Lecture 6 – repetition   Oct. 19

Particle physics

Leptons: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$

quarks: $\begin{pmatrix} u_i \\ d_i \end{pmatrix} \begin{pmatrix} c_i \\ s_i \end{pmatrix} \begin{pmatrix} t_i \\ b_i \end{pmatrix}$

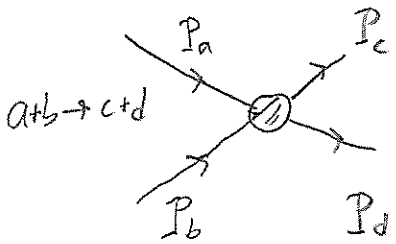
spin $\frac{1}{2}$ $\left(\frac{\hbar}{2}\right)$

$i = 1,2,3$ (colour)
+ antiparticles

Force carriers Z^0, W^+, W^- spin-1

Gives mass H^0 spin-0

Relativistic kinematics



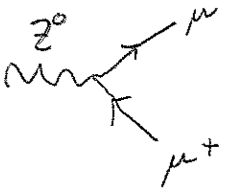
P: 4-momenta $P_i = \left(\frac{E_i}{c}, P_x, P_y, P_z\right)$

$E_i = \sqrt{m_i^2 c^4 + c^2 \bar{p}^2}$

$(P_i^\mu)^2 = \underbrace{m_i^2 c^2 + \bar{p}^2}_{E_i^2} - \bar{p}^2 = m_i^2 c^2$

rest mass = const
("Lorentz invariant")

E.g. Z^0 decay, $Z^0 \to \mu^+ \mu^-$



In Z^0 rest frame $P_{Z^0} = (m_{Z^0} c^2, \bar{0})$

$\Rightarrow P_{Z^0}^2 = m_{Z^0}^2 c^4 = \underbrace{(P_{\mu^-} + P_{\mu^+})^2}_{\text{Lorentz invariant}}$

Lorentz inv.

$\Rightarrow$ valid in any frame $(P_{\mu^-} + P_{\mu^+})^2$ is called the invariant mass^2 of the muon pair

Schrödinger equation $\quad \hat{\bar{p}} = -i\hbar\nabla \quad E \to i\hbar\frac{\partial}{\partial t}$ ②

$$\underbrace{\hat{H}}_{\text{Hamiltonian operator}} = \frac{\hat{\bar{p}}^2}{2m} + \hat{V}(x) = -\frac{\hbar^2\nabla^2}{2m} + \hat{V}(x)$$

$$\hat{H}\Psi(\bar{r},t) = E\Psi = i\hbar\frac{\partial\Psi(\bar{r})}{\partial t}$$ Non-relativistic eqn.

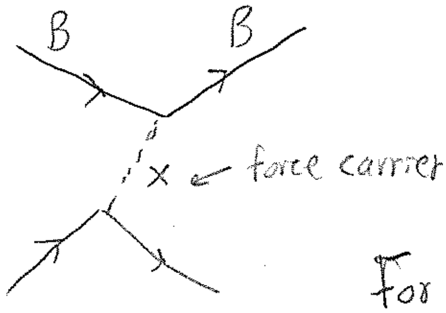
Oskar Klein: In relativity we have $E^2 = m^2c^4 + \bar{p}^2c^2$

$$\Rightarrow \left(i\hbar\frac{\partial}{\partial t}\right)\left(i\hbar\frac{\partial}{\partial t}\right)\phi(\bar{r},t) = m^2c^4\,\phi(\bar{r},t)\underbrace{-c^2\hbar^2\nabla^2}_{(c\hat{\bar{p}})\cdot(c\hat{\bar{p}})}\phi(\bar{r},t)$$

$$\Rightarrow \boxed{-\hbar^2\frac{\partial^2\phi(\bar{r},t)}{\partial t^2} = -c^2\hbar^2\nabla^2\phi(\bar{r},t) + m^2c^4\,\phi(\bar{r},t)}$$

Klein-Gordon equation for spin-0 particle (like Higgs)

