2018-10-09

# FK 5024: Particle & Nuclear Physics, Astrophysics & Cosmology

## Part III: Astrophysics & Cosmology, Lars Bergström

main topic, see book by A. Liddle "An introduction to Modern Cosmology"

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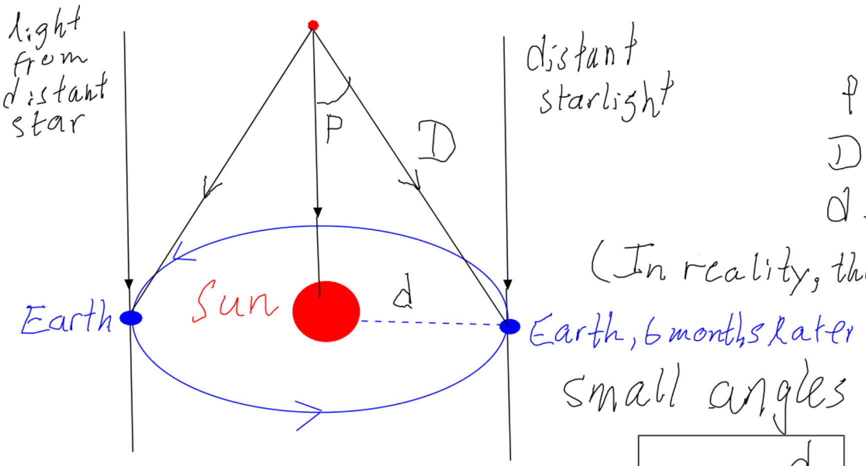
## How far away are the stars?

The Sun-Earth distance is know since ancient times. The modern value is $d = 1.496 \cdot 10^{11}$ m ($\approx$ 8 light minutes)

we can use this to measure distance to other stars by parallax:




p: parallax angle

D: distance to nearby star

d: Earth-Sun distance

(In reality, the angle p is <u>very</u> small)

small angles $\Rightarrow \sin(p) \approx p = \dfrac{d}{D}$

$$\Rightarrow \boxed{D = \frac{d}{p}} \quad \text{The law of parallax}$$

By using the fact that the star seems to be moving an angular distance p against distant stars, we can use the law of parallax to determine distance!

For very large distances we have to use other methods, like <u>supernovas</u> (see later) which are extremely bright.

Definition:

1 pc (parsec) is the distance which gives $p = 1''$ (one arcsec)

$1°$ (one degree) $= \frac{\pi}{180}$ rad, $1'$ (arcmin) $= \frac{1°}{60}$, $1''$ (arcsec) $= \frac{1'}{60}$

$\Rightarrow 1'' = \frac{\pi}{3600 \cdot 180}$ rad $\approx 4.85 \cdot 10^{-6}$ rad

Q: What is 1 pc in SI unit (m)?

A: $D = \frac{d}{p} = \frac{1.496 \cdot 10^{11}\,m}{4.85 \cdot 10^{-6}} = 3.09 \cdot 10^{16}\,m$

[see formula sheet: 1 pc = 3.261 lightyears $= 3.086 \cdot 10^{16}\,m$]

Nearest star (except the Sun!): Proxima Centauri

Distance $\approx 1.3$ pc $\Rightarrow p = \frac{1''}{1.3} \approx 0.8''$

$1''$ is approximate resolution of ground-based telescopes

Satellites: The new GAIA satellite has $p_{min} \approx 2 \cdot 10^{-5}$ arcsec

$\Rightarrow$ Can measure distance to 20 million stars with 1% accuracy!

How bright are stars?

Emittance $\mathcal{M} = \frac{\text{radiated energy}}{\text{time} \cdot \text{area}}$

Luminosity $L = \mathcal{M} \cdot \text{area} = \sigma_{SB} \cdot T^4$ (Stefan-Boltzmann law)

Formula sheet: $\sigma_{SB} = 5.67 \cdot 10^{-8} \frac{W}{m^2 K^4}$

The Sun: $T_{eff} = 5800\,K$, $R_\odot = 6.96 \cdot 10^8\,m$ (radius of the sun)

$\Rightarrow L_\odot = 4\pi R_\odot^2 \cdot \sigma_{SB} \cdot T^4 = \ldots = 3.9 \cdot 10^{26}\,W$ (check this!)

