Summary lecture 3

- We have covered the basics of nuclear models (Fermi-Gas-model, Shell model, Collective model)
- The shell model was able to predict magic numbers after (a) Woods-Saxon and (b) spin-orbit coupling was introduced.
- Excited states → vibrational/rotational degrees of freedom.

The shell model

- Assume spherical nuclei and potential
- Consider spin and angular momentum quantum numbers
- Properties of the nucleus determined only by the unpaired (”valence”) particle.

Spin orbit coupling

$$V_{\text{total}} = V_{\text{central}}(r) + V_{\ell s}(r)\mathbf{L} \cdot \mathbf{S},$$

The total potential is the Woods-Saxon potential with an added term that provides a coupling between the nucleons spin and it's angular momentum.

Differences between atomic and nuclear shell models

- Spin-orbit interaction much stronger in nuclei
- Opposite sign to the atomic case (spin-orbit coupling is attractive)
- Spin-orbit coupling not magnetic, but rather inherent to nuclear force.

The collective model

- The collective model combines the shell model with the liquid drop model.
- The outer valence nucleons are viewed as the surface molecules of a liquid drop
- Asphericity: vibrations and rotations can lead to additional excited states.

Lecture 4: Decays revisited

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Decay's considered

- α decay (strong force)
- β decay (weak force)
- γ decay (electro-magnetic)

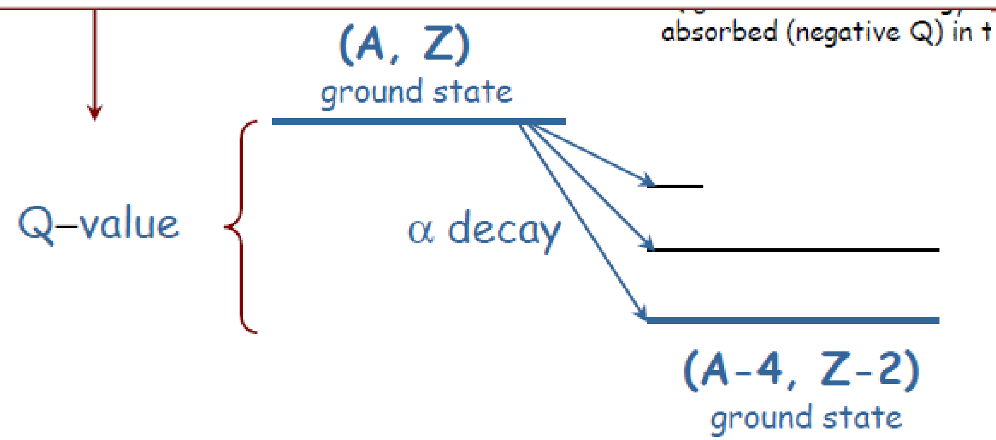
α decay

(typically for nuclei $82 < Z \leq 92$)