However, one has to interpret $\phi(\bar{r},t)$ as quantum field (contains also antiparticles)



potential energy $\quad V(r) = -\frac{g^2}{4\pi}\frac{e^{-\frac{r}{R}}}{r}$

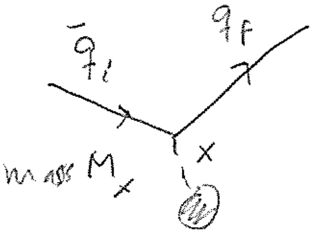
coupling, e.g. electric charge

range of potential

For electromagnetism, $g = \frac{e}{\varepsilon_0}$

limit $R \to \infty$ ("infinite range") $\Rightarrow V(r) = \frac{-e^2}{4\pi\varepsilon_0 r}$

Coulomb energy



$\bar{q} = \bar{q}_f - \bar{q}_i$

Amplitude

$$\mathcal{M}(\bar{q}) = \int d^3r\, e^{\frac{i}{\hbar}\bar{q}_f\cdot\bar{r}}\, V(r)\, e^{-\frac{i}{\hbar}\bar{q}_i\cdot\bar{r}} = \underbrace{\int d^3r\, e^{-\frac{i}{\hbar}\bar{q}\cdot\bar{r}}\, V(\bar{r})}_{\text{Fourier transform of } V(\bar{r}) \text{ at mom. transfer } \bar{q}}$$

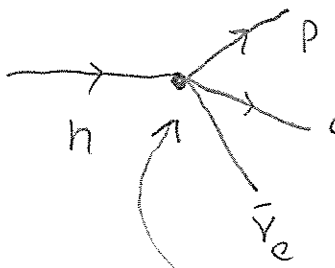
$$\Rightarrow \dots \Rightarrow \mathcal{M}(\bar{q}) = \frac{-g^2\hbar^2}{|\bar{q}|^2 + M_x^2c^2}$$

Relativistic treatment $\quad Q = Q_f - Q_i\,;\;\; Q_f = \left(\frac{E_f}{c}, \bar{q}_f\right)$ etc. (3)

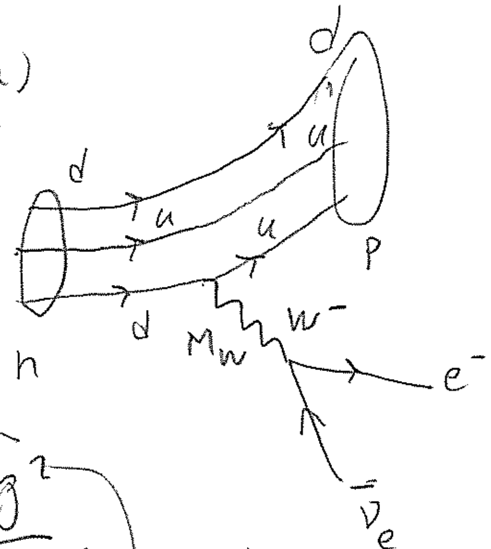
$$\Rightarrow \mathcal{M}(Q) = \frac{g^2}{Q^2 - M_\gamma^2 c^2} \qquad Q = \left(\frac{E_f - E_i}{c}, \bar{q}_f - \bar{q}_i\right)$$

$$Q^2 = \frac{1}{c^2}(E_f - E_i)^2 - (\bar{q}_f - \bar{q}_i)^2$$

Weak interaction: $\quad n = (ddu)$, $p = (uud)$



$\rightarrow$



$$\mathcal{M} \sim \frac{g^2}{(\cancel{\text{few MeV}})^2 - \underbrace{M_{W^-}^2 c^2}_{90\ \text{GeV}^2}} \rightarrow -\frac{g^2}{M_{W^-}^2 c^2}$$

point-like interaction (very short range)

However when energy & momentum transfer are $\gg 90$ GeV, then "weak" and e.m. interactions become of similar strength

Experimental particle physics: $\quad \underset{\text{interaction rate}}{R} = \overset{\text{luminosity}}{L} \cdot \sigma$ ← cross section

