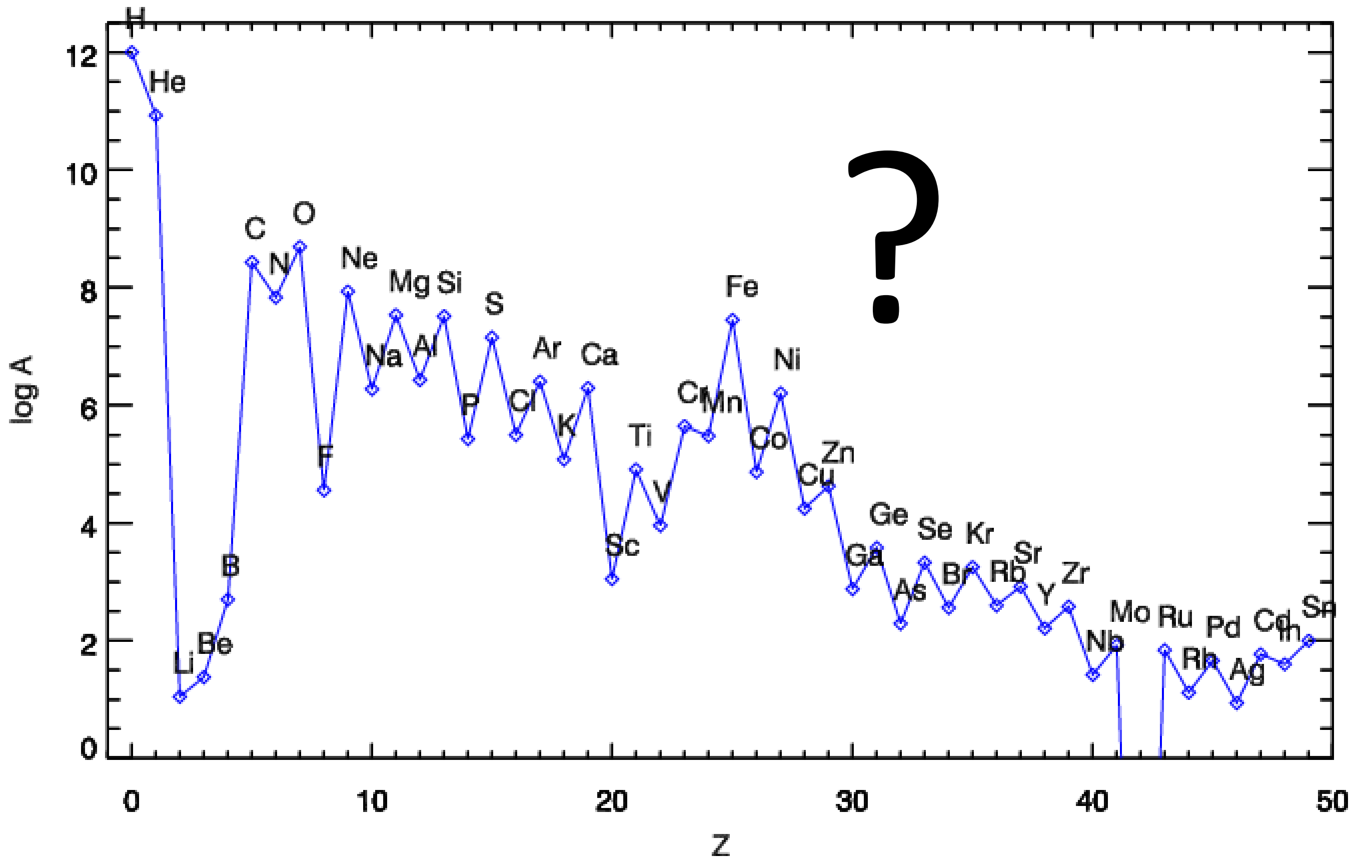
Summary of last lecture

- We discussed the impact of nuclear physics in the search for dark matter
- Otherwise: neutrino physics
- We discussed as applications: nuclear reactor and nuclear bombs

