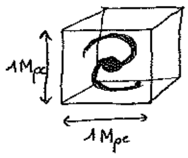
(2.1.) Suppose that galaxies contain ~ 10^{11} stars and are separated by 1 Mpc

→ Density of the universe ?

$$\begin{cases} 1\ M_\odot \simeq 2.10^{30}\ kg \\ 1\ pc \simeq 3.10^{16}\ m \end{cases}$$



$$\text{Density} = \frac{m}{V}$$

and we have 10^{11} stars / Mpc^3

$$\Rightarrow \underset{(\rho)}{\text{density}} = \frac{10^{11} \times M_\odot}{(1\ Mpc)^3} = 10^{11} \times 2.10^{30}\ kg/Mpc^3 = 2.10^{41}\ kg/Mpc^3$$

$$1\ Mpc = 10^6\ pc = 3.10^{22}\ m$$

$$\Rightarrow 1\ Mpc^3 = (3.10^{22})^3\ m^3 = 27.10^{66}\ m^3 \simeq 10^{67}\ m^3$$

$$\Rightarrow \rho \sim \frac{10^{41}\ kg}{10^{67}\ m^3} = 10^{41-67}\ kg.m^{-3}$$

$$\Rightarrow \boxed{\rho \sim 10^{-26}\ kg.m^{-3}} = \text{density of the Universe}$$

Density of the Earth: $\rho_{Earth} = 5,51\ g.cm^{-3} = 5,51.10^{-3}.10^{6}\ kg.m^{-3} = 5,51.10^{3}\ kg.m^{-3}$

So $\boxed{\rho_{Earth} \gg \rho_{universe}}$.

(2.3) If the Universe was not electrically neutral (ie. nb protons ≠ nb electrons), the electromagnetic force (much stronger than gravitational force) would dominate the structure of the Universe. (10^{39})

Evidence suggests that gravity is in fact the dominant force on the large scales.

(2.4) Frequence of a photon with energy 13,6 eV?

$$E = hf \Rightarrow f = \frac{E}{h}$$

Constants from the book and units:

$$\begin{cases} \hbar = \frac{h}{2\pi} = 1,055.10^{-34}\ m^2.kg.s^{-1} \\ 1\ kg.m^2.s^{-2} = 1\ J \\ 1\ eV = 1,602.10^{-19}\ J \end{cases}$$

$$\Rightarrow h = 2\pi \times \frac{1,055.10^{-34}}{1,602.10^{-19}}\ eV.s$$

$$\Rightarrow f = \frac{13,6 \times 1,602.10^{-19}}{2\pi \times 1,055.10^{-34}}\ s^{-1}$$

$$\boxed{f \simeq 3,29.10^{15}\ Hz}.$$

What is the temperature of the photon with mean energy 13,6 eV?

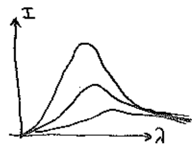
The mean energy of a photon in the black-body distribution is $E_{mean} \simeq 3\ k_B T$ (Fig 2.10)

$$k_B = 8,619.10^{-5}\ eV.K^{-1}$$

$$T \simeq \frac{13,6}{3 \times 8,619.10^{-5}}$$

$$\boxed{T \simeq 52600\ K}.$$

(2.5) Peak of the energy density distribution of a black-body is at $f_{peak} \simeq \frac{2,8\ k_B T}{h}$



$$\Rightarrow \frac{f_{peak}}{T} = cste$$

$$\frac{f_{peak}}{T} = \frac{2,8\ k_B}{h}$$

Constants: $h = 2\pi \times 1,055.10^{-34}\ m^2.kg.s^{-1}$

$k_B = 1,381.10^{-23}\ J.K^{-1} = 1,381.10^{-23}\ kg.m^2.s^{-2}.K^{-1}$

$$\Rightarrow \frac{f_{peak}}{T} = \frac{2,8 \times 1,381.10^{-23}}{2\pi \times 1,055.10^{-34}} \quad \left(\frac{\cancel{kg}.\cancel{m^2}.s^{-2}.K^{-1}}{\cancel{m^2}.\cancel{kg}.s^{-1}} = \frac{Hz}{K}\right)$$

$$\boxed{\frac{f_{peak}}{T} \simeq 5,8.10^{10}\ K^{-1}.s^{-1}\ (Hz.K^{-1})}.$$

The Sun: black-body at $T \simeq 5800\ K$

$$\Rightarrow \boxed{f_{peak}} \simeq 5800 \times 5,8.10^{10} \boxed{\simeq 3,4.10^{14}\ Hz}.$$

Where in the EM spectrum?

$$\boxed{\lambda} = \frac{c}{f} = \frac{299\ 792\ 458}{3,4.10^{14}} \simeq 8,82.10^{-7}\ m \boxed{\simeq 882\ nm} \rightarrow \boxed{\text{near IR}}$$

($\sim$750 nm → 100 µm)

(In the solution the author explains that he wanted it to be in the visible light, which is where the Sun is actually emitting, ie. yellow-green).

(2.6.) CMB = black-body spectrum at $T = 2{,}725$ K.

• peak frequency?

$$f_{peak} \simeq \frac{2{,}8\, k_B T}{h} = \frac{2{,}8 \times 1{,}381 \cdot 10^{-23}}{2\pi \times 1{,}055 \cdot 10^{-34}} \times 2{,}725$$

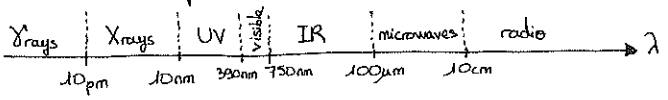
$$\boxed{f_{peak} \simeq 1{,}6 \cdot 10^{11} \text{ Hz}}.$$

• corresponding wavelength?

$$\lambda = \frac{c}{f} = \frac{299\,792\,458}{1{,}6 \cdot 10^{11}} \Rightarrow \boxed{\lambda \simeq 1{,}9 \text{ mm}}$$

microwaves ☺

Reminder EM spectrum:



• Compare to figure 2.5?

$\lambda \simeq 2$ mm means 1 wave every 2 mm

ie. ~ 5 waves per cm

cf. peak of the CMB curve in fig 2.5!

• Total energy density of the CMB?

given by Stefan-Boltzmann law (cf. lecture):

$$\mathcal{E}_{rad} = \alpha T^4 \quad \text{with } \alpha = \frac{\pi^2 k_B^4}{15\, \hbar^3 c^3} = 7{,}565 \cdot 10^{-16} \ \text{J} \cdot \text{m}^{-3} \cdot \text{K}^{4}$$

$$\mathcal{E}_{rad} = 7{,}565 \cdot 10^{-16} \times (2{,}725)^4$$

$$\boxed{\mathcal{E}_{rad} \simeq 4{,}17 \cdot 10^{-14} \ \text{J} \cdot \text{m}^{-3}}.$$

dimension analysis:

$$[\mathcal{E}_{rad}] = \frac{(kg \cdot m^2 \cdot s^{-2} \cdot \cancel{K^{-1}})^4}{(m^2 \cdot kg \cdot s^{-1})^3 (m \cdot s^{-1})^3} \cdot \cancel{K^4}$$

$$= \frac{kg^4 \cdot m^8 \cdot s^{-8}}{m^6 \cdot kg^3 \cdot s^{-3} \cdot m^3 \cdot s^{-3}}$$

$$= kg \cdot m^{-1} \cdot s^{-2}$$

$$= kg \cdot m^2 \cdot s^{-2} \cdot m^{-3}$$

$$= J \cdot m^{-3}$$