Fermi's golden rule

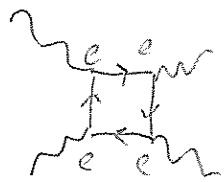
$$\underbrace{W}_{\text{Reaction rate per beam and target particle}} = 2\pi |\langle f | H_{int} | i \rangle|^2 \underbrace{\rho(E)}_{\text{number density of final states}}$$

Antiparticles: $\bar{n}$ antineutron $\quad \underset{+\frac{1}{3}+\frac{1}{3}-\frac{2}{3}}{(\bar{d}\bar{d}\bar{u})} \neq \underset{-\frac{1}{3}-\frac{1}{3}\;\frac{2}{3}}{(ddu)}$ since $\bar{d} \neq d$ etc

However, $\bar{\gamma} = \gamma$, $\bar{Z}^0 = Z^0$, $\bar{H}^0 = H^0$

Feynman diagrams

ex. $\gamma + \gamma \rightarrow \gamma + \gamma$



$\mathcal{M} \sim e^4 \sim \alpha^2$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137} \quad \text{fine structure const.}$$

<u>Nuclear physics</u>

$^{A}_{Z}(\text{Name})$ says the same thing (if you know the Z for all names)

$$A = N + Z$$

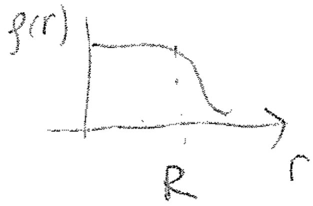
N: neutrons, Z: protons

Ex $^{16}_{8}O$ usually written ^{16}O for simplicity

number of neutrons $N = A - Z = 16 - 8 = 8$

Z define the <u>element</u> (and also number of electrons for a neutral atom)

N can vary by a few $\Rightarrow$ different <u>isotopes</u>



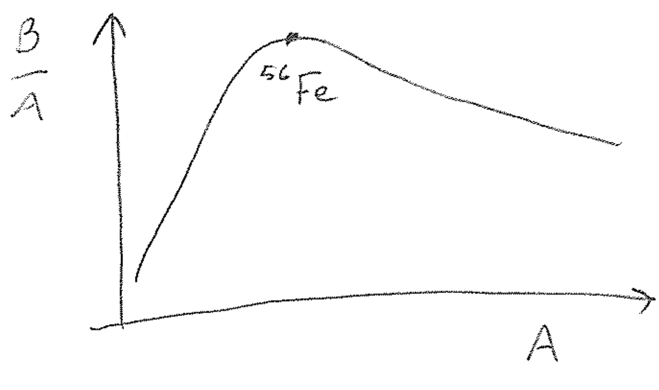
$$R \approx R_0 A^{1/3} \quad \Rightarrow \quad \frac{A}{\underbrace{\frac{4\pi}{3} R^3}_{\text{Volume}}} \sim \text{const.}$$

Mass $M_A = Z \cdot m_p + N \cdot m_n - \underbrace{\frac{B_A}{c^2}}_{\text{mass deficit}}$ for an atom

B_A: binding energy

For nucleus:

$$B = \left[Z m_p + N m_n - (m_A - Z m_e) \right] c^2$$



Semi-empirical mass formula:

$$B = \underbrace{a_v A}_{\text{volume}} - \underbrace{a_s A^{2/3}}_{\text{surface}} - \underbrace{a_c \frac{Z(Z-1)}{A^{1/3}}}_{\text{coulomb}} - \underbrace{a_{sym} \frac{(A-2Z)^2}{A}}_{\text{fermions}} + \underbrace{\delta(A,Z)}_{\text{pairing}}$$

ex: $a_v = 15.5$ MeV, $a_s = 17.8$ MeV, $a_c = 0.72$ MeV, $a_{sym} = 23$ MeV

(Many different versions)

$$\delta(A,Z) = \begin{cases} +\delta_0 & Z,N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ -\delta_0 & Z,N \text{ odd } (A \text{ even}) \end{cases} \qquad \delta_0 = \frac{+34 \text{ MeV}}{A^{3/4}}$$