Letcure 6: Stellar fusion and neutron stars: nucleosynthesis

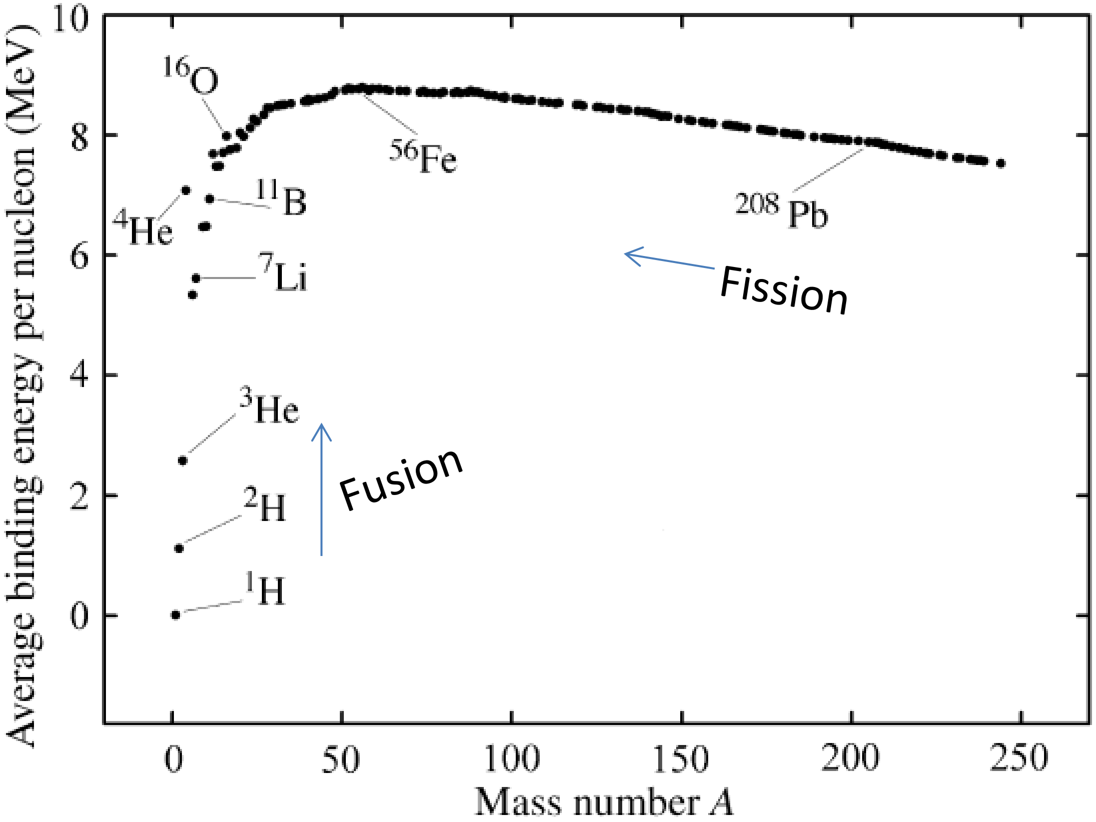
Jan Conrad

Abundance of elements in the solar system



?

Fusion


Average binding energy per nucleon (MeV)
Mass number A
10
8
6
4
2
0
0
50
100
150
200
250
16O
56Fe
208 Pb
4He
11B
7Li
3He
2H
1H
Fission
Fusion

Coulumb barrier

$$V_{\mathrm{C}} = \frac{1}{4\pi\varepsilon_0}\frac{ZZ'e^2}{R+R'},$$

$$V_{\mathrm{C}} = \left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\frac{\hbar c\,Z\,Z'}{1.2\left[A^{1/3}+(A')^{1/3}\right]\,\mathrm{fm}} = 1.198\frac{Z\,Z'}{A^{1/3}+(A')^{1/3}}\mathrm{MeV}.$$

$$V_{\mathrm{C}} \approx 0.15A^{\frac{5}{3}}\ \mathrm{MeV}.$$

A ~8 → V_c ~ 4.8 MeV

Overcoming the Coulomb barrier

- Colliding beams would be elastically scattering
- Confined, heated plasma were the thermal energy can be large enough to overcome the Coulomb Barrier.

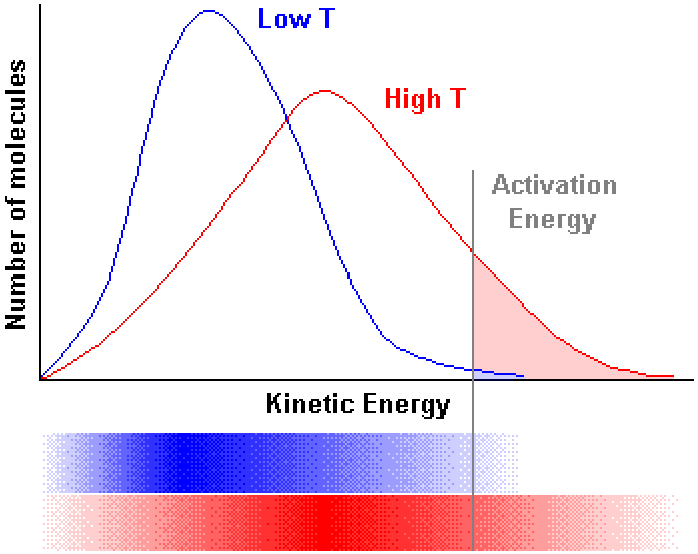
$T \sim 8$ MeV/k → $T \sim 10^{10}$ K

Stellar interiors seem not hot enough (10^{7} K)

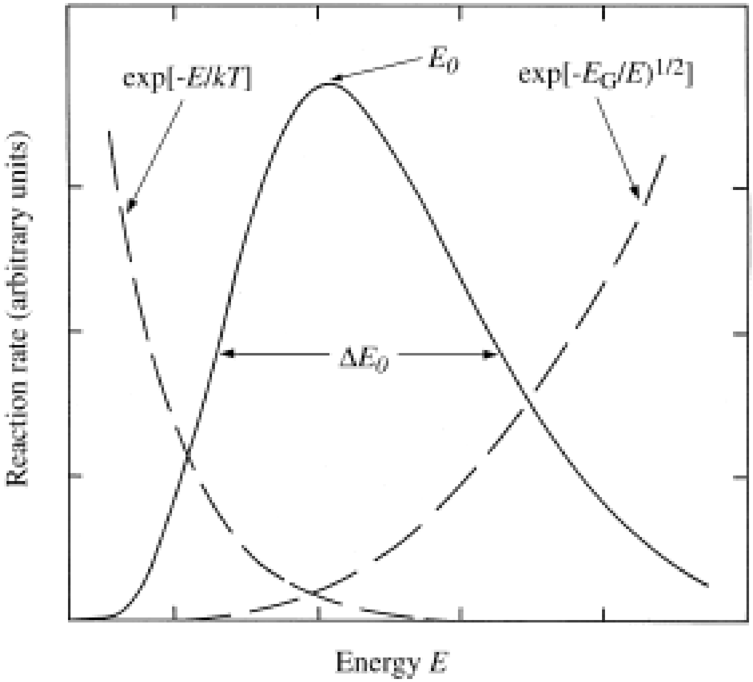
Eddington: ”…. go and find a hotter place”.

Anything to the rescue?

- Tunneling → Gamow-factor
- Maxwell-Boltzman distribution→ high E tail.

