Lecture 16

Friedmann eqn:

$$H^2(t) = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\left[\rho_m(t) + \rho_r(t) + \rho_\Lambda(t)\right] - \frac{k}{a^2}$$

$$\overbrace{H^2(t_0)}^{\equiv H_0^2} = \overbrace{\frac{8\pi G}{3}\rho_c^0}^{= H_0^2}\left[\Omega_M + \Omega_R + \Omega_\Lambda\right] - \frac{k}{a^2}$$

$$\rho_c^0 = \frac{3H_0^2}{8\pi G}$$

Observations: Ω_M: 0.3 $\quad \Omega_R$: 10^{-5} $\quad \Omega_\Lambda$: 0.7

$$\Omega_K \equiv -\frac{k}{H_0^2 a^2(t_0)} \sim 0 \pm 10^{-3}$$

$$\Rightarrow \Omega_M + \Omega_R + \Omega_\Lambda \simeq 1$$

$$\rho_m(t) \sim \frac{\rho_m(t_0)}{a^3(t)} = \rho_m(t_0)(1+z)^3$$

$$\rho_r(t) \sim \frac{\rho_r(t_0)}{a^4(t)} = \rho_r(t_0)(1+z)^4$$

$$\rho_\Lambda(t) = \rho_\Lambda(t_0) \text{ (const.)} \Rightarrow \rho_\Lambda(t) = \rho_\Lambda(t_0)\underbrace{\cancel{(1+z)^0}}_{1}$$

$\Rightarrow$ Modern form of Friedmann eqn.

$$H^2(t) = H_0^2\left[\Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_\Lambda\right]\left(-\frac{k}{a^2}\right)$$

can be directly measured!

usually neglected

(or $\Omega_K(1+z)^2$)

$k \simeq 0 \Rightarrow$ Flat (or "critical") universe

$$\Rightarrow \Omega_M + \underbrace{\Omega_R}_{\sim 10^{-5}} + \Omega_\Lambda = 1$$

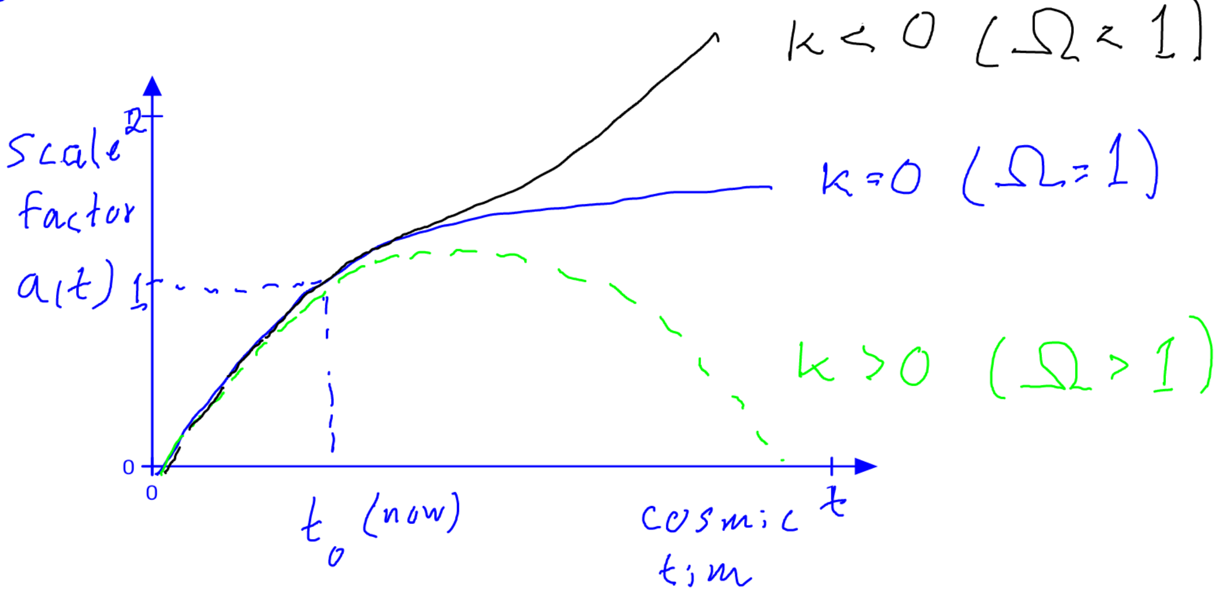
$$\Rightarrow \boxed{\Omega_M + \Omega_\Lambda \simeq 1}$$

