Summary of last lecture

- We had an initial discussion of decays and reactions.
- Introduced: half-life (decay-constant) and decay chains and Q-value
- Discussed initially: β decay (weak), γ decay (e.-m.), spontaneous fission and α decay (strong)

Summary of last lecture

- Is arguably the most important decay mode

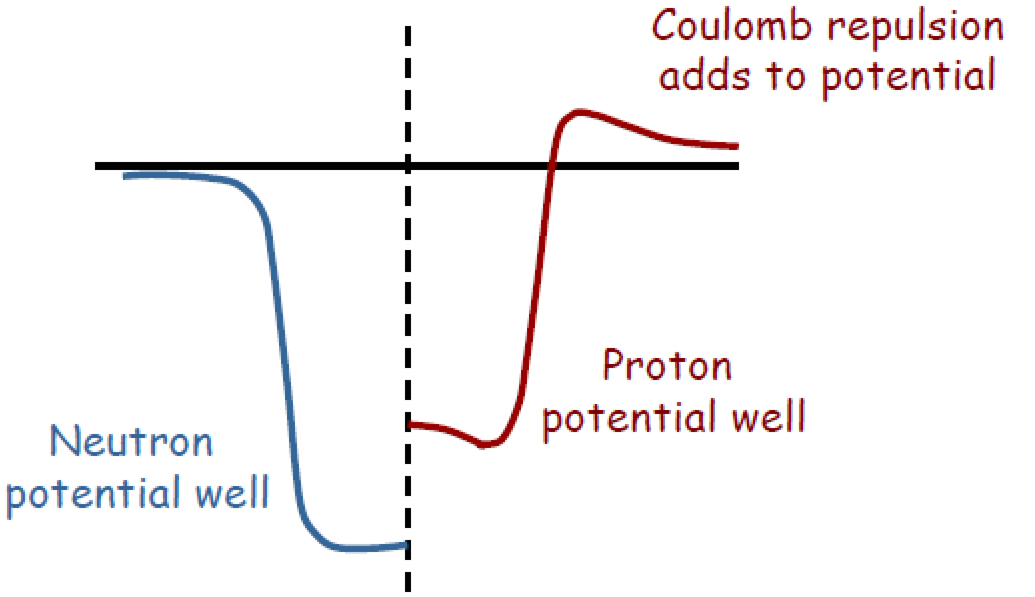
- Using the B-W formula and β-decay we can derive the valley of β stability (← mass parabolas)

Summary of last lecture

- α decay: prevalent to high mass nuclei, can be derived from B-W formula
- Spontaneous fission: can be derived from considering deviations of nuclei from spherical shape.
- Nuclear reactions: direct (like in particle physics, new particles can be found), capture (compound reactions).

Oppen issue last lecture

- Why does ^{7}Li not α-decay?
- Sum of binding energies:
  - 5.60 MeV (^{4}He), 3.01 MeV (^{3}He)



Lecture 3: Nuclear models

Jan Conrad

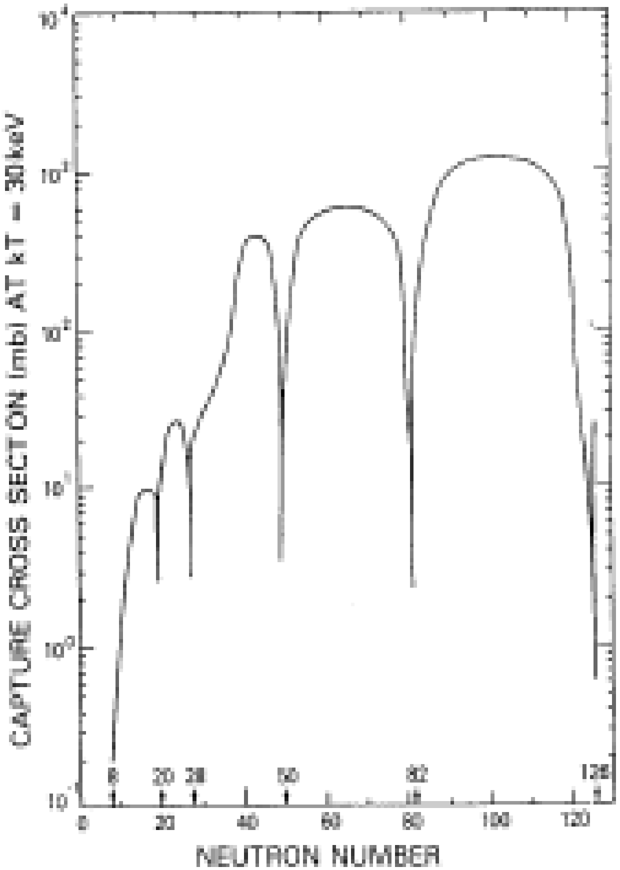
The story

- We introduced the Bethe-Weizsäcker formula which broadly was able to explain the binding energy of nuclei and explain basic phenomenology of β-decay and α-decay.

Ideally:

- A nuclear model should be able to predict the coefficents of the B-W formula
- A nuclear model should be able explain phenomena that the B-W formula does not explain.

Evidence for shell effect



Magic numbers and other discrete phenomena.

How to improve?