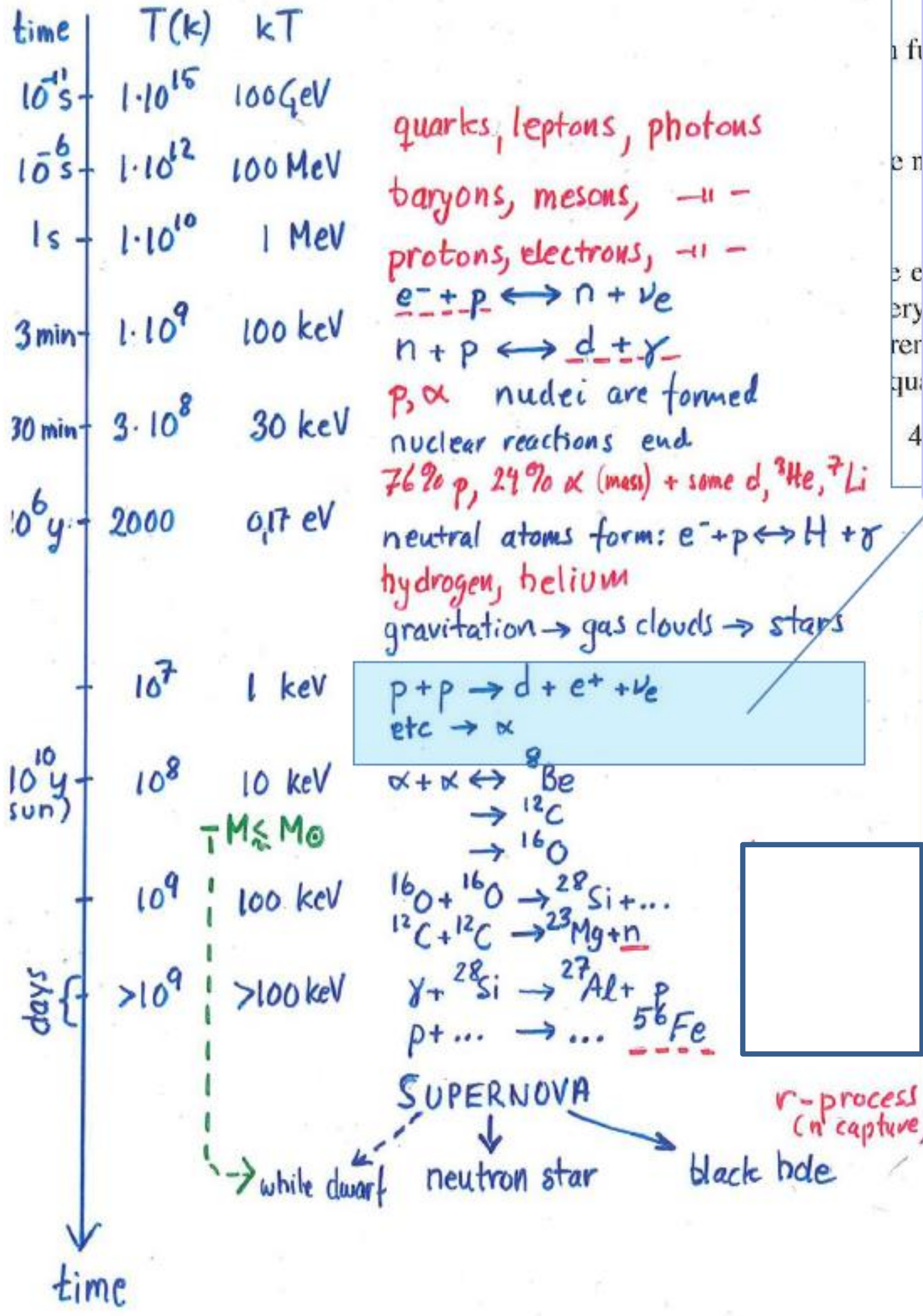


Tunneling and Maxwell-tail



- Tunneling effect rises for high energies, Maxwell tail gets enhanced

  → Appreciable fusion rates possible in stars



time	T(k)	kT
10^{-11} s	$1\cdot10^{15}$	100 GeV
10^{-6} s	$1\cdot10^{12}$	100 MeV
1 s	$1\cdot10^{10}$	1 MeV
3 min	$1\cdot10^{9}$	100 keV
30 min	$3\cdot10^{8}$	30 keV
10^{6} y	2000	0,17 eV
	10^{7}	1 keV
10^{10} y (sun)	10^{8}	10 keV
	10^{9}	100 keV
days	$>10^{9}$	>100 keV

quarks, leptons, photons

baryons, mesons, –"–

protons, electrons, –"–

$e^- + p \leftrightarrow n + \nu_e$

$n + p \leftrightarrow d + \gamma$

p, α nuclei are formed

nuclear reactions end

76% p, 24% α (mass) + some d, ^3He, ^7Li

neutral atoms form: $e^- + p \leftrightarrow H + \gamma$

hydrogen, helium

gravitation → gas clouds → stars

$p + p \to d + e^+ + \nu_e$

etc → α

$\alpha + \alpha \leftrightarrow {}^{8}Be$

$\to {}^{12}C$

$\to {}^{16}O$

$M \lesssim M_\odot$

${}^{16}O + {}^{16}O \to {}^{28}Si + \dots$

${}^{12}C + {}^{12}C \to {}^{23}Mg + n$

$\gamma + {}^{28}Si \to {}^{27}Al + p$

$p + \dots \to \dots$ ${}^{56}Fe$

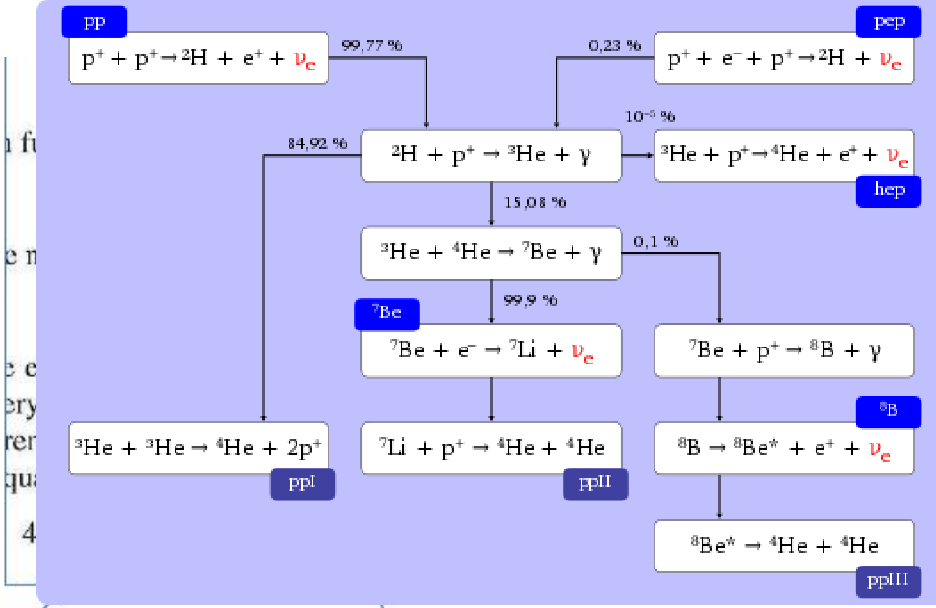
SUPERNOVA

white dwarf

neutron star

black hole

r-process (n capture)