For constant A, $M(Z,A)$ is a parabola

Spontaneous fission $C \to D + E$

Q-value $\quad Q = (M_C - M_D - M_E)c^2$

$$N(t) = N(0)\, e^{-\lambda t} \qquad t_{1/2} = \frac{\ln 2}{\lambda}$$

β-decay $\quad (Z,A) \to (Z+1, A) + e^- + \bar{\nu}_e \quad (\beta^-)$

$\quad (Z,A) \to (Z-1, A) + e^+ + \nu_e \quad (\beta^+)$

$\underbrace{\alpha}_{^4\text{He nucleus}}$-decay $\quad (Z,A) \to (Z-2, A-4) + \underbrace{(2,4)}_{^4_2\text{He}}$

Cosmology

Homogeneous, isotropic expansion

$\Rightarrow$ radial symmetry (indep. of θ and φ)

$r(t) = a(t) \cdot x \qquad x = \frac{r(t)}{a(t)}$ (so $a(t)$ follows the physical distance)

physical distance ↑ $r(t)$; scale factor ↑ $a(t)$; comoving distance ↖ x

$$v(t) = \frac{d}{dt} r(t) = \dot{a}(t) \cdot x = \underbrace{\frac{\dot{a}(t)}{a(t)}}_{= H,\ \text{Hubble parameter}} r(t)$$

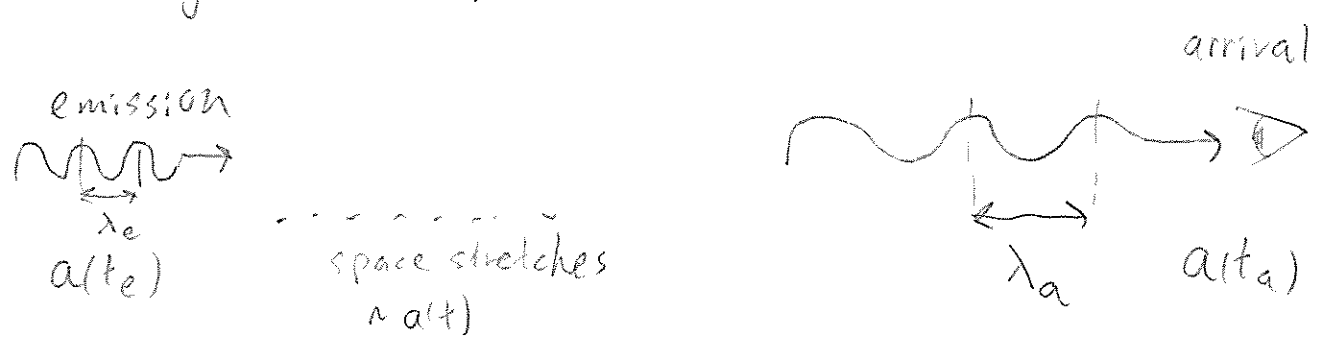
$$t = \text{today} = t_0 \Rightarrow \underbrace{H(t_0)}_{H_0} = \frac{\dot a(t_0)}{a(t_0)} = \{\text{measurements}\} =$$

t_0: 13.8 billion years

$$= h \cdot 100 \text{ km/s per Mpc}$$

$$h \sim 0.7$$

Cosmological redshift:



$$\frac{\lambda_a}{\lambda_e} = \frac{a(t_a)}{a(t_e)}$$

$$z = \frac{\lambda_a - \lambda_e}{\lambda_e} = \frac{\lambda_a}{\lambda_e} - 1 \Rightarrow \frac{\lambda_a}{\lambda_e} = 1 + z$$

Thus $\boxed{1 + z = \frac{\lambda_a}{\lambda_e}}$

Friedmann equation: $H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3} \underbrace{\rho}_{\rho_m + \rho_\gamma} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$

Define $\rho_\Lambda = \frac{\Lambda}{8\pi G} \Rightarrow H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho_{tot} - \frac{kc^2}{a^2}$

$\rho_c(t) = \frac{3H^2}{8\pi G}$ ← critical

$$\rho_{tot} = \rho_m + \rho_\gamma + \rho_\Lambda \qquad \rho_{now} = \rho_M + \rho_R + \rho_\Lambda$$