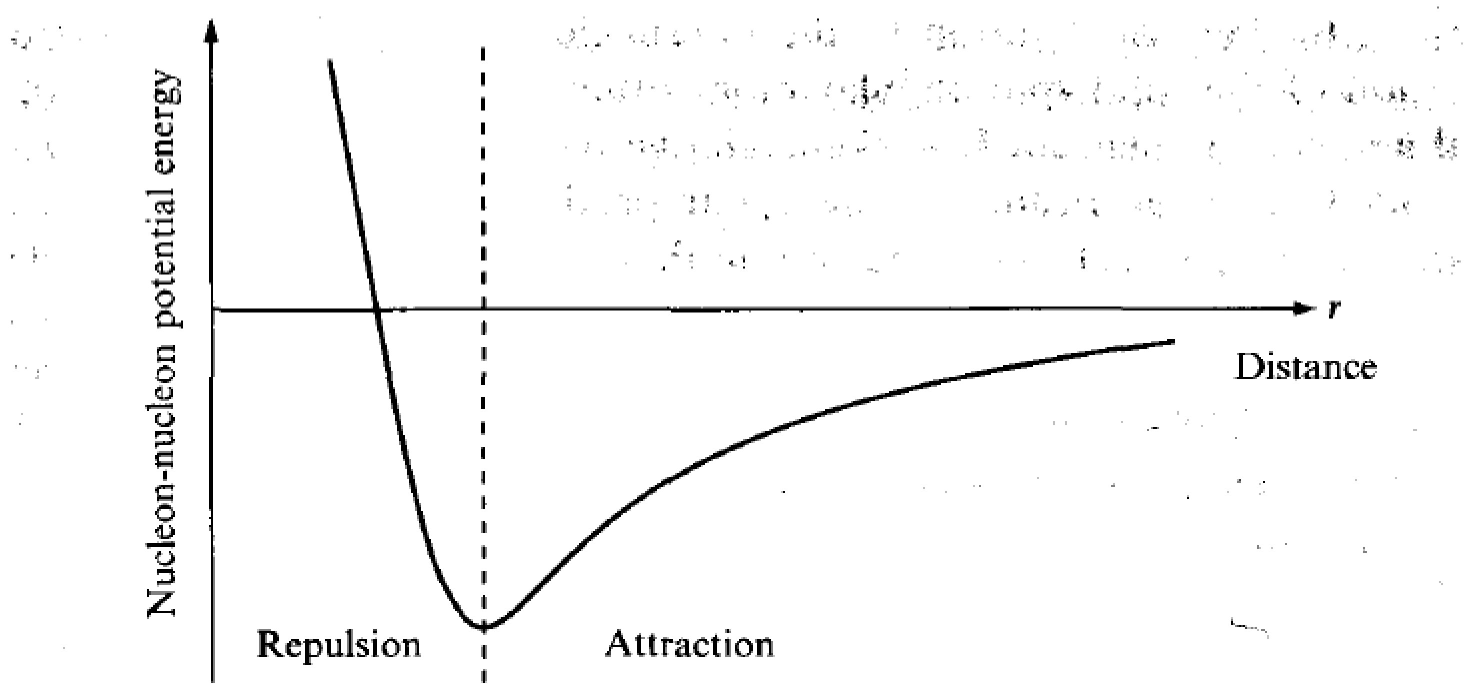
- Consider the quantum-mechanical states of a nucleus in a potential.
- The simplest approach is to fill states in a box-like potential well (neutrons → protons with some modification) applying only Pauli-principle constraints → **Fermi-gas model**

Did the B-W formula make use of the nuclear potential and quantum mechanics?

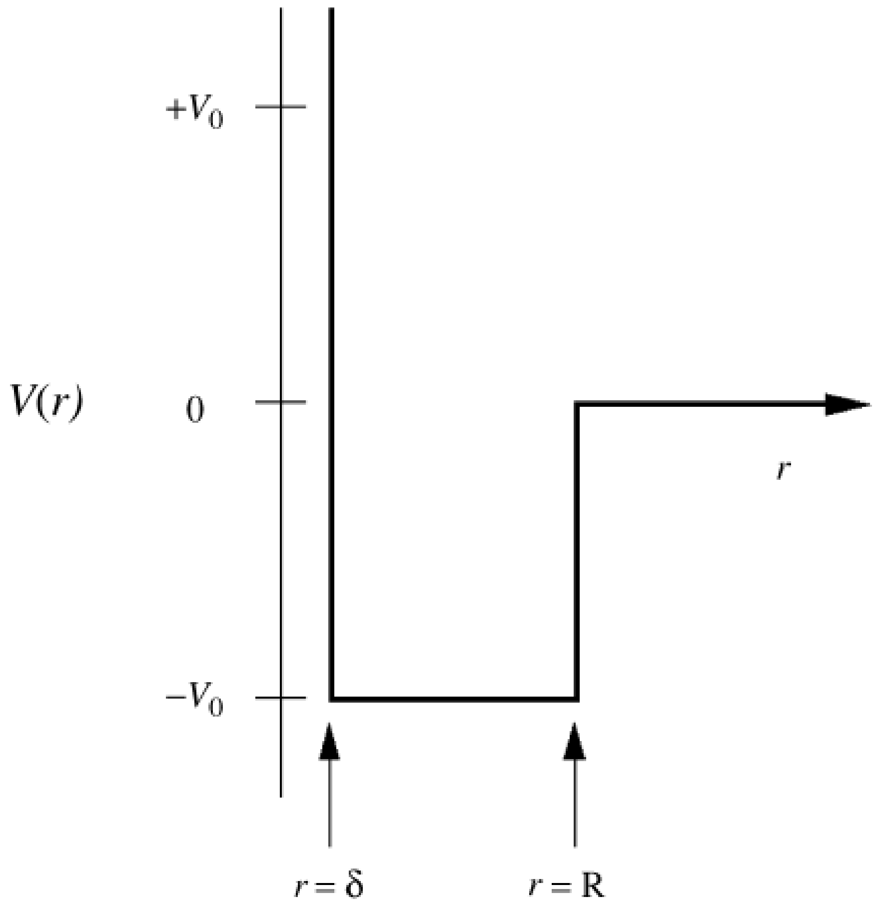
How to improve?

- The next step is to include other quantum numbers accounting for the spin and angular momentum dependence of the nuclear force → **shell model**
- Finally, we’ll have to account for multi-particle correlated states (instead of viewing the nucleuons as independent) → **collective (excitations) model.**

Nuclear potential



First approximation: box



The Fermi-gas model

Protons and neutrons move freely inside the volume or the nucleus, subject to the Pauli principle → can predict the depth of the nuclear potential and the contribution of the assymetry term to the B-W formula