$$(A,Z) \rightarrow (A-4,\ Z-2) + \alpha$$

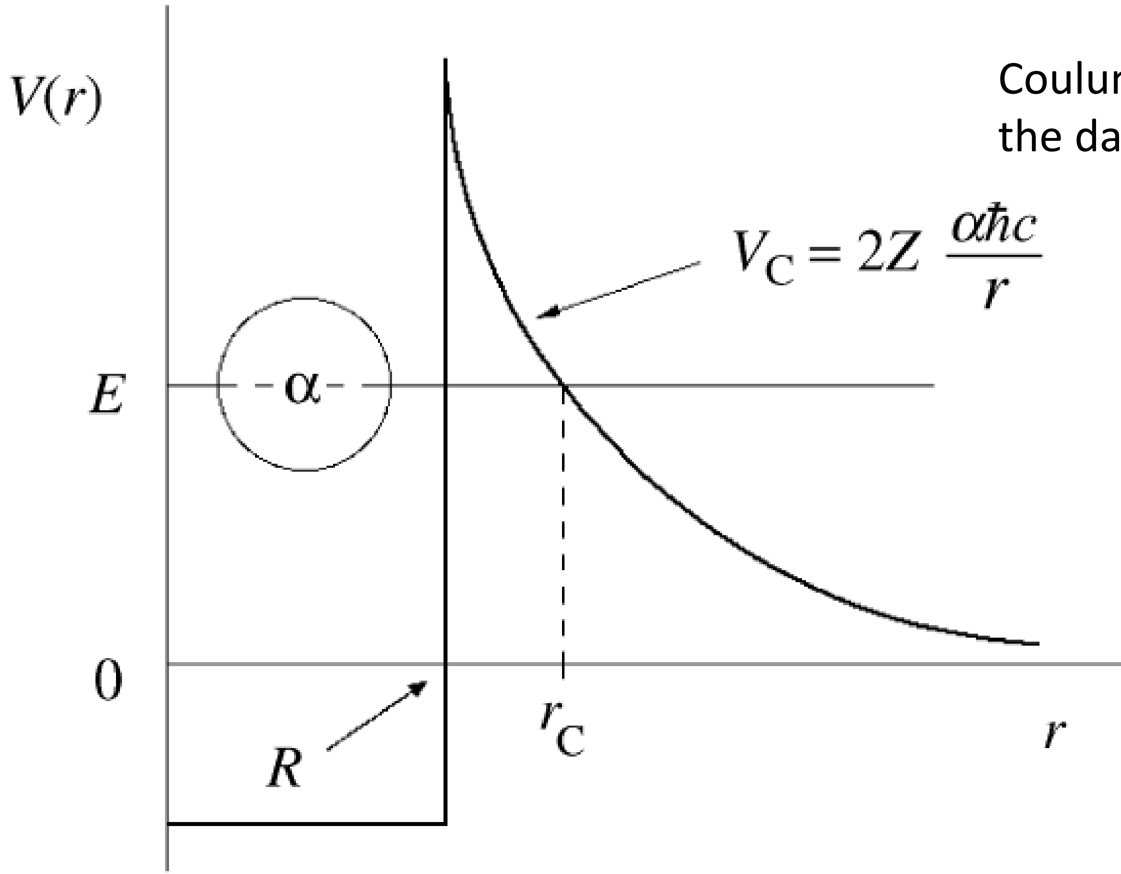
Needs to be energetically allowed:

$$Q_\alpha = [M(A,Z) - M(A-4,\ Z-2) - M(^4He)]c^2 > 0$$



Large spread in half-lives: ns ~ 10^{17} years. Especially the longer half-llives motivate modelling as a tunneling processa tunneling process.