pp

$p^+ + p^+ \to {}^2H + e^+ + \nu_e$

99,77 %

pep

$p^+ + e^- + p^+ \to {}^2H + \nu_e$

0,23 %

84,92 %

${}^2H + p^+ \to {}^3He + \gamma$

10^{-5} %

${}^3He + p^+ \to {}^4He + e^+ + \nu_e$

hep

15,08 %

${}^3He + {}^4He \to {}^7Be + \gamma$

0,1 %

99,9 %

${}^7Be$

${}^7Be + e^- \to {}^7Li + \nu_e$

${}^7Be + p^+ \to {}^8B + \gamma$

${}^3He + {}^3He \to {}^4He + 2p^+$

ppI

${}^7Li + p^+ \to {}^4He + {}^4He$

ppII

${}^8B$

${}^8B \to {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \to {}^4He + {}^4He$

ppIII

6.55 MeV per consumed proton → heat

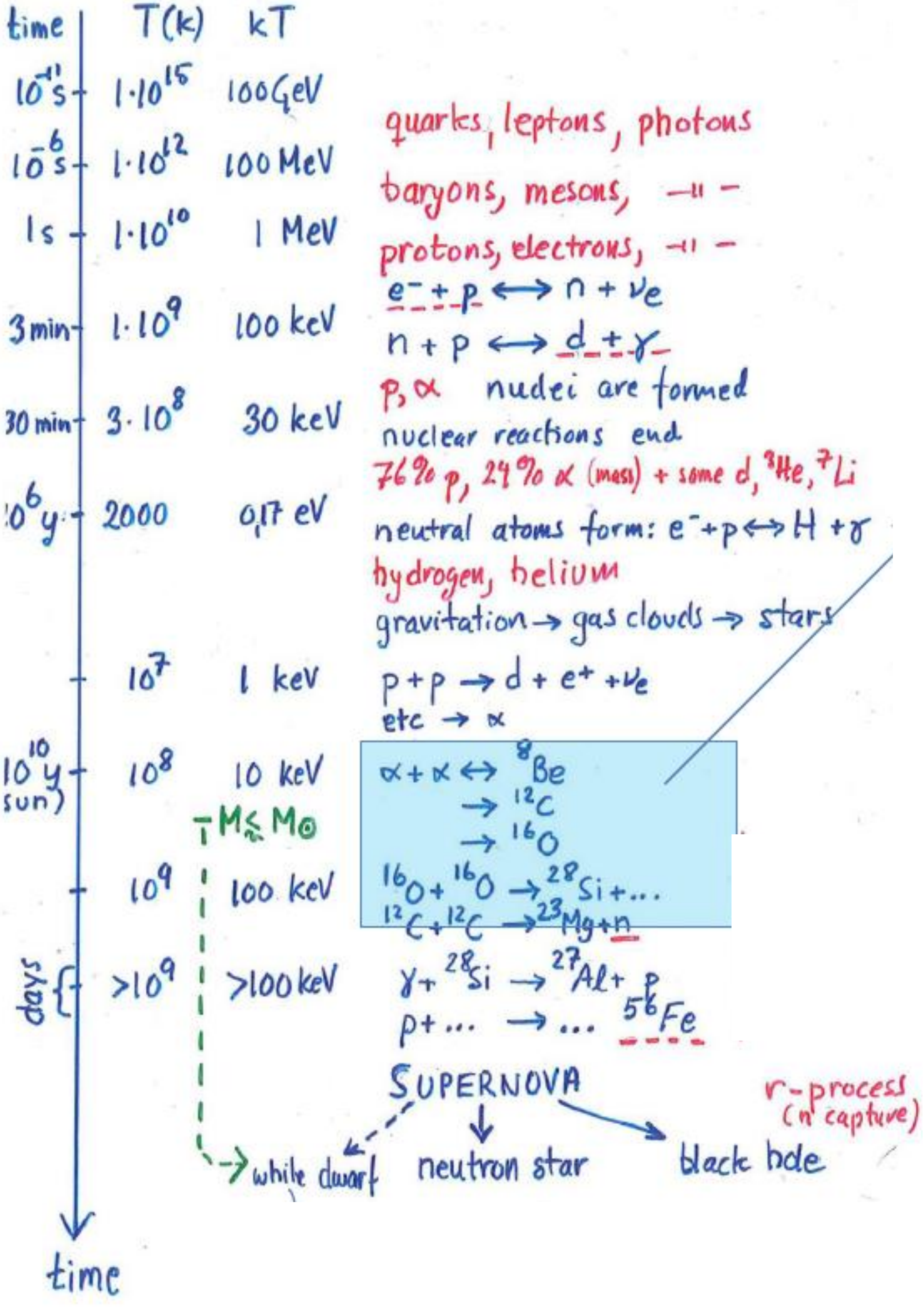
Total power of the sun: $P_\odot = 3.92\times10^{26}$ W

So $dN_p/dt = P_\odot / 6.55\ \text{MeV} \approx 3.7\times10^{38}\ \text{s}^{-1}$