$$n(p)\mathrm{d}p = \mathrm{d}n = \frac{4\pi V}{(2\pi\hbar)^3}p^2\mathrm{d}p,$$

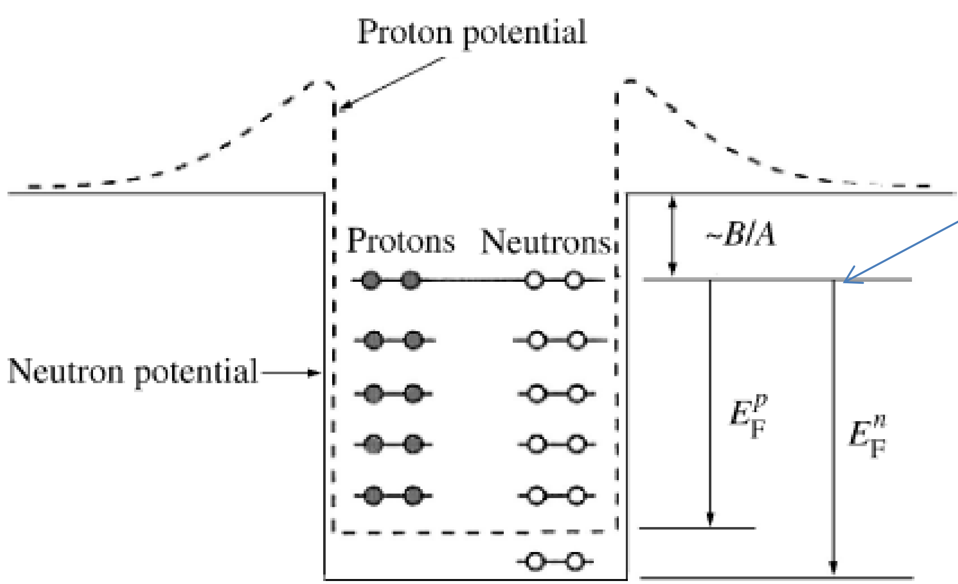


Figure 7.2 Proton and neutron potentials and states in the Fermi gas model

Fermi-level: $p_F = (2ME_F)^{1/2}$

$$N = \frac{V(p_F^n)^3}{3\pi^2\hbar^3} \quad \text{and} \quad Z = \frac{V(p_F^p)^3}{3\pi^2\hbar^3}$$

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A,$$

$$E_F = \frac{p_F^2}{2M} \approx 33\,\text{MeV}.$$

$$V_0 = E_F + \tilde{\boldsymbol{B}} \approx 40\,\text{MeV}.$$

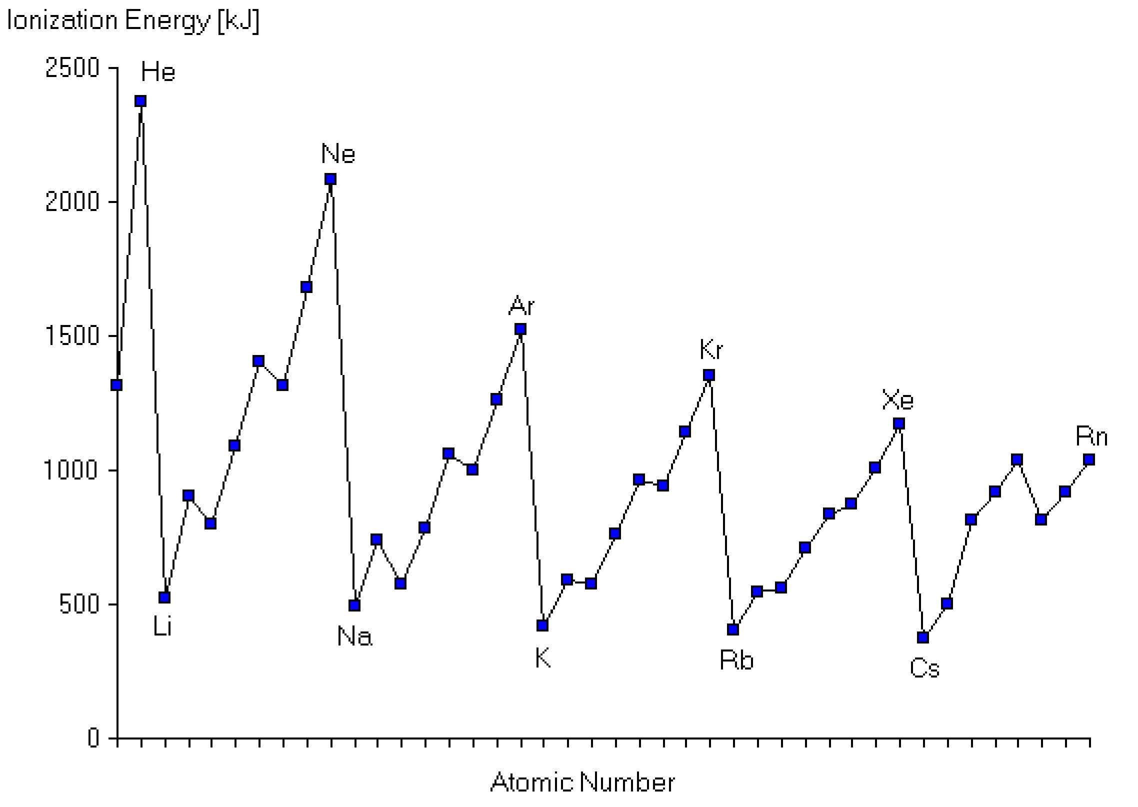
with B ~ 7-8 GeV

Would this nucleus β-decay?

The shell model

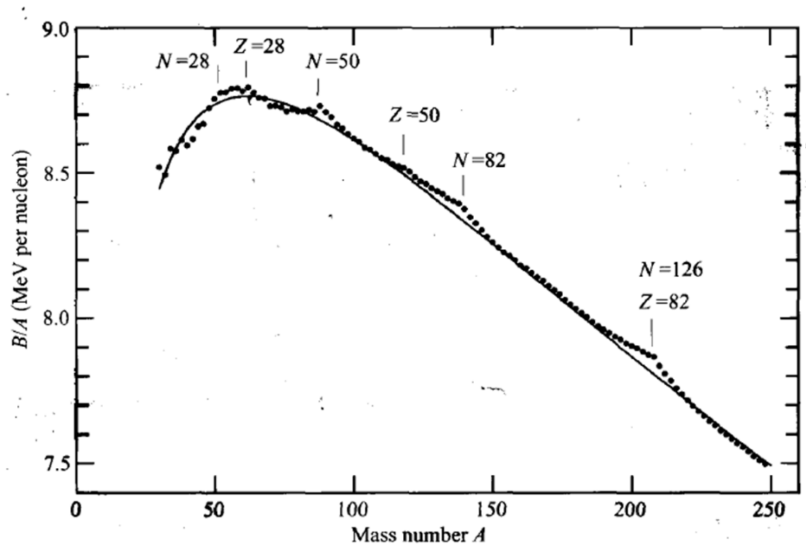
- Assume spherical nuclei and potential
- Consider spin and angular momentum quantum numbers
- Properties of the nucleus determined only by the unpaired (”valence”) particle.

