Tutorial 1

1 (i) 23.5 GeV/c Momentum

SI $23.5 \times \frac{10^9 \times 1.602 \times 10^{-19}}{3 \times 10^8} = 1.25 \times 10^{-17}$ kgms^{-1}

15 keV Energy

SI 15 $\times 10^3 \times 1.602 \times 10^{-19} = 2.4 \times 10^{-15}$ J

(ii) 5×10^{-27} kg Mass $\Rightarrow$ unit $\frac{\text{eV}}{c^2}$

$$1 \quad \frac{\text{eV}}{c^2} = \frac{1.602 \times 10^{-19}}{\left(3 \times 10^8\right)^2} = 1.78 \times 10^{-36} \text{ kg}$$

$$\Rightarrow \quad 5 \times 10^{-27} \text{ kg} = \frac{5 \times 10^{-27}}{1.78 \times 10^{-36}} \frac{\text{eV}}{c^2} = 2.81 \times 10^9 \frac{\text{eV}}{c^2} = 2.81 \frac{\text{GeV}}{c^2}$$

3×10^{-21} J Energy $\Rightarrow$ unit eV

1 eV = 1.602×10^{-19} J

$$\Rightarrow \quad 3 \times 10^{-21} \text{ J} = \frac{3 \times 10^{-21}}{1.602 \times 10^{-19}} \text{ eV} = 0.0187 \text{ eV}$$

2 Second postulate - light speed is the same regardless of inertial frame.
In the rest frame of the passenger and thus the spaceship the laser light moves at light speed and triggers an explosion when it hits the other side of the spaceship.

In an inertial frame in which the spaceship moves faster than light, the passenger would still fire the laser but the light, according to the 2nd postulate, must travel at a speed less than than that of the spaceship. Therefore it would not trigger an explosion.

Physics is invariant to a change of inertial frame. Here we have an example in which the events (spaceship destroyed or not) is frame-dependent!

3 (i) Classical physics

$L = 10^4$ m

Time $= \dfrac{10^4}{0.98 \times 3 \times 10^8} = 34 \times 10^{-6}$ s

$\Rightarrow$ Survival fraction $= e^{-\frac{t}{\tau}} = e^{-\frac{34\times10^{-6}}{2.197\times10^{-6}}} = 1.9 \times 10^{-7}$

(ii) Relativistic physics

Mean lifetime $= \gamma\tau$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-0.98^2}} = 5.025$$

Survival fraction $= e^{-\frac{t}{\gamma\tau}} = e^{-\frac{34\times10^{-6}}{5.025\times2.197\times10^{-6}}} = 4.6 \times 10^{-2}$

A huge difference...

4 Threshold : $p_C = p_D = 0$

$P_A + P_B = P_C + P_D$

As ever, the trick is to square them and see what happens.

$$\left(P_A + P_B\right)^2 = \left(\frac{E + m_B c^2}{c}, \vec{p}_A\right)^2 = \left(\frac{E + m_B c^2}{c}\right)^2 - \vec{p}_A^{\,2} = \frac{E^2}{c^2} + 2Em_B + m_B^2 c^2 - \vec{p}_A^{\,2}$$

$$\vec{p}_A^{\,2} = \frac{E^2}{c} - m_A^2 c^2 \quad \Rightarrow \left(\vec{P}_A + \vec{P}_B\right)^2 = 2Em_B + m_B^2 c^2 + m_A^2 c^2$$

$\left(P_C + P_D\right) = \left(m_C c + m_D c, 0\right)$ at threshold $E_C = m_C c^2$; $E_D = m_D c^2$

$$\Rightarrow \left(P_C + P_D\right)^2 = \left(m_C c + m_D c\right)^2$$

$$\left(P_A + P_B\right)^2 = \left(P_C + P_D\right)^2 \Rightarrow 2Em_B + m_B^2 c^2 + m_A^2 c^2 = \left(m_C c + m_D c\right)^2$$

$$\Rightarrow E = c^2 \frac{\left(m_C + m_D\right)^2 - m_A^2 - m_B^2}{2m_B}$$

(5) (i) $E_C = \left(\left(p_f c\right)^2 + \left(m_C c^2\right)\right)^{\frac{1}{2}}$; $E_D = \left(\left(p_f c\right)^2 + \left(m_C c^2\right)\right)^{\frac{1}{2}}$

$$\frac{dE_C}{dp_f} = \frac{c^2 p_f}{\left(\left(p_f c\right)^2 + \left(m_C c^2\right)\right)^{\frac{1}{2}}} = \frac{c^2 p_f}{E_C} \quad ; \quad \frac{dE_D}{dp_f} = \frac{c^2 p_f}{\left(\left(p_f c\right)^2 + \left(m_D c^2\right)\right)^{\frac{1}{2}}} = \frac{c^2 p_f}{E_D}$$

$$E = E_C + E_D$$

$$\Rightarrow \frac{dE}{dp_f} = \frac{dE_C}{dp_f} + \frac{dE_D}{dp_f} = c^2 p_f \left(\frac{1}{E_C} + \frac{1}{E_D}\right) = c^2 p_f \frac{E_C + E_D}{E_C E_D} = c^2 p_f \frac{E}{E_C E_D}$$

(ii) $p_f = \gamma_C m_C v_C \quad E_C = \gamma_C m_C c^2 \quad E_D = \gamma_D m_D c^2$

$$\frac{dE}{dp_f} = \frac{c^2 p_f E}{E_C E_D} = c^2 \gamma_C m_C v_C \frac{\gamma_C m_C c^2 + \gamma_D m_D c^2}{\gamma_C m_C c^2 \gamma_D m_D c^2} = v_C \frac{\gamma_C m_C c^2 + \gamma_D m_D c^2}{\gamma_D m_D c^2} = \frac{v_C \gamma_C m_C}{\gamma_D m_D} + v_C$$

$$v_C \gamma_C m_C = v_D \gamma_D m_D \quad \Rightarrow \quad v_D = \frac{v_C \gamma_C m_C}{\gamma_D m_D}$$

$$\Rightarrow \frac{dE}{dp_f} = v_D + v_C$$

(6)

$$P_\rho = P_{\pi_1} + P_{\pi 2} \Rightarrow P_\rho^2 = m_\rho^2 c^2 = \left(P_{\pi_1} + P_{\pi 2}\right)^2 = 2m_\pi^2 c^2 + 2P_{\pi_1} \bullet P_{\pi_2} = 2m_\pi^2 c^2 + 2\frac{E_{\pi_1} E_{\pi 2}}{c^2} - 2\vec{p}_{\pi_1} \bullet \vec{p}_{\pi 2}$$

$\vec{p}_{\pi_1} c \sim E_{\pi_1}, \vec{p}_{\pi_2} c \sim E_{\pi 2}$ since $E_{\pi_1}, E_{\pi 2} \gg m_\pi$

$$\Rightarrow m_\rho^2 c^2 \approx 2m_\pi^2 c^2 + 2\frac{E_{\pi_1} E_{\pi 2}}{c^2} - 2\frac{E_{\pi_1} E_{\pi 2}}{c^2} \cos 24^o$$

$m_\pi = 0.14$ GeV/c^2

$\Rightarrow m_\rho = 0.74$ GeV/c^2.