Solar mass: $M_\odot = 1.99\times10^{30}$ kg, 75% H

So $N_p \approx 8.9\times10^{56}$

$N_p/(dN_p/dt) \approx 2.4\times10^{18}\ \text{s} \approx 8\times10^{10}$ years



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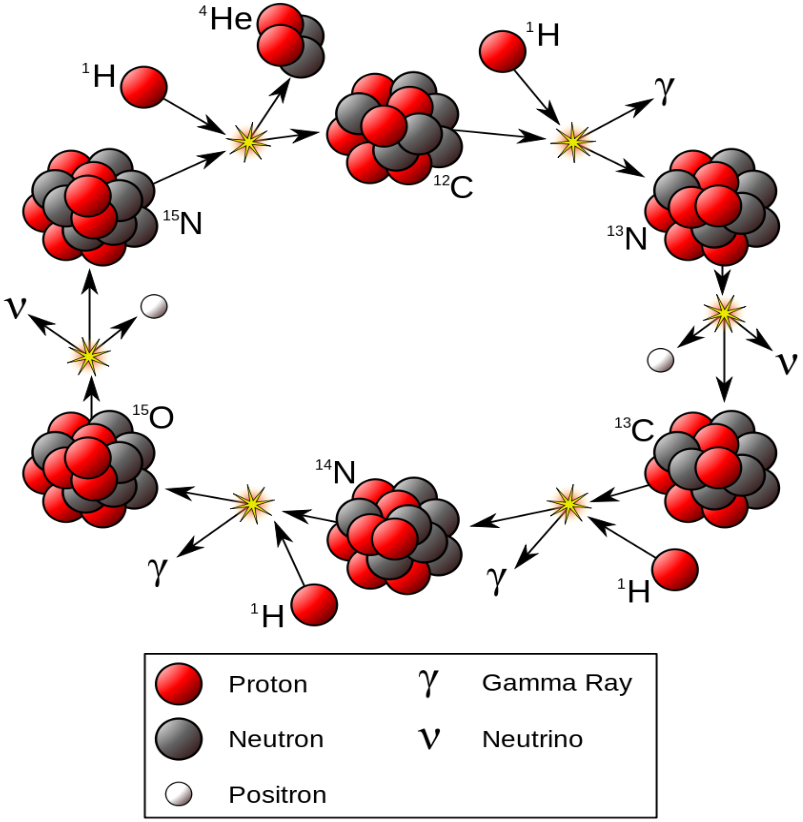
3min	$1\cdot10^{9}$	100 keV	$n + p \leftrightarrow d + \gamma$ p, α nuclei are formed
30 min	$3\cdot10^{8}$	30 keV	nuclear reactions end 76% p, 24% α (mass) + some d, ^3He, ^7Li
10^6y	2000	0,17 eV	neutral atoms form: $e^- + p \leftrightarrow H + \gamma$ hydrogen, helium gravitation → gas clouds → stars
	10^7	1 keV	$p + p \to d + e^+ + \nu_e$ etc → α
10^{10}y sun)	10^8	10 keV	$\alpha + \alpha \leftrightarrow {}^8Be$ $\to {}^{12}C$ $\to {}^{16}O$
	$M \lesssim M_\odot$		
	10^9	100 keV	${}^{16}O + {}^{16}O \to {}^{28}Si + \dots$ ${}^{12}C + {}^{12}C \to {}^{23}Mg + n$
days	$>10^9$	>100keV	$\gamma + {}^{28}Si \to {}^{27}Al + p$ $p + \dots \to \dots$ ${}^{56}Fe$

SUPERNOVA

white dwarf   neutron star   black hole

r-process (n capture)



Recap

- Big bang creates ^{1}H,^{4}He and some ^{3}Li
- Massive stars create all elements (in a fusion process) from ^{7}Li to ^{56}Fe.
- Once ^{56}Fe is produced no more energy can be gained → stellar core collapse → Core Collapse Supernova
- 99% of gravitational energy radiated in neutrinos
- Remnant: white-dwarf, neutron star or black hole

Neutron star properties

- 1.2-2 M_{solar}
- R= 8-13 km
- Densities: ~10^{14} g/cm^3 → nuclear matter
- White dwarfs: supported by ”electron-degeneracy pressure”
- Neutron stars: supported by ”neutron-degeneracy pressure”

Electron degeneracy pressure

- Pauli principle: electrons can not occupy identical states.
- Chandrasekar limit: for M < 1.4 M_{solar} electron-degeneracy pressure halts collapse → white dwarf.

$$P_g = -\frac{\partial E}{\partial V} = -\frac{1}{5} G (N M_N)^2 \left(\frac{4\pi}{3}\right)^{\frac{1}{3}} V^{-\frac{4}{3}}$$

Fermi-pressure:
P dV = dE_Fermi

$$P_e = \frac{\pi^3 \hbar^2}{15 m_e} \left(\frac{3 N_e}{\pi}\right)^{\frac{5}{3}} V^{-\frac{5}{3}}$$

Neutron degeneracy pressure

- Above the Chandrasekar limit, the gravitation collapse can continue and form neutron stars

- How?

Neutron star formation

$\Delta p \, \Delta x \geq \hbar$

$p_{min} \geq \hbar / \Delta x$

$\Delta x \sim n^{-\frac{1}{3}}$

$p_F \geq \hbar \, n^{\frac{1}{3}}$

$\rightarrow E = \frac{p_F^2}{2M} \sim n^{\frac{2}{3}}$ non relativiste

$\approx pc \sim n^{\frac{1}{3}}$ relativist.

Neutron star formation

$n \rightarrow p + e^- + \bar{\nu}_e$

Neutron star formation

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$$m_n > m_p + m_e \qquad \approx 1.5\ \mathrm{MeV}$$

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but Fermi energy: (scales as density$^{\frac{2}{3}}$!)

Neutron star formation

$n \to p + e^- + \bar{\nu}_e$

$m_n > m_p + m_e$ ≈ 1.5 MeV

but Fermi energy: (scales as density$^{2/3}$!)

$m_n \overset{!}{<} m_p + m_e + E_F^e$

$\rho(E_F) \approx \rho(511\,\text{keV}) \approx 10^6\,g/cm^3$

for densities larger than this
you have to use $n^{1/3}$ scaling.

Neutron star formation

$\Rightarrow \rho \sim 10^{7} g/cm^3 \quad (E_F \sim 1.5 \text{ MeV})$

$p + e \rightarrow n + \nu_e$ becomes energetically favoured.

$\Rightarrow$ more and more neutron rich nuclei

Neutron star formation

At some point, nuclei become so neutron-rich that the neutrons separate from the nucleus. (neutron drip) ⇒

$$\sim O\left(10^{11} \frac{g}{cm^3}\right)$$

⇒ nuclei "swimming" in the sea of neutrons.

We are still far above the densities observed in neutron stars → collapse continues → as you confine neutrons in a volume their Fermi-energy increases → this Fermi-energy will (a) counteract the gravitational collapse and (b) allow neutrons to form new types of particles (e.g. hyperons) → neutron-only stars is a simplification.