Shell structure in the atom



- Chemical properties determined by valence electrons
- Closed shell structures especially stable (noble gases).

Again… magic numbers



Assumption: neutrons and protons move freely in the potential well
→ V(r) determines energy levels

→ Fill states according to quantum statistics → reproduce magic numbers: 2, 8, 28, 50 ….

→ Nuclei with magic numbers in both neutrons and protons separately → doubly magic numbers, e.g. ^{208}Pb (82p, 126n)

Quantum numbers

- As the nucleons are fermions the have spin, s=½.

$$m_s = \pm 1/2.$$

- In addition we should consider the angular momentum ℓ

$$m_\ell = -\ell, -\ell+1, \ldots, 0, 1, \ldots, \ell-1, \ell.$$

- So unless we have a spin-orbit splitting, the number of (degenerate) states is:

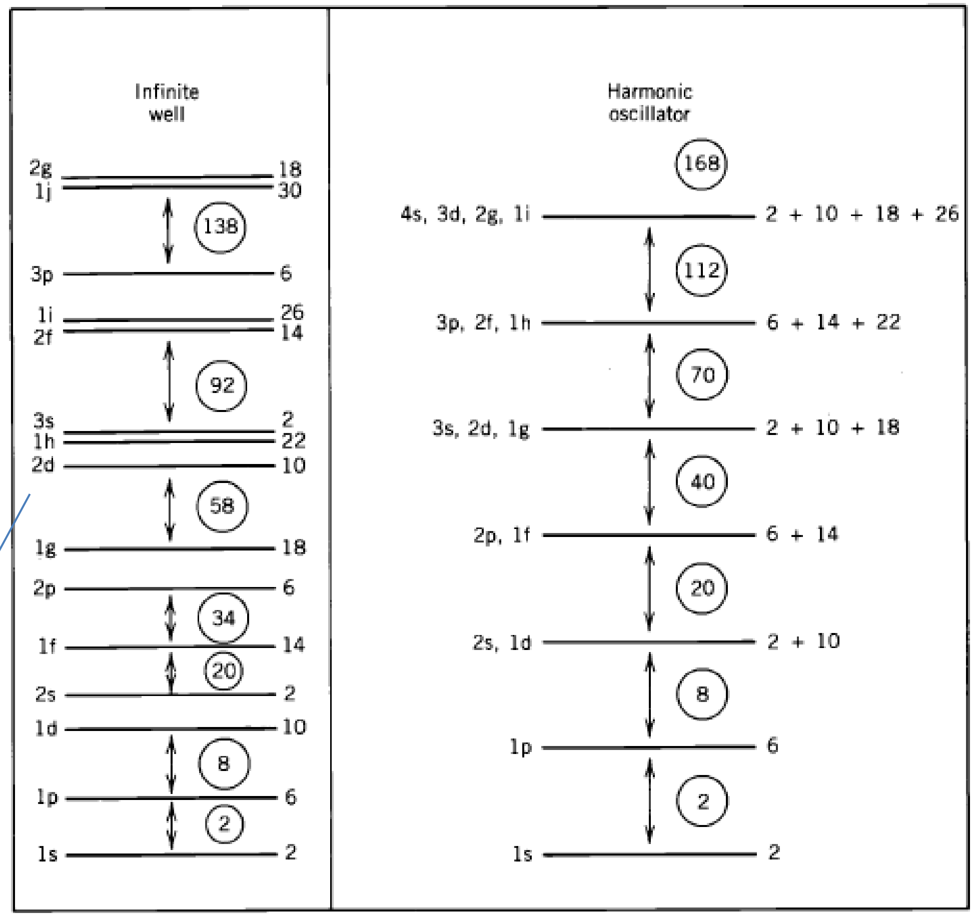
$$2(2\ell+1)$$

Straw man shell model

$N = 2, 8, 20, 28, 50, 82, 126$

$Z = 2, 8, 20, 28, 50, 82$

$$2(2\ell + 1)$$



quasi - degenerate

We don’t recvoer 28 and above?

More realisic potential?

- Coulumb, box or harmonic oscillator do not represent a realistic potential

→ short range of the nuclear force
→ potential following the charge distribution (Fermi-distribution),

$$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

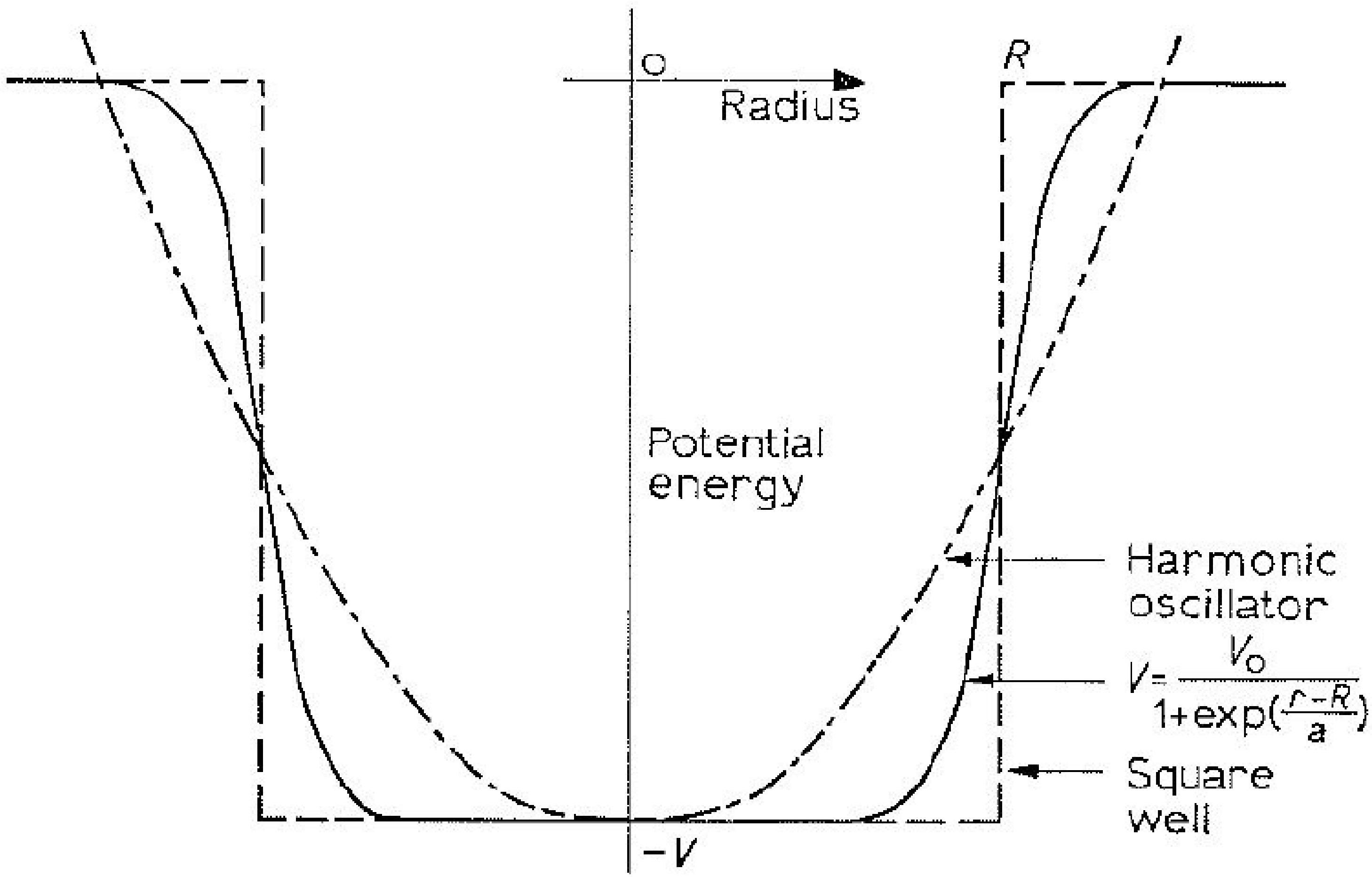
R = Radius at half density
a = diffuseness parameter
ρ_0 ~ central density