$\Rightarrow$ Flux near Earth $F = \frac{L_\odot}{4\pi d^2} = \frac{3.9 \cdot 10^{26}\,W}{4\pi \cdot (1.496 \cdot 10^{11}\,m)^2} = \frac{1400\,W}{m^2}$

The solar constant!

Astronomers use magnitudes, $m$

$$m = -2{,}5 \log_{10}\left(\frac{F}{1\,W/m^2}\right) + \text{const}$$

Ex: Vega star, measured flux $\frac{F_\odot}{F_{vega}} = 5 \cdot 10^{10}$

$$\Rightarrow m_\odot - m_V = -2.5 \log_{10}\left(\frac{F_\odot}{F_{vega}}\right) = -2.5 \cdot 10.7 = -26.7$$

Absolute magnitude: magnitude exactly 10 pc from star

$$\Rightarrow m_x - M = -2{,}5 \log_{10}\left(\frac{F_x}{F_M}\right) = -2.5 \log_{10}\left(\frac{(10\,pc)^2}{D_x^2}\right)$$

[since Flux $\sim \frac{1}{D^2}$ ]

$$\Rightarrow m_x - M = -2 \cdot 2{,}5 \log_{10}\left(\frac{10\,pc}{D_x}\right) = 5 \log_{10}\left(\frac{D_x}{10\,pc}\right) =$$

$$= 5 \log_{10}(D_x) - 5 \underbrace{\log_{10}(10)}_{=1} = 5\left[\log_{10}(D_x) - 1\right]$$