Neutron star

- If we assume neutron stars to be bound nuclear matter, we should be able to calculate how large mass ”nucleus” you need to get a bound system → Bethe-Weizsäcker formula

Look at SEMF, case $Z=0$, $A=N$

$$B = a_v A - a_s A^{2/3} - a_c \cdot \overset{\overset{\text{no charge}}{\swarrow}}{0} - a_a \cdot \frac{(A/2)^2}{A}$$

add a gravitational term $B_G = \frac{3}{5} G \frac{M^2}{R}$

$M = A \cdot M_n$, $R = R_0 A^{1/3}$ $\Rightarrow M^2/R = \frac{A^2 M_n^2}{R_0 A^{1/3}} = \frac{M_n^2}{R_0} \cdot A^{5/3}$

$$B = a_v A - a_s A^{2/3} - a_a \frac{1}{4} A + \frac{3}{5} \frac{G}{R_0} M_n^2 A^{5/3}$$

$\uparrow$ small in comparison. when A large.

$$= \underbrace{\left(a_v - \frac{1}{4} a_a\right)}_{\sim 7{,}73 \text{ MeV}} A + \underbrace{\frac{3}{5} \frac{G}{R_0} M_n^2}_{} A^{5/3}$$

$$\frac{3}{5} \frac{6{,}67 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2}{1{,}2 \cdot 10^{-15} \text{ m}} \cdot \left(1{,}67 \cdot 10^{-27} \text{ kg}\right)^2 = 9{,}3 \cdot 10^{-50} \text{ J}$$

$$= 5{,}8 \cdot 10^{-37} \text{ MeV}$$

We see that $B < 0$ (unbound) until A becomes

(solve for $B=0$):

$$-7{,}73\,A + 5{,}8\cdot 10^{-37} A^{5/3} = 0$$

$$-7{,}73 + 5{,}8\cdot 10^{-37} A^{2/3} = 0 \Rightarrow A = \left(\frac{7{,}73}{5{,}8\cdot 10^{-37}}\right)^{3/2} =$$

$$\approx 10^{56}$$

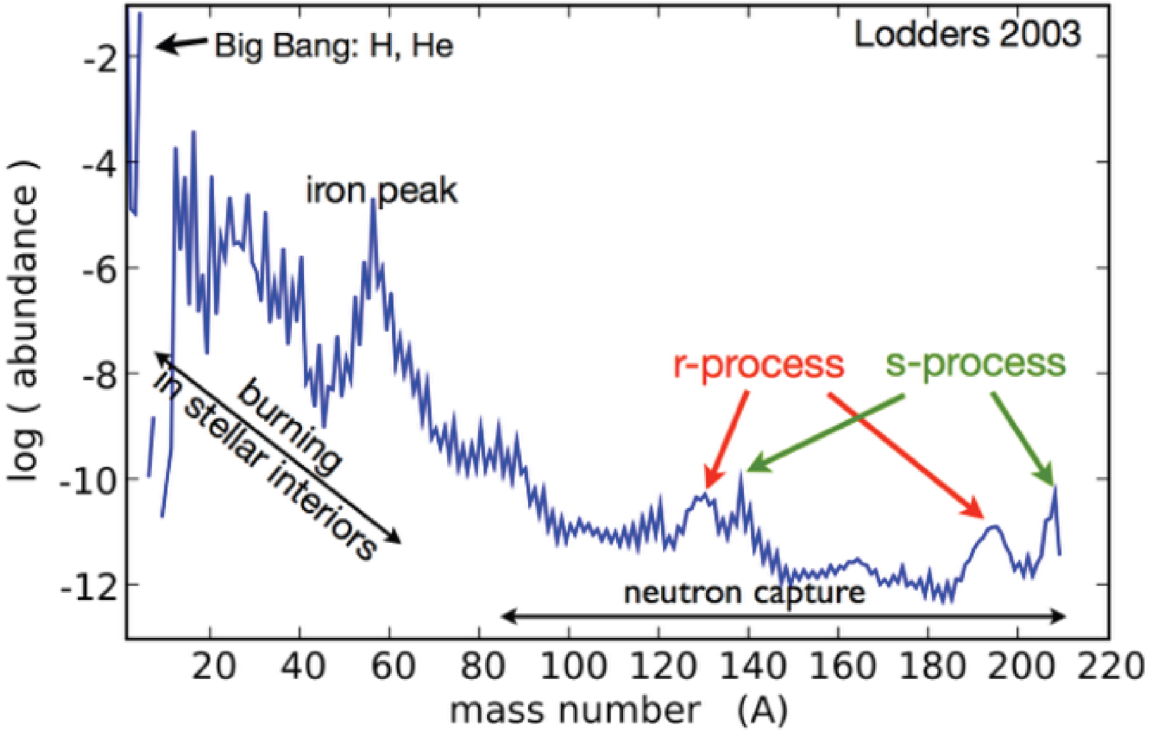
So, the SEMF (parameters fitted to nuclei with $A = 20$ to ~ 240) predicts nuclei with only neutrons to be bound if $A \gtrsim 10^{56}$

s-process/r-process

- The standard process of heavy-element nucleosynthesis (beyond A ~ 60) is the so called s-process (slow neutron capture)

- ***r***apid neutron capture (r-process) occurs in environments with high density of free neutrons (temperature dependent) (10^{24}/cm^3)

Solar abundance: nucleo-synthesis



two neutron capture processes in nature:

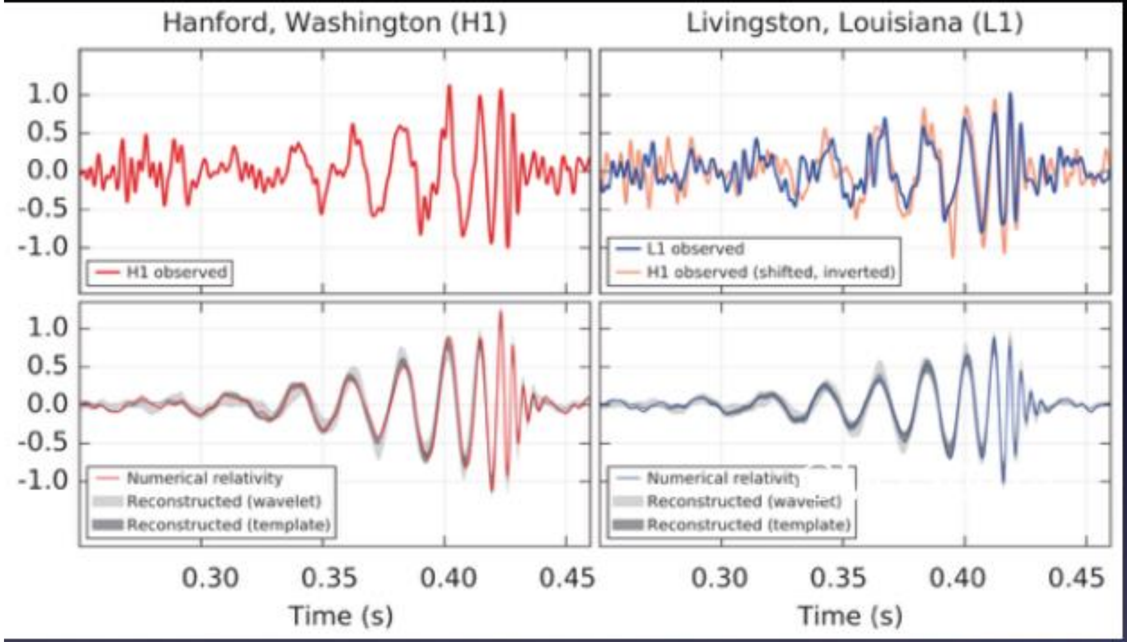
- rapid n-capture (“r-process”)
- slow n-capture (“s-process”)

Do we have environments that provide neutron-rich environments, where the r-process could happen?

Neutron stars seem a very good candidate.

What does the 2017 detection of gravitational waves have to do with the nucleosynthesis of heavy elements?

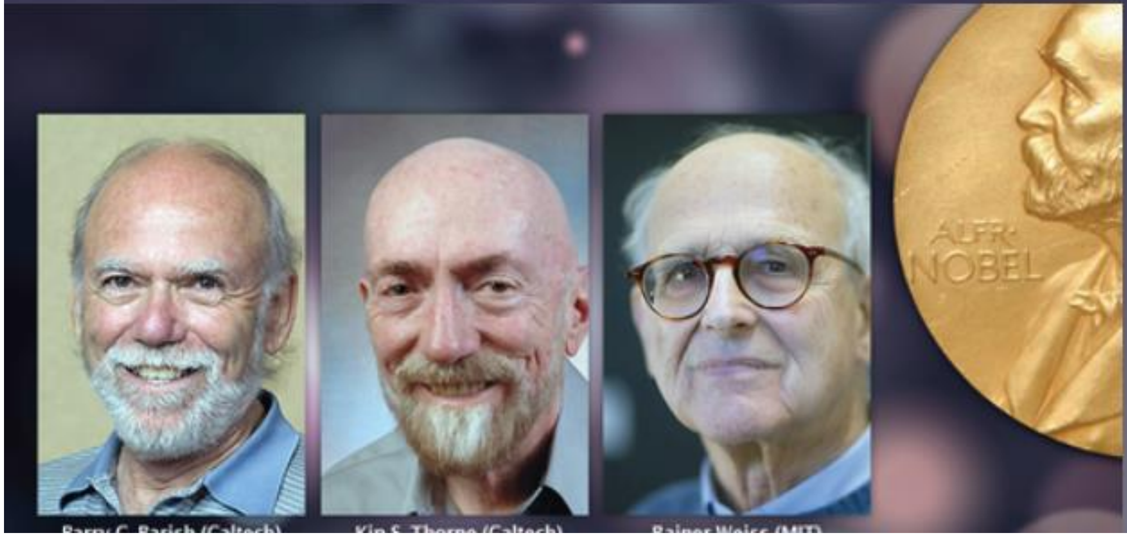
GW+EM 170817: the first multi-messenger observation of a neutron star merger