nuclei ~ const. density

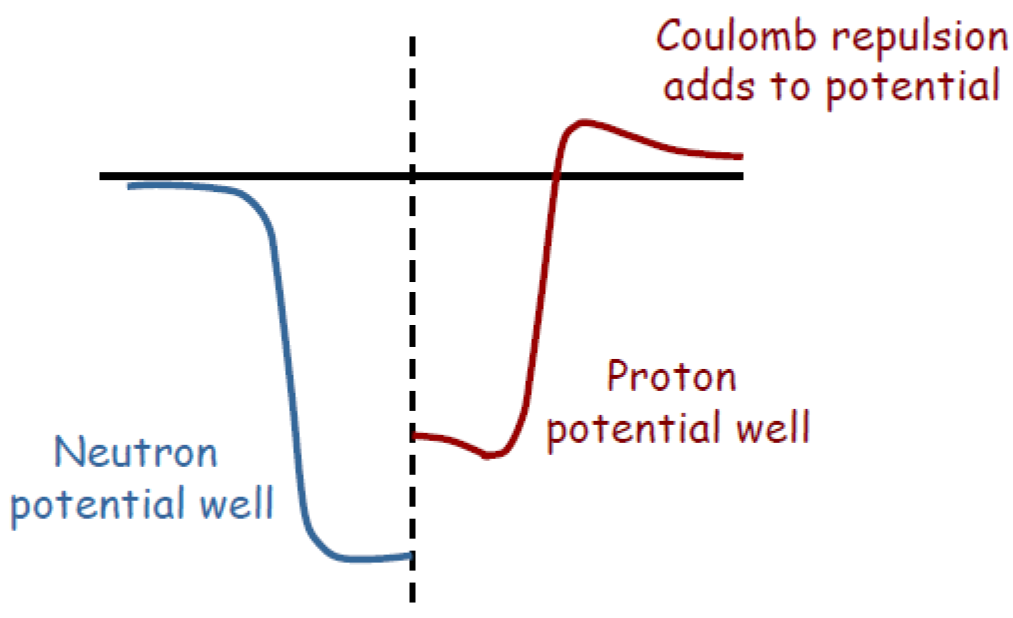
→

$$V_{\mathrm{central}}(r) = \frac{-V_0}{1+\mathrm{e}^{(r-R)/a}}.$$

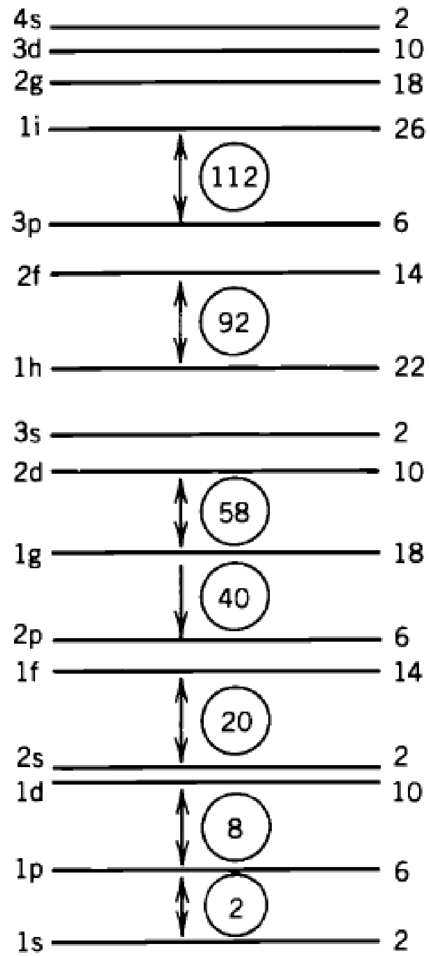
Woods-Saxon potential

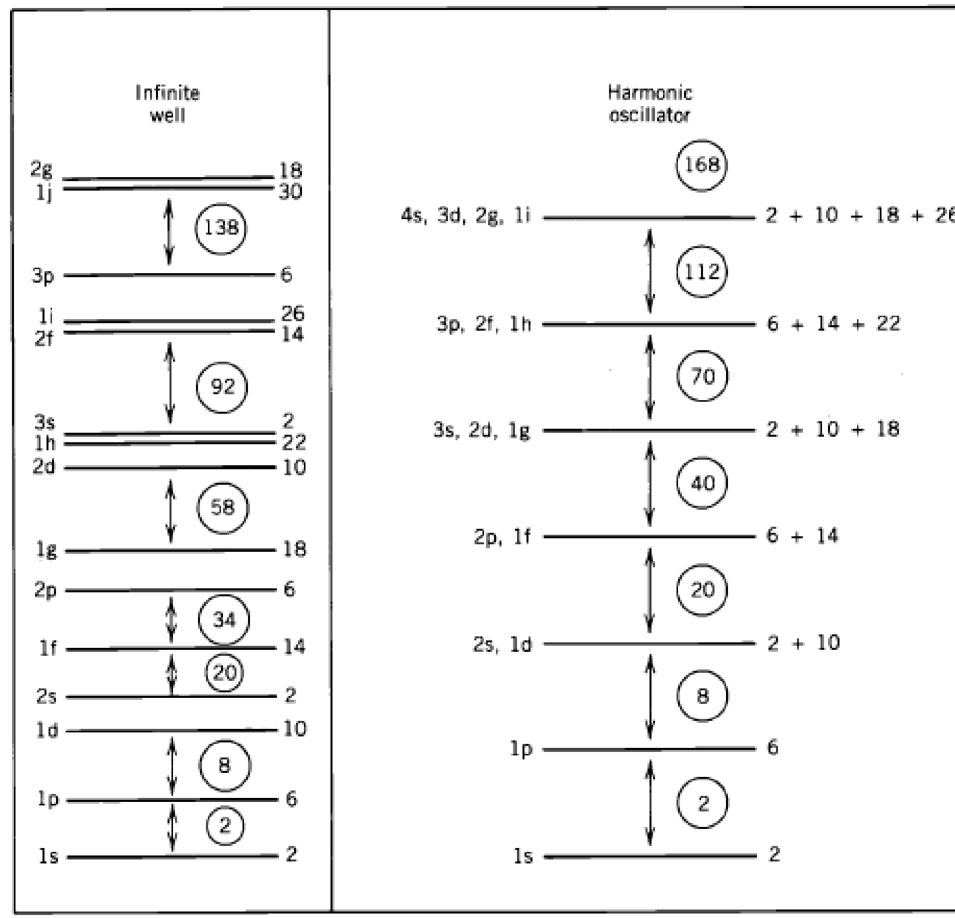


Difference protons/neutrons



Levels in the Woods-Saxon potential



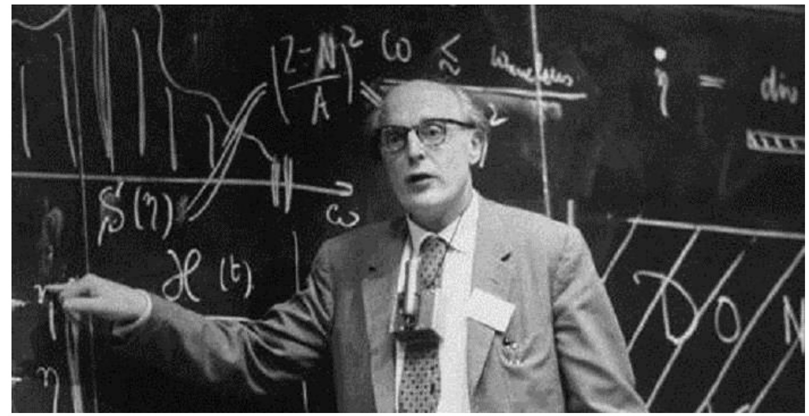


$N = 2, 8, 20, 28, 50, 82, 126$

$Z = 2, 8, 20, 28, 50, 82$

Still doesn't work … what is missing?

Spin-orbit coupling



Hans Jensen (1907-1973)

Maria Goeppert- Mayer
(1906-1972)

Nobel prize 1963: Goeppert-Mayer, Maria
Jensen, Hans
Wigner, Jenő
Nobel prize 2018: October 2

Cf. Atomic physics.

- Fine structure due to spin-orbit interaction (electron spin→magnetic moment) interacts with the magnetic field produced by the moving electron (orbital angular momentum)

How large are these effects in atomic physics?

Spin orbit coupling

$$V_{\text{total}} = V_{\text{central}}(r) + V_{\ell s}(r)\mathbf{L}\cdot\mathbf{S},$$

The total potential is the Woods-Saxon potential with an added term that provides a coupling between the nucleons spin and it’s angular momentum.

