Tutorial 2

1

(a) $\vec{P}_X = \vec{P}_e + \vec{P}_p$

$E_{CM}^2 = m^2c^4 = \vec{P}_X^2 = \left(\vec{P}_e + \vec{P}_p\right)^2 = m_e^2c^4 + m_p^2c^4 + 2\vec{P}_e \bullet \vec{P}_p = m_e^2c^4 + m_p^2c^4 + 2\left(E_eE_p - c^2\vec{p}_e \bullet \vec{p}_p\right)$

$\vec{p}_e = -\vec{p}_p$; neglect masses $\Rightarrow$ $m_e \approx m_p \approx 0$; $E_e \approx p_e$c ; $E_p \approx p_p$c ; $E_e \approx E_p$

$\Rightarrow E_{CM}^2 = 2\left(E_eE_e + E_eE_e\right) = 4E_e^2 \quad \Rightarrow E_{CM} \approx 2E_e \quad ; E_{CM} = 500$ GeV $\Rightarrow$ $E_e = 250$ GeV

(b) (i)

Lorentz transformation:

$$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \frac{v}{c}p_x\right)$$

$E = 250$ GeV

Frame in which proton is at rest: $p_x' = 0, E' = m_pc^2$

$$E = \gamma mc^2 \quad 250 = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}0.938 \quad \Rightarrow \gamma = 266.5,$$

$$\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = 0.99999296 \sim 1$$

Electron: $E = 250$ GeV, $p_x = -250$ GeV/c

In proton rest frame:

$$E' \approx 266.5c\left(\frac{250}{c} + \frac{250}{c}\right) = 130000 \text{ GeV}$$

Very big! Much cheaper to build a collider at the appropriate centre-of-mass energy.

(ii) $\vec{P}_X^2 = \left(\vec{P}_e + \vec{P}_p\right)^2 = m_e^2c^4 + m_p^2c^4 + 2\vec{P}_e \bullet \vec{P}_p = m_e^2c^4 + m_p^2c^4 + 2\left(E_eE_p - c^2\vec{p}_e \bullet \vec{p}_p\right)$

$\vec{p}_p = 0$; $E_p = m_pc^2 \Rightarrow E_{CM}^2 = m_e^2c^4 + m_p^2c^4 + 2E_em_p$

$$\Rightarrow E_e = \frac{E_{CM}^2 - m_e^2c^4 + m_p^2c^4}{2m_pc^4} \approx \frac{E_{CM}^2}{2\times 0.939} = 130000 \text{ GeV.}$$

Amplitude $M\left(q^2\right)=\dfrac{k}{q^2+m_X^2c^2}$; $\dfrac{d\sigma}{d\Omega}=\alpha\left|M\left(q^2\right)\right|^2$

Cross section must change with q^2 if it is a weak process. However, it doesn't change within the experiment's sensitivity

$\Rightarrow$ it can't have changed by an amount more than is necessary to shift the cross section by > 1%.

$$\frac{d\sigma}{d\Omega}=\alpha k^2\left(\frac{1}{q^2+m_X^2c^2}\right)^2 \Rightarrow \frac{d\sigma}{d\Omega}\Big|_{q=0.5\text{ GeV}/c}=\alpha k^2\left(\frac{1}{0.25/c^2+m_X^2c^2}\right)^2 \quad ; \quad \frac{d\sigma}{d\Omega}\Big|_{q=5\text{ GeV}/c}=\alpha k^2\left(\frac{1}{25+m_X^2c^2}\right)^2$$

$$\Rightarrow r=\frac{\frac{d\sigma}{d\Omega}\Big|_{q=0.5\text{ GeV}/c}}{\frac{d\sigma}{d\Omega}\Big|_{q=5\text{ GeV}/c}}=\frac{\left(25/c^2+m_X^2c^2\right)^2}{\left(0.25/c^2+m_X^2c^2\right)^2}$$

Measurement $r=1.00\pm0.01$

$$\Rightarrow \frac{\frac{d\sigma}{d\Omega}\Big|_{q=0.5\text{ GeV}/c}}{\frac{d\sigma}{d\Omega}\Big|_{q=5\text{ GeV}/c}}=\frac{\left(25/c^2+m_X^2c^2\right)^2}{\left(0.25/c^2+m_X^2c^2\right)^2}\approx\frac{\left(25/c^2+m_X^2c^2\right)^2}{m_X^4c^4}<1.01\Rightarrow\frac{25/c^2}{m_X^2c^2}+1<1.01^{\frac{1}{2}} \quad \Rightarrow \quad m_X^2c^2>\frac{25/c^2}{1.01^{\frac{1}{2}}-1}=5012\text{ GeV}$$

$$\Rightarrow \quad \frac{25}{m_X^2c^4}+1<1.01^{\frac{1}{2}}\Rightarrow m_X^2c^4>\frac{25}{1.01^{\frac{1}{2}}-1}>5012\text{ GeV}^2\Rightarrow m_X>71\text{ GeV}/c^2$$

$\Rightarrow$ consistent with weak force $M_W\sim80$ GeV/c^2, $M_Z\sim90$ GeV/c^2

3

$\sigma L = N$

For at least 5 events : $L = \dfrac{5}{40000} = 1.25 \times 10^{-4}$ fb^{-1}

However, some channels are easier to see than others and each decay channel has its own branching ratio.

Eg $H \to \gamma\gamma$ $BR \sim 0.001$ $\Rightarrow$ scale up by factor 1000

(knowledge on the values of BRs not expected)

Also, detector inefficiencies and background $\Rightarrow$ more lumi needed.

~ 5fb^{-1} needed to observe Higgs and make first measurements at the LHC.

$$\tau = 4\times10^{-25}\ \text{s}$$

$$\Delta E \Delta t \sim \hbar$$

$$\Rightarrow \quad \Delta E \sim \frac{\hbar}{\Delta t} = \frac{10^{-34}}{4\times10^{-25}} = 2.5\times10^{-10}\ \text{J} = \frac{2.5\times10^{-10}}{1.6\times10^{-10}} \sim 2\ \text{GeV}$$