α decay: tunneling



Coulumb potential of the daughter nucleus

How does this relate to the large spread o

α decay: transition probability

$$T = e^{-G}$$

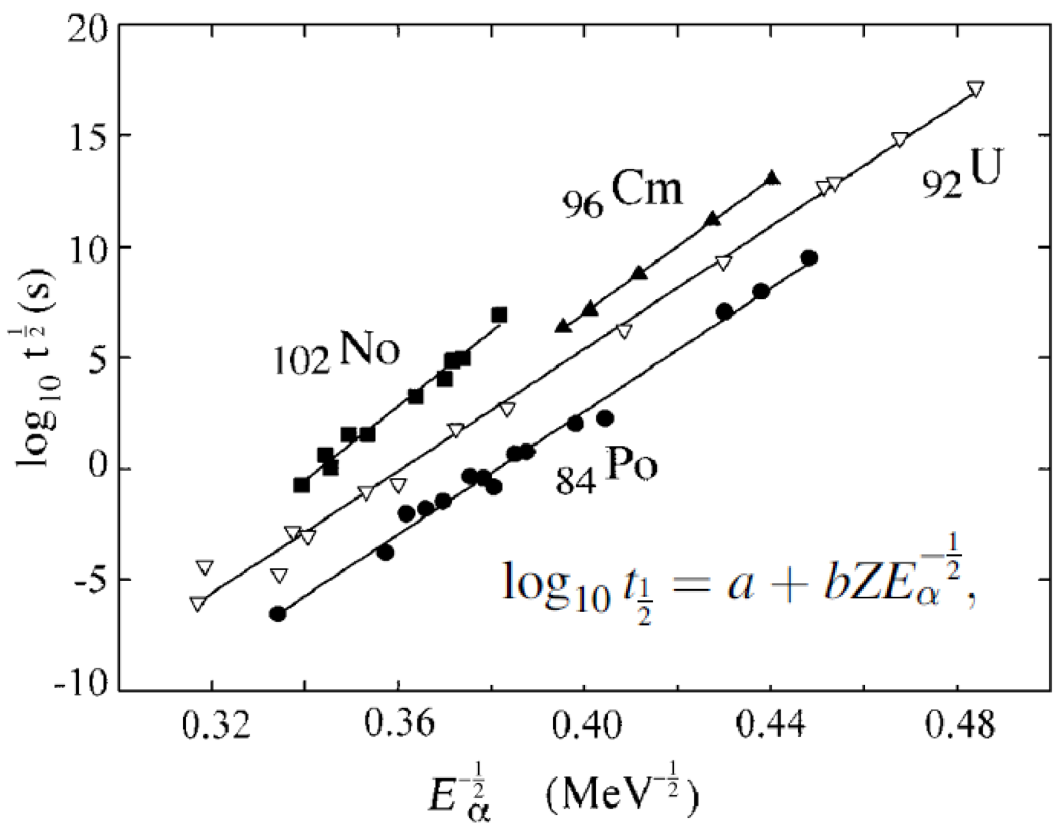
- Gamow factor:

$$G = \frac{2}{\hbar} \int_{R}^{r_C} [2m|V_C(r) - E_\alpha|]^{1/2} dr,$$

- Derived by considering the Coulumb-barrier as a succession of thin barriers of height V(r).

α decay: Geiger Nutall relation

The transition probability (which is a function of the Coulomb barrier height and α particle energy)can be used to find a relation between lifetime (~ transition probability) and energy of the α particle (~Q value).



Geiger and Nutall (1911)
Gamows explanation: (1928)

What nuclear model did we use to arrive at this relation?

G. Gamov (1904-1968) in W.H. Braggs group 1931 (Cambridge)

β decay: Transition rates and spectrum

- The transition rates are determined by the ”Fermi Golden Rule”

$$\omega = \frac{2\pi}{\hbar}|M_{fi}|^2 n(E),$$

Matrix element

Density of states

β decay: Transition rates and momentum spectrum

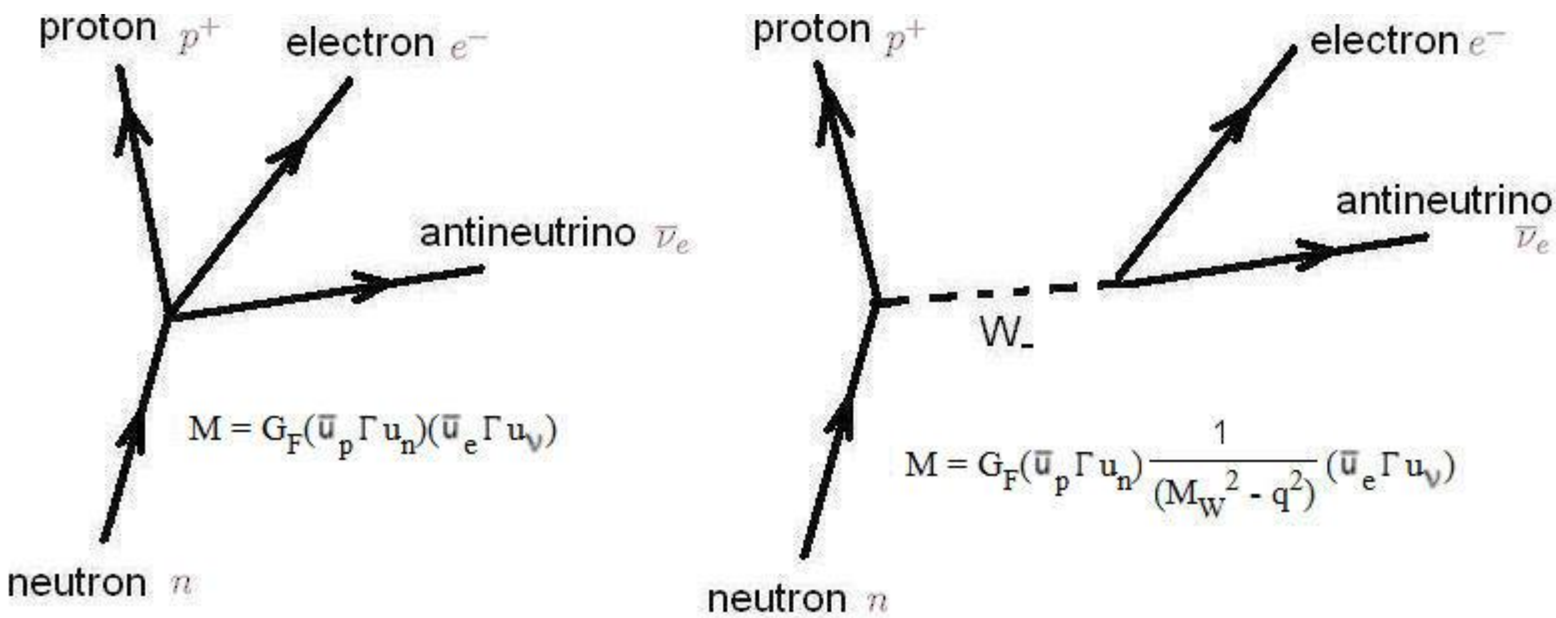
$$M_{fi} = \int \Psi_f^*(g\hat{\mathbf{O}})\Psi_i \, d^3\mathbf{x},$$