Spin-orbit coupling

$\vec{j} = \vec{l} + \vec{s}$

$j^2 = (\vec{l} + \vec{s})^2 = l^2 + 2\vec{l}\vec{s} + s^2$

$\rightarrow \vec{l} \cdot \vec{s} = \frac{1}{2}(j^2 - l^2 - s^2)$

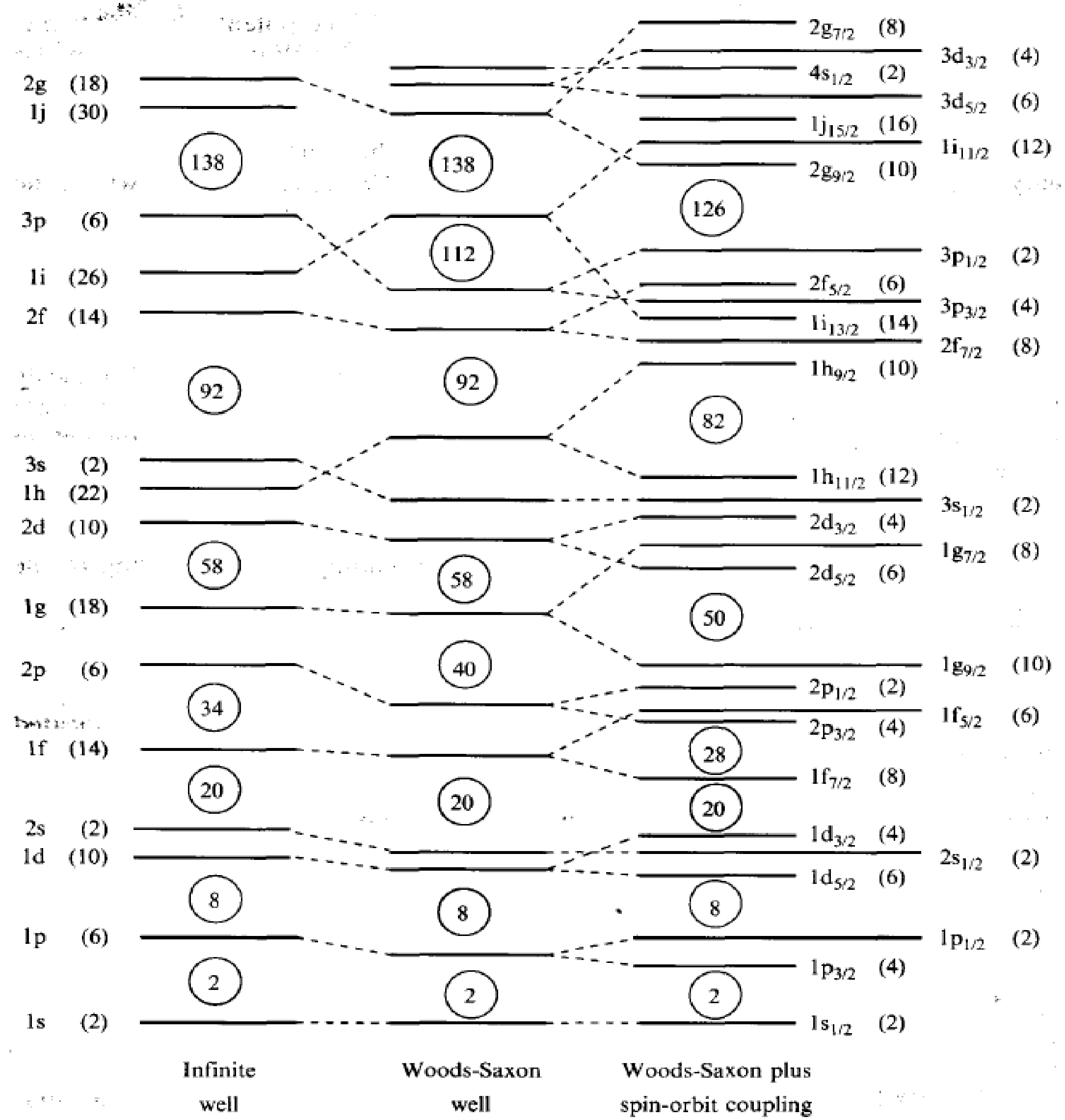
$\rightarrow \langle \vec{l} \cdot \vec{s} \rangle = \frac{1}{2}[j(j+1) - l(l+1) - s(s+1)]\hbar^2$

For given value of $l \rightarrow j = l + \frac{1}{2}$; $j = l - \frac{1}{2}$

$\langle \vec{l} \cdot \vec{s} \rangle_{j=l+\frac{1}{2}} - \langle \vec{l} \cdot \vec{s} \rangle_{j=l-\frac{1}{2}} = \frac{1}{2}(2l+1)\hbar^2$

$\rightarrow$ additional splitting!

Energy levels and occupancies