first detection GW150914: a merging black hole binary system with (36 + 29) M⊙

+

four more BH binaries



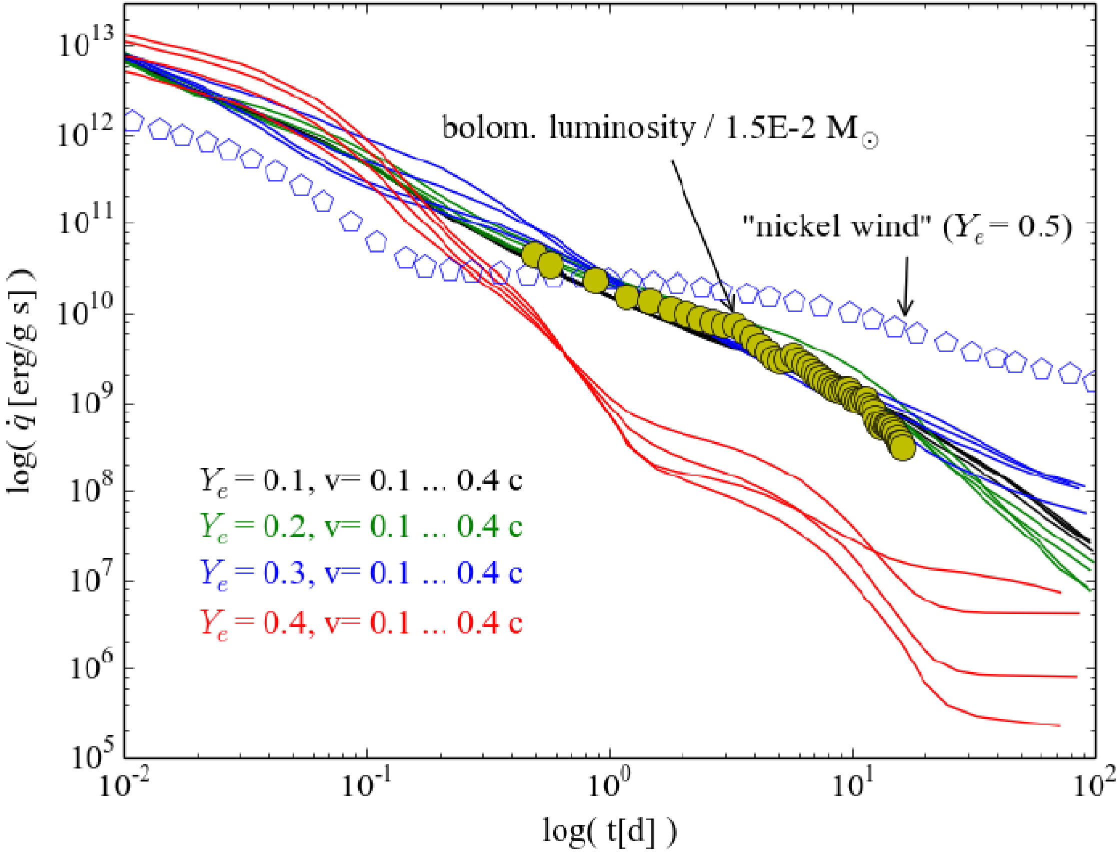
2017 Nobel Prize in Physics

Neutron star merger

- If we could identify a neutron star merger and observe it, could we learn about the r-process?

- Observation of electromagnetic radiation from the radioactive decay of r-process elements! In the best case, we’ll have the *bolometric lightcurve*

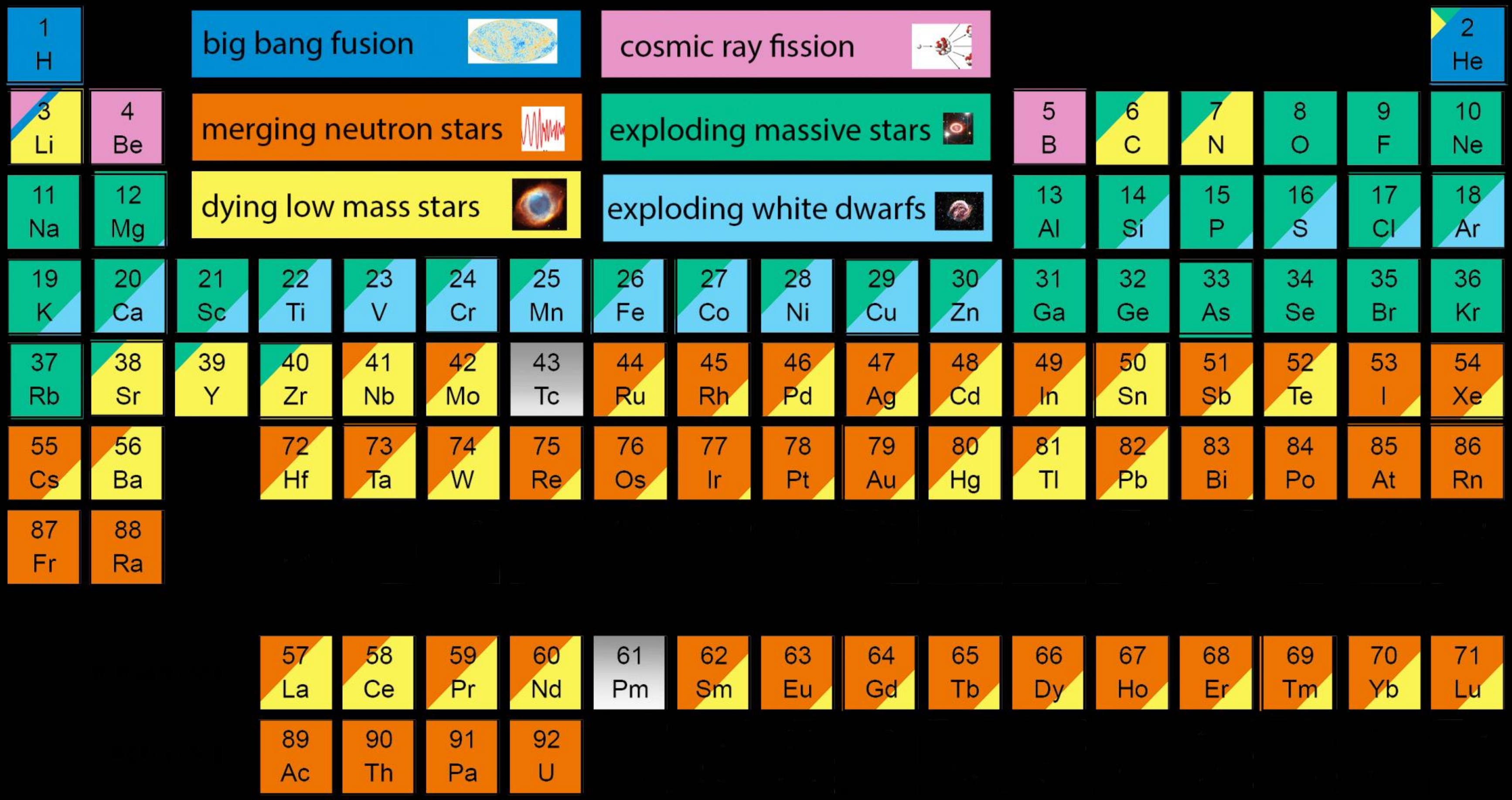
Gravitational wave detection and the synthesis of heavy elements



S. Rosswog et al. (2017)

$$\text{“electron fraction”} \quad Y_e = \text{”} \frac{\#\ \text{protons}}{\#\ \text{nucleons}} = \frac{\#\ \text{electrons}}{\#\ \text{nucleons}} \text{”}$$

The Origin of the Solar System Elements



big bang fusion

cosmic ray fission

merging neutron stars

exploding massive stars

dying low mass stars

exploding white dwarfs

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra																
			57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
			89 Ac	90 Th	91 Pa	92 U											

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Summary of today's lecture

- We discussed nucleosynthesis: fusion and neutron stars.
- Most elements up to iron are produced by fusion in massive stars.
- Most elements above iron require som kind of neutron capture: either slow or rapid neutron capture.
- Neutron star collisions as site of significant rapid neutron capture have just been recently proven.