- Type of interaction reflects in transformation of operator. In a scalar interaction the operator will be a simple an integral over the potential that affects the state.

$$= \int \psi_f^* V \psi_i \, dv$$

Name	Symbol	Current	Number of components	Effect under Parity
Scalar	S	$\overline{\psi}\psi$	1	+
Vector	V	$\overline{\psi}\gamma^\mu\psi$	4	(+,-,-,-)
Tensor	T	$\overline{\psi}\sigma^{\mu\nu}\psi$	6	
Axial Vector	A	$\overline{\psi}\gamma^\mu\gamma^5\psi$	4	(+,+,+,+)
Pseudo-Scalar	P	$\overline{\psi}\gamma^5\psi$	1	-

Fermi-theory



a. Fermi's 4-point Interaction, 1934 b. Weak Interaction mediated by boson, 1938

Fermi theory

- We consider a point-interaction → extremely short range Yukawa potential → constant

$$M_{fi} = \frac{G_F}{V},$$

Fermi Coupling constant

Normalisation of wave function

$$G_F = \frac{4\pi(\hbar c)^3 \alpha_W}{(M_W c^2)^2}.$$

$$G_F/(\hbar c)^3 = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}.$$

Measured in muon decay

Fermi theory

- Since the matrix element is a constant, the crucial quantity is the density of the states factor

β decay: spectrum

final states of electron :

$$dn_e = \frac{4\pi p^2 dp V}{\hbar^3}$$

final states of ν :

$$dn_\nu = \frac{4\pi q^2 dq V}{\hbar^3}$$

$$\Rightarrow \quad d\lambda = \frac{2\pi}{\hbar} g^2 |M_{fi}|^2 (4\pi)^2 \frac{p^2 dp\, q^2 dq}{\hbar^6 \quad dE_f}$$