Differences between atomic and nuclear shell models

- Spin-orbit interaction much stronger in nuclei
- Opposite sign to the atomic case (spin-orbit coupling is attractive)
- Spin-orbit coupling not magnetic, but rather inherent to nuclear force.

Are we happy now?

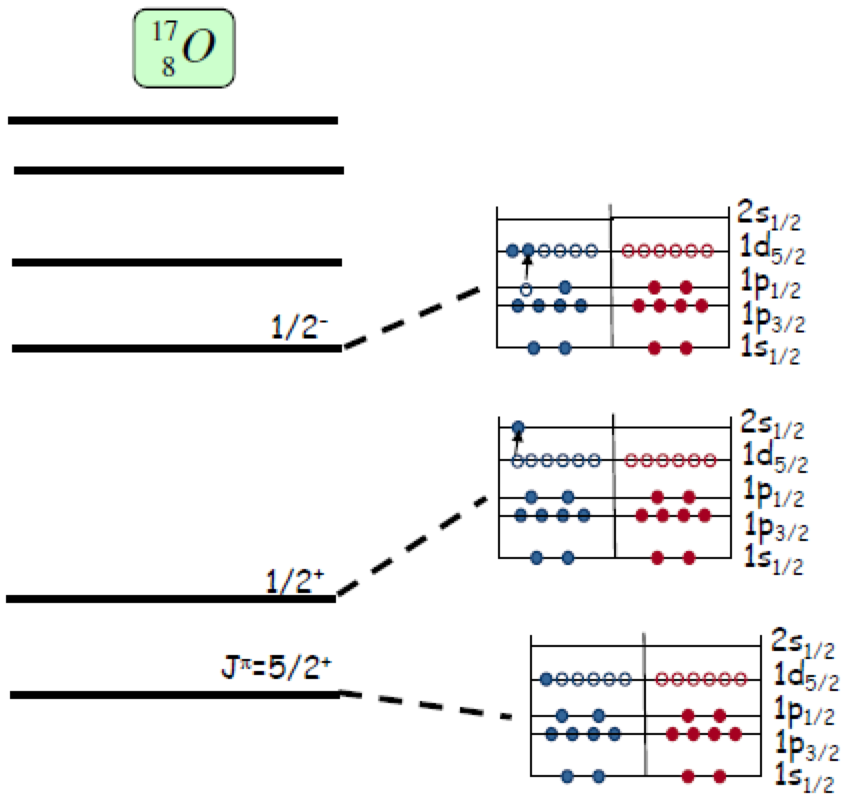
Shortcomings of shell model

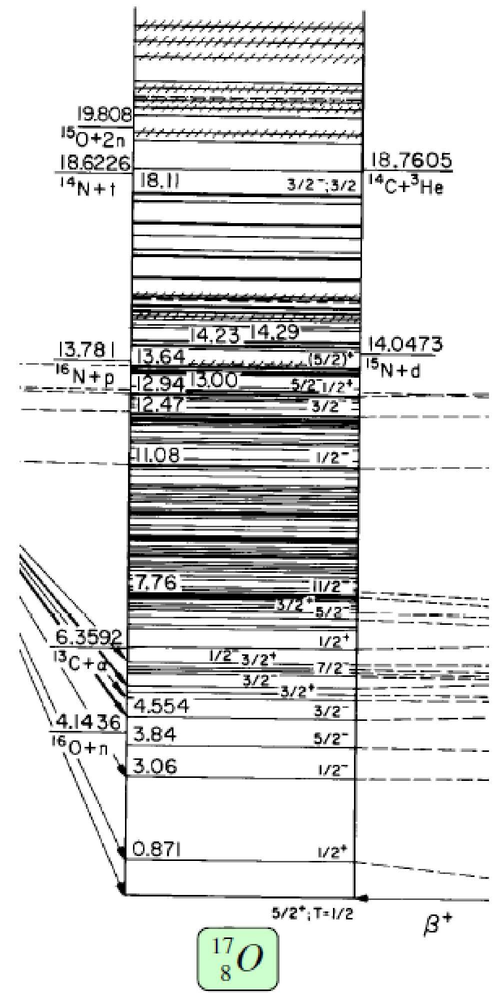
- Excited states: add energy (e.g. in scattering) will produce an excited state

- Within shell model → lift nucleon into next available state of higher energy

- Experimentally: some excited states can be explained, some others not.