$\Omega_M = \frac{\rho_M}{\rho_c(t_0)}$ etc.

$$\Rightarrow H^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_R (1+z)^4 + \Omega_\Lambda \right] + \underbrace{\mathcal{O}(k)}_{\sim 10^{-3} \text{ term}}$$

Eqn. of state $p = w \cdot \rho$

Fluid eqn. $\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(\rho + \underbrace{p}_{= w\rho}) = 0$

$$\Rightarrow \quad \dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)\rho(1+w) = 0 \quad \Rightarrow \quad \frac{\dot{\rho}}{\rho} = -3(1+w)\left(\frac{\dot{a}}{a}\right)$$

$$\Rightarrow \quad \ln(\rho) = -3(1+w)\ln(a)$$

$$\Rightarrow \quad \rho = \frac{\rho_0}{a^{3(1+w)}}$$

matter: $p \approx 0 \Rightarrow w = 0 \Rightarrow \rho_m \sim a^{-3}$

radiation: $p = \frac{\rho}{3} \Rightarrow w = \frac{1}{3} \Rightarrow \rho_R \sim a^{-4}$

Λ: $p = -\rho \Rightarrow \rho_\Lambda = \text{const}$

$$\Lambda: \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \Rightarrow$$

$$\Rightarrow \quad \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3}} \quad \Rightarrow \quad a(t) \sim e^{\sqrt{\frac{\Lambda}{3}} \cdot t}$$

exponential growth!

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \frac{\rho_0}{a^{3(1+w)}}$$

Make Ansatz

$$a(t) \sim t^q \Rightarrow t^{(2q-2-2q)} = t^{-3(1+w)q}$$

$$\Rightarrow \quad -2 = -3(1+w)q$$

$$\Rightarrow \quad q = \frac{2}{3(1+w)}$$

$w = 0$ matter $a \sim t^{2/3}$

$w = \frac{1}{3}$ radiation $a \sim t^{1/2}$

$w = -1$ Λ diverges $(a \sim e^{\sqrt{\frac{\Lambda}{3}} t})$

Early Universe

matter-radiation equality ~ $z = 10\,000$

For $z \gg 10\,000$ $\quad \rho_m, \rho_\Lambda, \rho_k$ can be neglected

$$\Rightarrow H^2 = \frac{8\pi G}{3}\rho_R \,, \quad \text{High density}, \ \rho_R = \alpha T^4$$
$$\left(\alpha = \frac{4}{c}\cdot\sigma_{SB}\right)$$

$$\Rightarrow \ \ldots \ \Rightarrow T(\text{MeV}) \sim \frac{1}{\sqrt{t(\text{sec})}}$$

MeV temperatures $\Leftrightarrow$ nuclear binding energies

$\Rightarrow$ BBN big bang nucleosynthesis

$$R_n = \frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{\frac{-(m_n-m_p)c^2}{k_B T}} \sim e^{\frac{-1.3\,\text{MeV}}{0.8\,\text{MeV}}} \sim \frac{1}{5} = 0.2$$

$k_B \cdot$ (temp. when neutrons leave equilibrium)

After that, neutrons decay $n \to p + e^- + \bar{\nu}_e$ ($\tau_n \sim 880\,s$) until they are "saved" by ^4He at ~ 300 sec

$\Rightarrow$ Neutron rate diminished by factor $e^{-\frac{t}{\tau_n}} = e^{-\frac{300s}{880s}} \sim 0.7$

$$\Rightarrow \frac{n_n}{n_p} = 0.2\cdot 0.7 = 0.14$$

2 neutrons in each ^4He $\Rightarrow$

$\Rightarrow$ number of $^4He = \frac{1}{2}$(number of n)

$$\Rightarrow \text{mass fraction } \frac{4\cdot n_{^4He}}{n_n+n_p} = \frac{2n_n}{n_n+n_p} = 0.25 \sim 25\%$$

The rest is protons (hydrogen) ~ 75% plus very small abundances of D, ^3He, ^7Li

Heavier elements were synthesized later in the interior of stars, and spread in the galaxies through violent processes (e.g. supernova explosions)