β decay: spectrum

$$\frac{m_\nu}{m_e} < 10^{-7} \quad \rightarrow \quad m_\nu = 0$$

$$qc = E_\nu \quad \rightarrow \quad E_f = E_e + E_\nu = E_e + qc$$

$$\frac{dq}{dE_f} = \frac{1}{c}$$

electrons with momentum $p \pm dp$

$$\Rightarrow N(p)\,dp = \text{const} \cdot p^2 q^2 \, dp$$

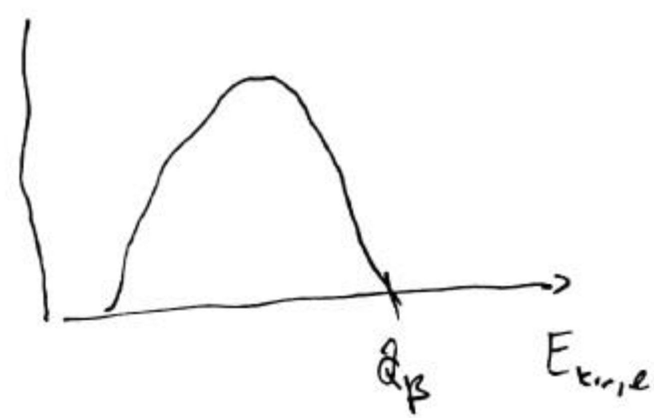
$$q = \frac{Q_\beta - E_{kin,e}}{c} = \frac{\left(Q_\beta - \sqrt{p^2c^2 + m_e^2c^4} - m_e c^2\right)}{c}$$

$$N(p) = \frac{\text{const}}{c^2}\left[Q_\beta - \sqrt{p^2c^2 + m_e^2c^4} + m_e c^2\right]^2 p^2$$

Now we rewrite this equation in terms of $E_{kin,e}$

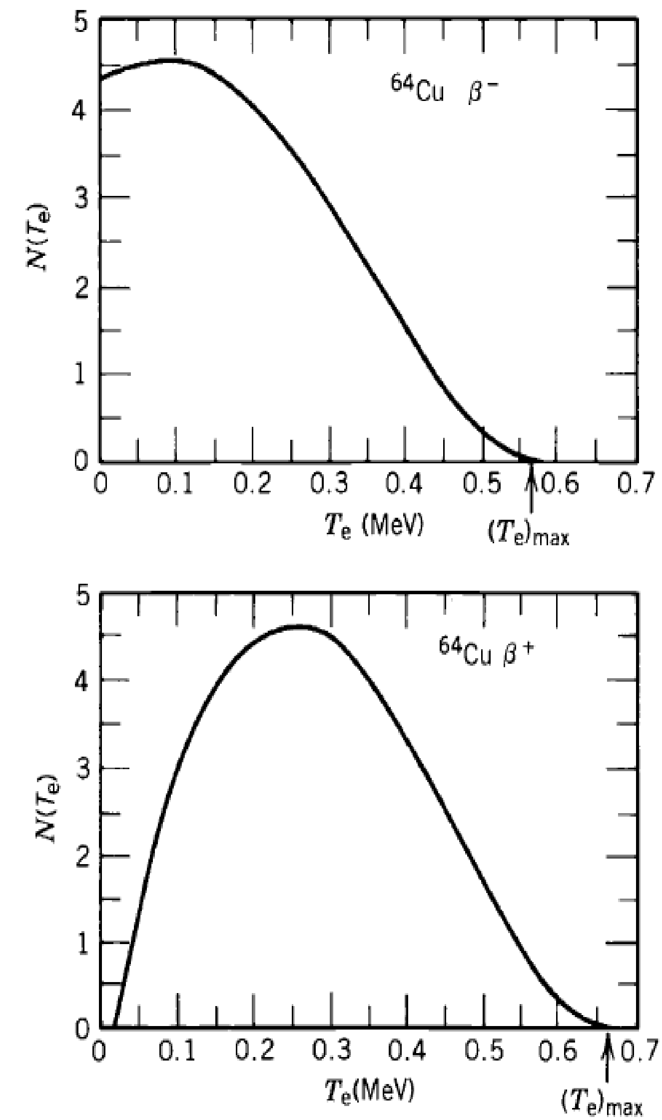
$$N(E_{kin,e}) = \frac{const}{c^5}\left(E_{kin,e}^2 + 2E_{kin}m_ec^2\right)^{\frac{1}{2}} \cdot \left(Q_\beta - E_{kin,e}\right)^2\left(E_{kin,e} + m_ec^2\right)$$