$$\Omega_M + \Omega_\Lambda \approx \Omega_{tot}$$

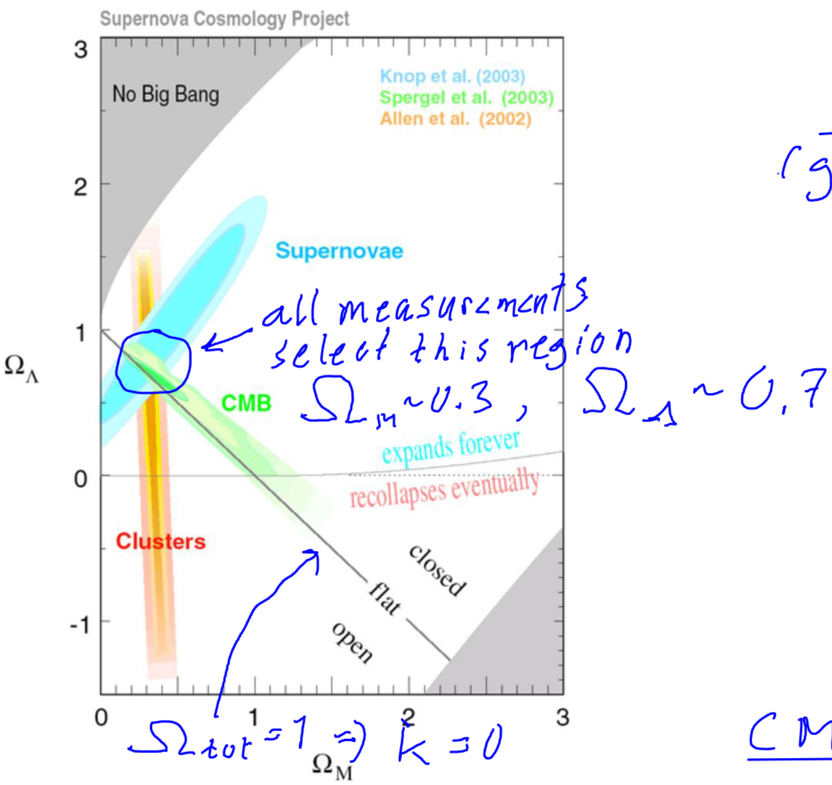
History: ~30 years ago

"$\Omega = \Omega_{tot}$" $= \Omega_M$, but k unknown, $\underbrace{\Lambda := 0}_{\text{assumed}}$

Figure from old books:



Then ≈ 20 years ago, acceleration was discovered:



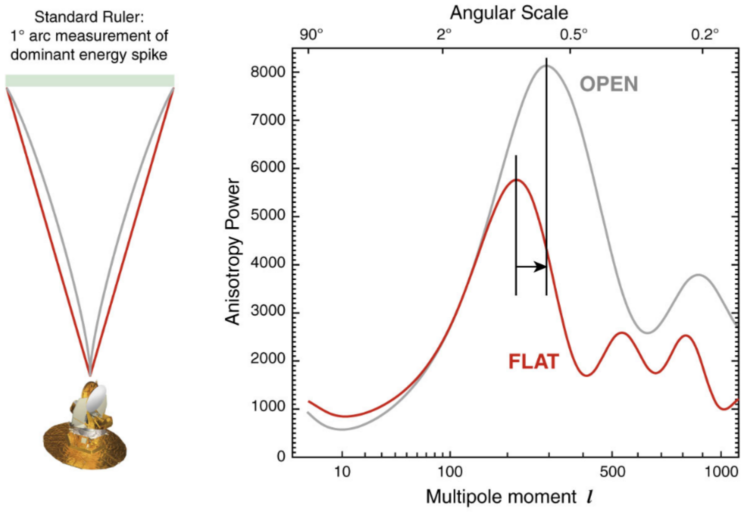
Clusters: Measure of motion (gas & stars) in galaxy clusters $\Rightarrow \Omega_M$

Supernovae: use of supernovae as "standard candles" going much further out than cepheids.

