Tutorial on Rutherford scattering

August 29, 2018

1 Relation between the impact parameter and the scattering angle

An alpha particle of charge $q = 2e$ and mass m is thrown with initial velocity $\boldsymbol{v}_0$ against a heavy nucleus of charge $Q = Ze$ at rest. They interact via the electrostatic force ; see figure 1. The distance b is called the impact parameter while θ is named scattering angle.

1) Write the equation of motion for the alpha particle ; keep vector notations. What can we assume for the motion of the nucleus ? Otherwise, how to deal with the simultaneous motion of two objects ?

2) Show that the orbital momentum J is conserved and equal to $-mbv_0$.

3) Why is the motion constrained in a plane like in the picture ? Project the equation of motion on y-axis and derive the following ODE :

$$\frac{dv_y}{d\phi} = -\frac{qQ}{4\pi\epsilon_0 mbv_0}\,\sin(\phi) \tag{1}$$

4) Separate variables an integrate it along the trajectory viz. ϕ goes from π to θ. Infer the following relation between the impact parameter and the scattering angle :

$$b = \frac{qQ}{8\pi\epsilon_0 E_0}\,\cot\left(\frac{\theta}{2}\right) \quad \text{where} \quad E_0 = \frac{1}{2}mv_0^2 \tag{2}$$

2 Differential cross section

The Rutherford experiment is repeated a great number of times and at each time, the alpha particle has a different impact parameter and a different scattering angle. Or equivalently, one uses a flux of of alpha particles. This is the reason why we need the concept of differential cross section $\frac{d\sigma}{d\Omega}$.

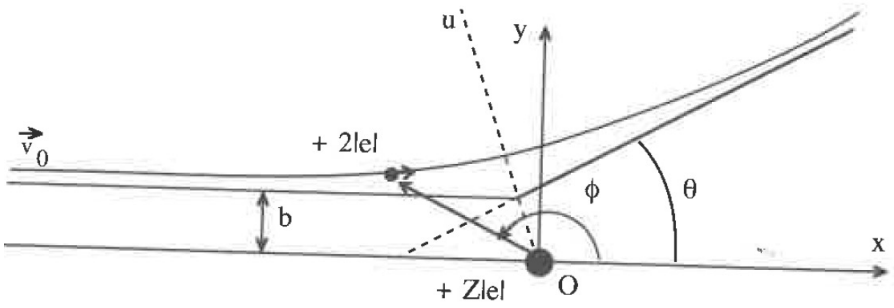

Figure 1: Kinematics of Rutherford scattering

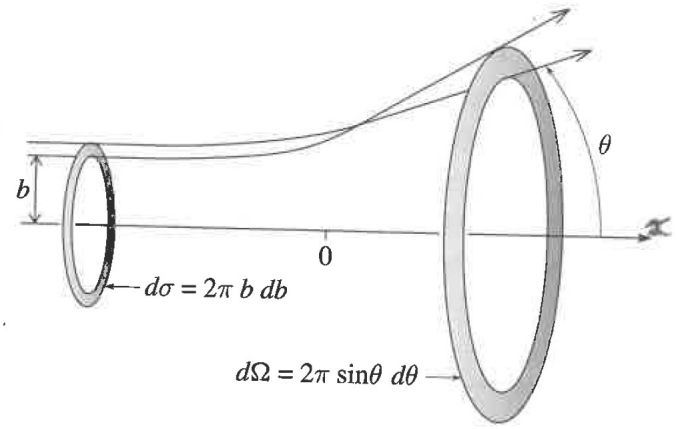

Figure 2: Symmetry of the Rutherford scattering

1) Why is the Rutherford scattering symmetric with respect to x-axis ? Justify that $d\sigma = 2\pi b db$; figure 2 might be helpful.

2) Infer the following general expression of the differential cross section for symmetric scattering :

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin(\theta)} \left| \frac{db}{d\theta} \right| \tag{3}$$

3) Use your results to derive the famous Rutherford formula :

$$\frac{d\sigma}{d\Omega} = \left(\frac{qQ}{16\pi\epsilon_0 E_0 \sin^2\left(\frac{\theta}{2}\right)} \right)^2 \tag{4}$$