$\Rightarrow N(E_{kin,e}) = 0$ for $E_{kin,e} = 0$ and for $E_{kin,e} = Q_\beta$



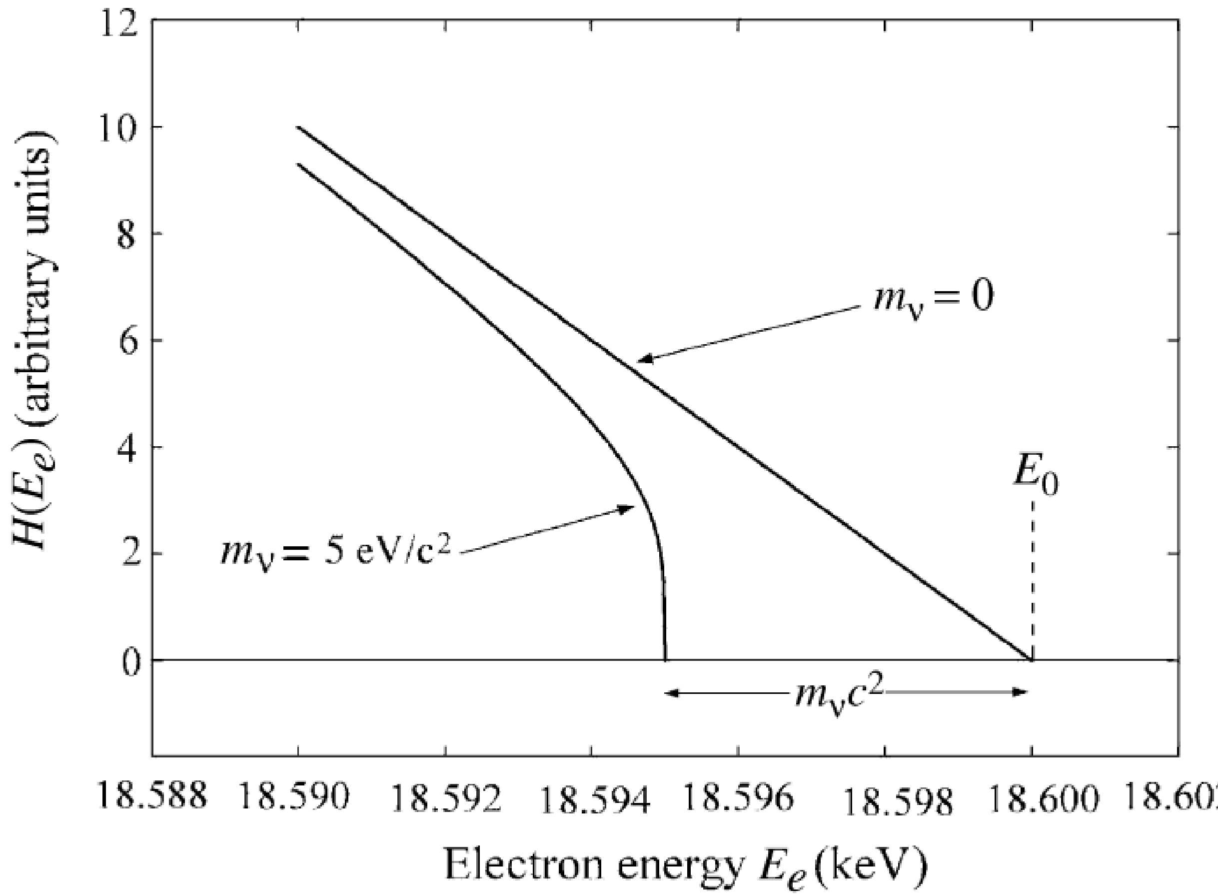
β decay

- Neglected Coulomb interaction→ Fermi "screening" –F(Z_d,E_{kin})