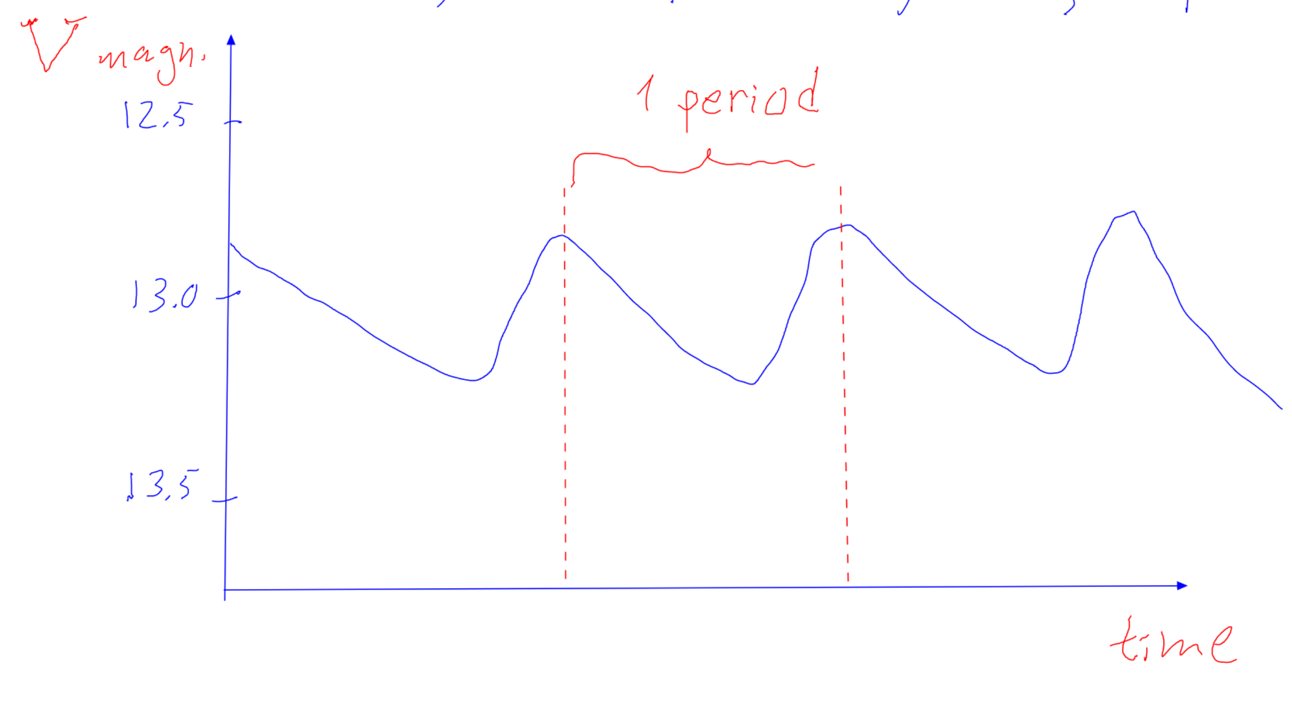
with $D_x$ distance in parsec

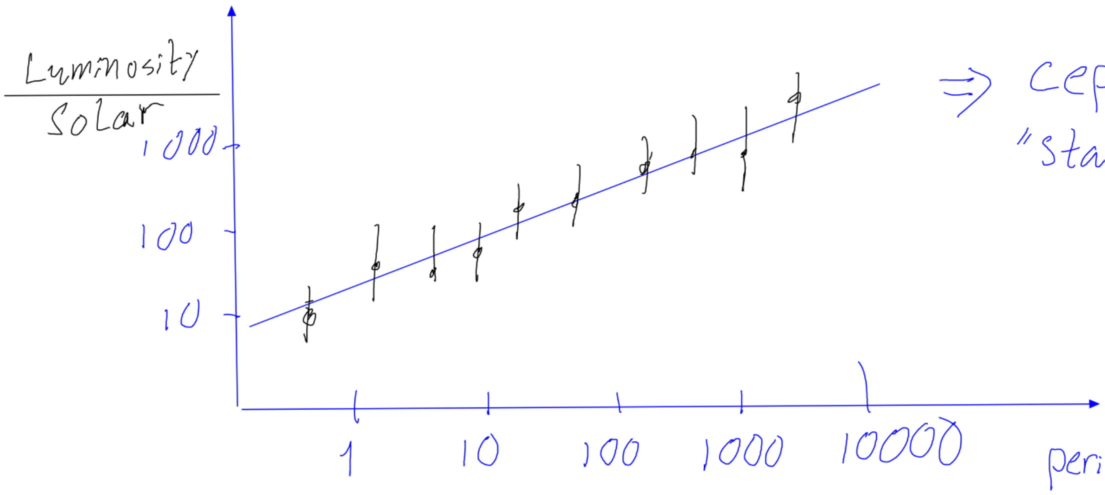
Magnitudes can also be defined in colour intervals

e.g. B, V, U, I

blue visible UV IR

Henrietta Leavitt, 1912: pulsating stars, "cepheids"

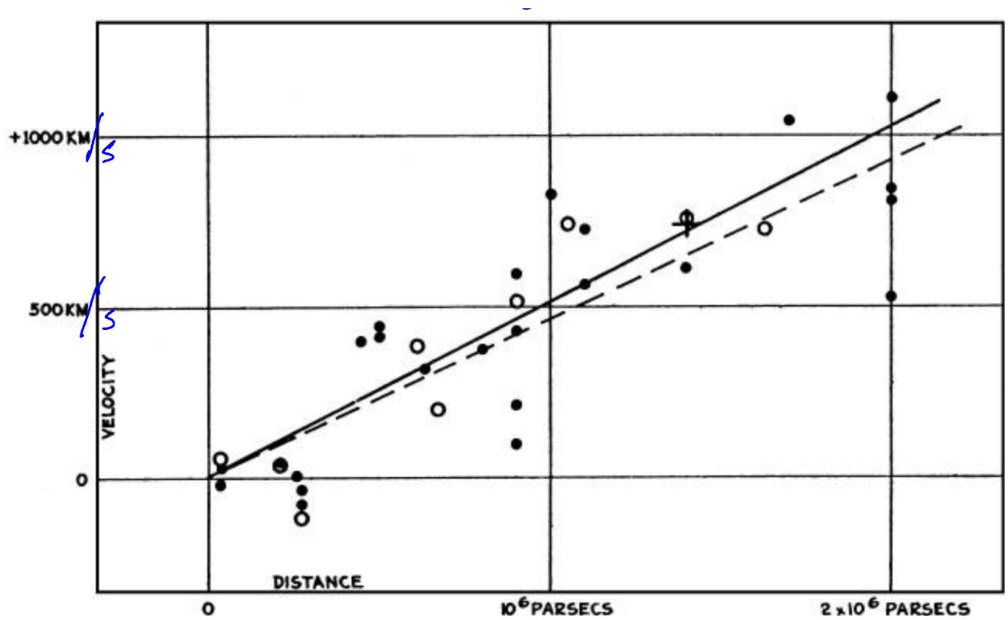





$\Rightarrow$ cepheids are "standard candles"

