Nuclear physics tutorial 2

August 15, 2018

1 Shell model

Give the spin and parity predicted by the shell model for the ground states of ^7Li, ^{11}B and ^{15}C.

2 Deuteron : the simplest bound state

A proton and a neutron interact through the potential $V(r) = -V_0\ \theta(R-r)$ where θ is the Heaviside step function and $r \geqslant 0$.

a) Write down the 1D stationnary Schrödinger equation in the center of mass for the wave function $\psi(r)$.

b) What is a bound state ? Give another kind of state.

c) Determine the wave function of a bound state imposing :

i) No divergence at infinity

ii) Continuity and differentiability at $r = R$

iii) Normalizing condition : $\int |\psi(r)|^2 dr = 1$

Bonus : Rigorously, this calculation should be done in 3D. Try to do it assuming spherical symmetry.

3 Fermi model

When numbers of protons and neutrons are equal, the Fermi model yields :

$$\frac{A}{2} = \frac{2V}{h^3}\int d^3p \quad \text{where integration is performed over the Fermi sphere} \tag{1}$$

a) Compute the Fermi energy in this case, assuming that protons and neutrons have approximately the same mass. Express the volume in terms of A.

b) Explain why protons and neutrons have a different Fermi momentum p_F when their number is not equal viz. when $Z \neq \frac{A}{2}$.

c) According to your answer of the previous question, one has :

$$Z = \frac{2V}{h^3}\int d^3p \quad \text{and} \quad N = \frac{2V}{h^3}\int d^3p \quad \text{where integrations are performed up to different Fermi levels} \tag{2}$$

From these relations, you can infer the Fermi momenta of protons and nucleons respectively (use a calcualtion done in the first question). Then compute the average kinetic energy :

$$E = \frac{2V}{h^3}\int \frac{p^2}{2m_p}\, d^3p + \frac{2V}{h^3}\int \frac{p^2}{2m_n}\, d^3p \tag{3}$$

where the first integration is performed over the Fermi sphere of protons and the second integration is performed over the Fermi sphere of neutrons. Once again, express the volume in terms of A. In the end, you can also assume that protons and neutrons have approximately the same mass.