write the Fourier series for $f(x)$ in terms of $e^{2i\pi nx/a}$, giving the expressions for the Fourier coefficients.

2. (a) Show that

$$\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = -\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$$

   where $\vec{\nabla}$ acts on the components of $\vec{r}$.

   (b) Show that

$$\vec{\nabla}'\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = -\vec{\nabla}\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)$$

   where $\vec{\nabla}'$ acts on the components of $\vec{r}\,'$.

   (c) Discuss the relation between $\vec{\nabla}(1/|\vec{r}-\vec{r}\,'|)$ and the $\phi = 1/|\vec{r}-\vec{r}\,'| = constant$ surfaces.

3. Show that

$$\nabla^2(\frac{1}{|\vec{r}-\vec{r}\,'|}) = -4\pi\delta(\vec{r}-\vec{r}\,')$$

4. Prove the identities $\vec{\nabla}\times(\vec{\nabla}\phi) = 0$ and $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$ (either by explicit calculation or using the integral theorems).

5. Using the divergence and the Stokes's theorems, write the Maxwell's equations,

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho\,, \qquad \vec{\nabla}\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J} \qquad \vec{\nabla}\times\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} = 0\,, \qquad \vec{\nabla}\cdot\vec{B} = 0$$

   in integral form.