# Nuclear physics tutorial 1

August 13, 2018

## 1 Form factor

We have defined the form factor and the mean square charge radius by :

$$F(\boldsymbol{q}) \triangleq \frac{1}{Ze} \int \rho(\boldsymbol{r})\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}} d^3r \quad (1)$$

$$\left\langle r^2 \right\rangle \triangleq \frac{1}{Ze} \int \rho(\boldsymbol{r})\, r^2 d^3r \quad (2)$$

a) Derive the simplified expression of the form factor for distribution with spherical symmetry :

$$F(\boldsymbol{q}) = \frac{4\pi}{Ze} \int \rho(r)\, \frac{\sin(qr)}{q}\, rdr \quad (3)$$

b) Compute the form factor and the mean square charge radius for the following charge density profiles :

i) $\rho(\boldsymbol{r}) = \rho_0\, \theta(R - r)$ where $\theta$ is the Heaviside step function.

ii) $\rho(\boldsymbol{r}) = \rho_0\, e^{-\ln(2)\frac{r^2}{R^2}}$

## 2 Fermi distribution

A nuclear distribution that is more realistic than the uniform one is the Fermi distribution :

$$\rho(\boldsymbol{r}) = \frac{\rho_0}{1 + e^{\frac{r-R}{a}}} \quad (4)$$

a) Plot this distribution.

b) Show that this distribution is symmetric with respect to the point $(R, \frac{1}{2})$ and has an inflection point.

c) Find the parameter $a$ if the skin thickness is $t = 2.9\ fm$.

## 3 Semi-empirical formulae

a) Show that the potential energy of a sphere of radius $R$ with uniform charge distribution is :

$$E(R) = \frac{3}{5}\, \frac{Q^2}{4\pi\epsilon_0 R}, \quad Q \text{ is the total charge of the sphere} \quad (5)$$

b) Considering that the Coulomb energy is given by (5) evaluated in the so-called nuclear radius, estimate it for $^{21}Ne,\ ^{57}Fe$ and $^{209}Bi$.

c) Estimate the binding energy of $^{21}Ne,\ ^{57}Fe$ and $^{209}Bi$ after the liquid drop model.