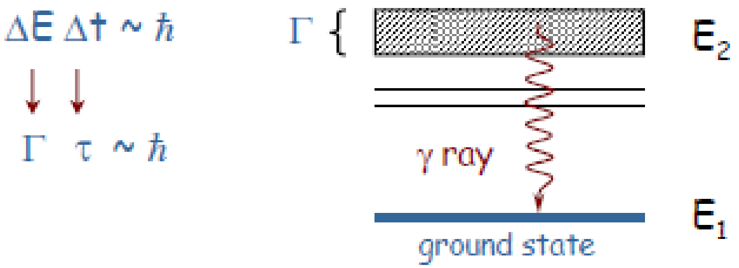
β decay and the neutrino: the Kurie plot

$$H(E_e) \equiv \left[\left(\frac{d\omega}{dp_e}\right)\frac{1}{p_e^2 K(Z, p_e)}\right]^{\frac{1}{2}} = E - E_e,$$



γ decay

- As γ decay is a electromagnetic process, total angular momentum and parity are conserved



- electric (E) transitions (oscillating charges)
- magnetic (M) transitions (varying currents)

γ decay: selection rules

- Photon carries away momentum L.

- $J_i = 0 \rightarrow J_f = 0$ is strictly forbidden, because photon is a spin-1 vector meson (minimum L ==1)

- L=1: dipole, L=2: quadrupole, L=3: octupole ..
M2: magnetic dipole radiation, E2: electric dipole

$$\mathbf{J}_i = \mathbf{J}_f + \mathbf{L}.$$

$$J_i + J_f \geq L \geq |J_i - J_f|.$$

- electric radiation: $(-1)^L$, magnetic radiation $(-1)^{L+1}$

γ selection rules

Table 7.1 Selection rules for γ emission

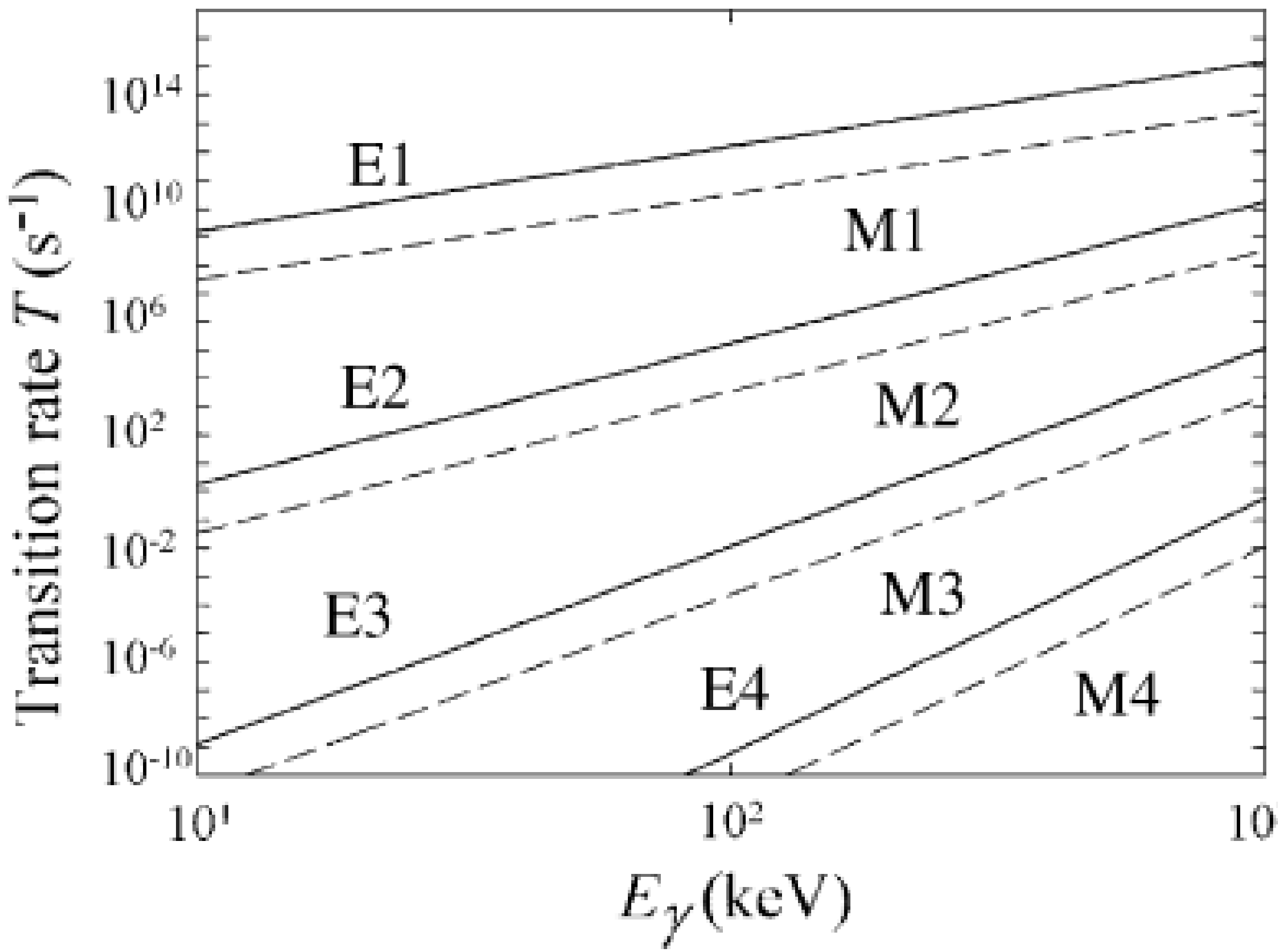
Multipolarity	Dipole		Quadrupole		Octupole	
Type of radiation	E1	M1	E2	M2	E3	M3
L	1	1	2	2	3	3
ΔP	Yes	No	No	Yes	Yes	No