For longer distances, have to use supernovas as standard candles

Edwin Hubble used cepheids to estimate distances




Hubble's measurements show recession velocities increasing with distance & $v = H_0 \cdot r$ $H_0$: Hubble constant

Solid line gives $H_0 \sim 500$ km/s per Mpc

(To big by almost factor of 10, because cepheid distances were underestimated at the time)

The measured quantity was really redshifts, $z$

$$z = \frac{\lambda_{obs} - \lambda_{lab}}{\lambda_{lab}} = \frac{\Delta\lambda}{\lambda_{lab}} \approx \frac{v}{c} \quad \text{(Doppler-effect)}$$

For small velocities, then $v = c \cdot z$

$\uparrow$ $3 \cdot 10^8 \, m/s$

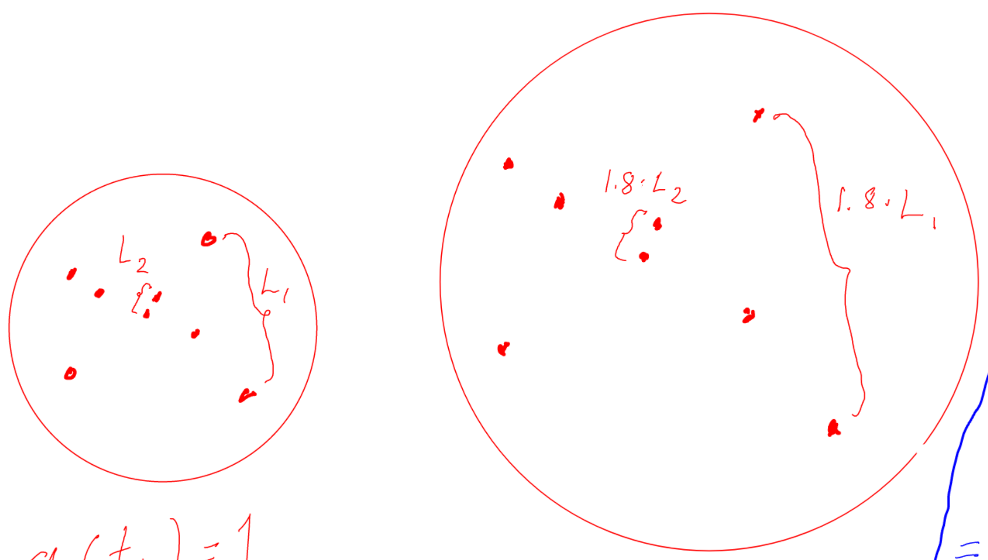
However, we need a general formula. Some distant galaxies have $z \approx 6$, so we need a formula for cosmological (very large) redshifts.

Hubble's discovery implies that the universe is expanding.

Let $r(t)$ be distance to a far-away galaxy

Write $\underset{\substack{\uparrow \\ \text{physical distance}}}{r(t)} = \underset{\substack{\uparrow \\ \text{scale factor}}}{a(t)} \cdot \underset{\substack{\uparrow \\ \text{co-moving distance}}}{x}$

Assumption: On large scales, the universe is isotropic and homogeneous. Analogy: a balloon of changing radius (2-dim example)




$a(t_0) = 1$ $\quad$ $a(t_1) = 1.8$

"velocity" between two points:

$$v = \frac{\Delta L}{(t_1 - t_0)} = \frac{[a(t_1) - a(t_0)] \cdot L}{t_1 - t_0} = c \cdot L$$

$\uparrow$ expansion rate

Hubble's law!

For small time intervals,

$$\lim_{t_1 \to t_0} \frac{a(t_1) - a(t_0)}{t_1 - t_0} = \left. \frac{da(t)}{dt} \right|_{t = t_0} \equiv \dot{a}(t_0)$$