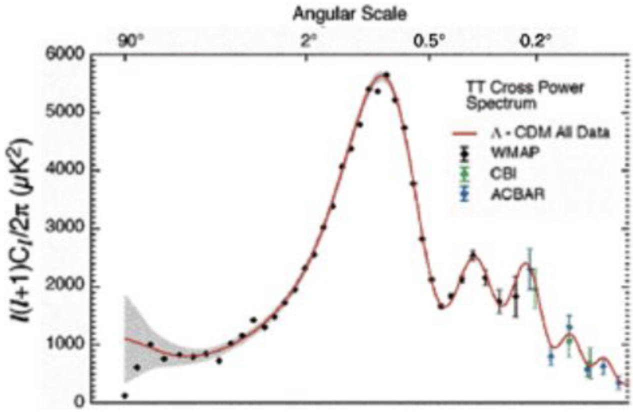
CMB: Cosmic microwave backgrund ⇒

$\Rightarrow$ geometry of universe

$\Rightarrow \Omega_M + \Omega_\Lambda = \Omega_{tot} = 1$



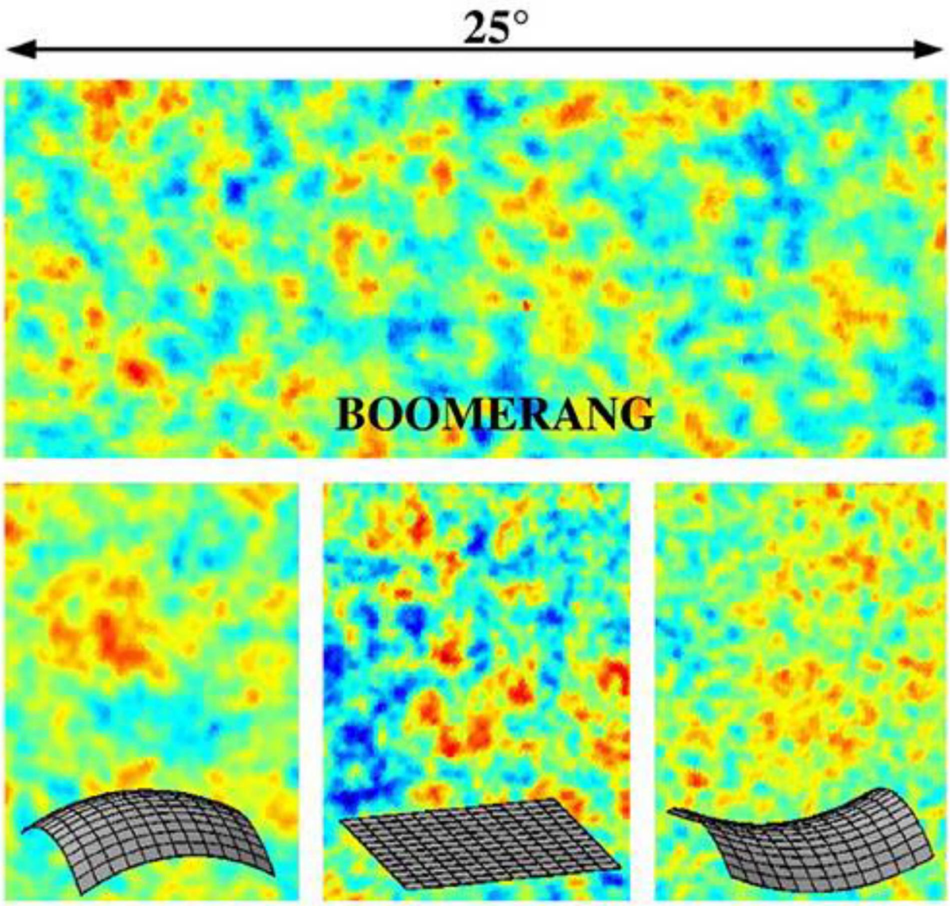
Theory predictions for WMAP satellite



WMAP measurements

$\Rightarrow$ flat cosmology ($k \sim 0$, $\Omega_{tot} \sim 1$)