γ decay: transition rates

- Transition rates can be calculated in the shell model.
- General features: rates decrease with L, electric transitions several dex. more probable than magnetic.

$$\Gamma_\gamma(\text{E1}) = 0.068 E_\gamma^3 A^{2/3}; \quad \Gamma_\gamma(\text{M1}) = 0.021 E_\gamma^3; \quad \Gamma_\gamma(\text{E2}) = (4.9 \times 10^{-8}) E_\gamma^5 A^{4/3},$$

γ decay: transition rates



Virtual γ

- Energy conversation can be violated for a very short time (Heisenberg uncertainty principle).

→**Internal conversion** (electron emitted)

and **internal pair production**

(electron-positron pair emitted)

E0 de-excitation

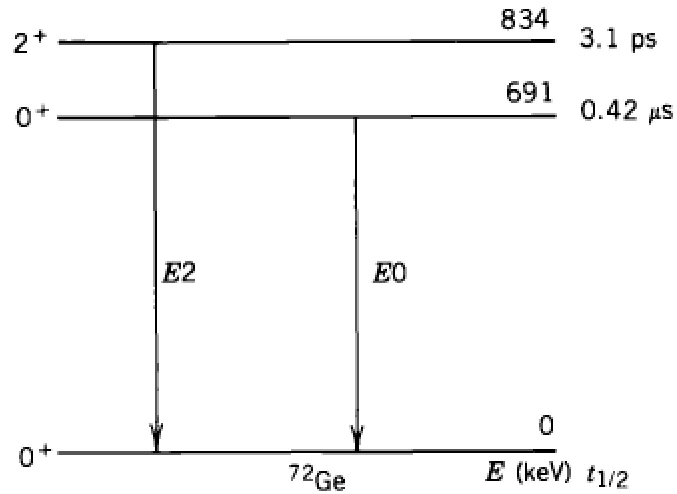


Figure 10.10 Energy levels in ^{72}Ge.

Summary of today's lecture

- We discussed α-decay theory, β-decay theory (Fermi-theory) and γ-ray theory

- We scetched the derivation of the electron spectrum for β-decay from Fermi-theory (main assumption: point-like interactions, short range Yukawa coupling, no Fermi screening), which is determined by the phase space factor.

Summary of today's lecture

- Geiger-Nutall relation for α-decay
  - relates half lives to Q-value
- Fermi-Kurie relation for β-decay
  - deviation can point to ”new” physics, e.g. neutrino mass
- Selection rules for γ decays
  - According to angular momentum and Parity conservation (n.b. internal conversion, internal pair production)