$\Omega_{tot} \sim 1$ had already been indicated by BOOMERANG balloon measurements:



This fits best for size of structures

Inflation

$$\rho_m(t) \sim \frac{1}{a^3(t)}, \quad \rho_r \sim \frac{1}{a^4(t)}, \quad \rho_\Lambda = \text{const.}$$

Expanding universe $\Rightarrow$ $a(t)$ increases with time.
$\Rightarrow$ As universe expands, ρ_m and ρ_r will become smaller and smaller, but ρ_Λ will be constant, and larger in comparison.

$\Rightarrow$ cosmological constant domination!

(Already, Ω_Λ is the largest component)

$$\Rightarrow \quad H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G_N}{3}(\cancel{\rho_m} + \cancel{\rho_r}) + \frac{\Lambda}{3} \quad (k=0)$$

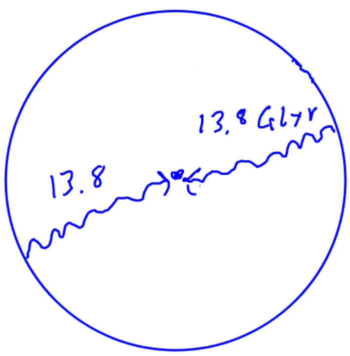
neglect in the future

$$\Rightarrow \quad H^2 = \frac{\Lambda}{3} \Rightarrow \left(\frac{\dot a}{a}\right)^2 = \frac{\Lambda}{3} \Rightarrow \frac{\dot a}{a} = \sqrt{\frac{\Lambda}{3}} = H$$

$$a(t) \sim e^{\sqrt{\frac{\Lambda}{3}}\cdot t} = e^{H\cdot t}$$ exponential growth!

However, Λ may be time-dependent, for instance "quintessence" ~ time-dependent dark energy

Isotropy problem of CMB



(Distance ≥ 26 billion light years)

How come that the radiation from diametrically opposite sides of the universe has exactly the same temperature $T_{CMB} = 2.73K$?

These two regions cannot have been in thermal contact

The only solution found so far:

Cosmological <u>inflation</u>:

Assume that there was a very early epoch when $\rho_{vacuum} \sim$ const (indep. of time)

$$\Rightarrow H_{infl}^2 = \frac{\Lambda_{infl}}{3} = \text{const} = \frac{\dot{a}(t)}{a(t)}$$

$$\Rightarrow a(t) \sim e^{\sqrt{\frac{\Lambda_{infl}}{3}} \cdot t} = e^{H_{infl} \cdot t}$$

$$r(t) = a(t) \cdot x = e^{H_{infl} \cdot t} \cdot x$$

$$v = \frac{d}{dt} r(t) = \dot{a}(t) \cdot x = \underbrace{H_{infl}\, e^{H_{infl} \cdot t} \cdot x}_{\text{This can be} \gg c}$$

$\Rightarrow$ superluminal expansion of the universe!

[However, material particles and radiation still travel with $v \leq c$, See Liddle 3.3]

Curvature: $$\Omega_k = \frac{-kc^2}{a(t)^2} \sim \underbrace{-kc^2 \frac{1}{e^{2H_{infl} \cdot t}}}_{\equiv k_{eff} \rightarrow 0 \text{ very fast as time goes on}}$$

Thus, a successful prediction of inflation is that $\rho_{tot} = \rho_{crit}$ to excellent accuracy ($k_{eff} = 0$)

A small region, where thermal equilibrium may have been maintained suddenly "blew up" and became exponentially larger $\Rightarrow$ explains why temperature of CMB is everywhere $\sim$ the same.

Also, inflation <u>predicts</u> that $\Omega_{tot} = 1$.