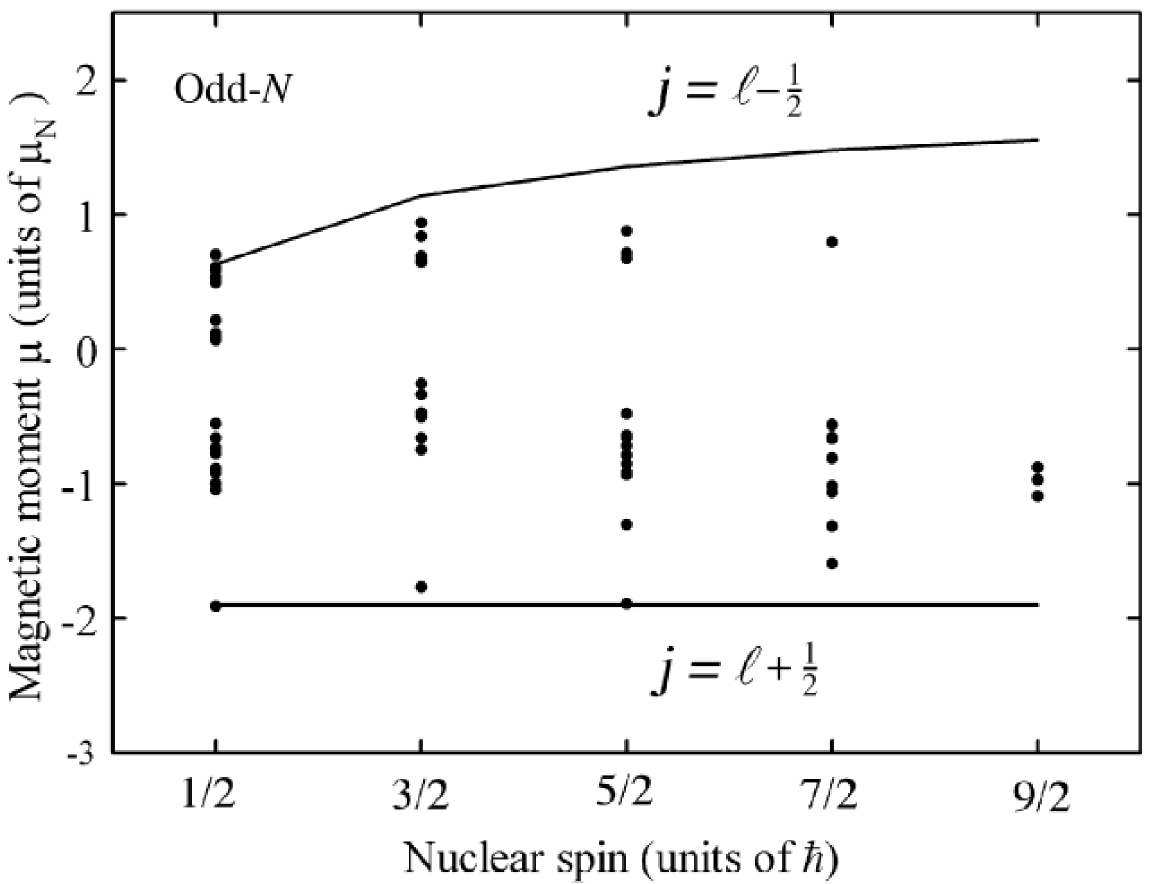
Excited states in shell model





Shortcomings of shell model

- Magnetic moments:

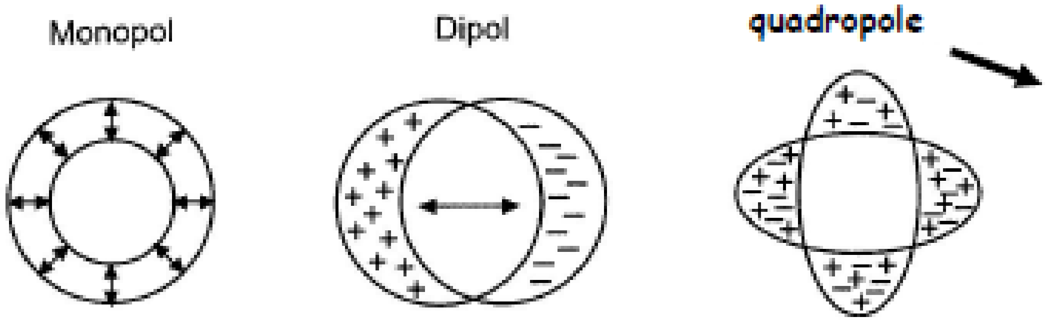


See ch. 7.3.3

The collective model

- The collective model combines the shell model with the liquid drop model.
- The outer valence nucleons are viewed as the surface molecules of a liquid drop
- Asphericity: vibrations and rotations can lead to additional excited states.

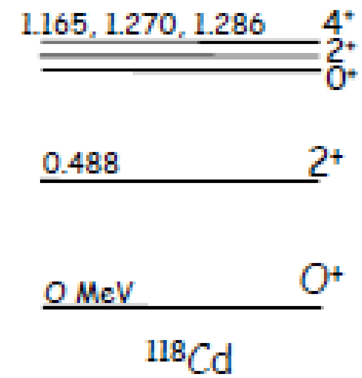
Vibration



Expansion and contraction of R - need to compress nuclear matter - only observe at high excitation energy

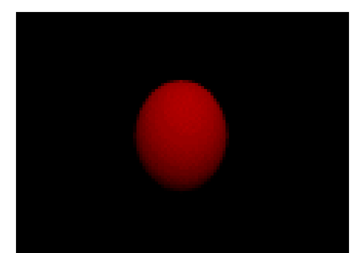
Separation of protons And neutrons against the strong force- only observe at high excitation energy

Lowest vibrational mode which requires no compression or separation - gives low lying vibrational states

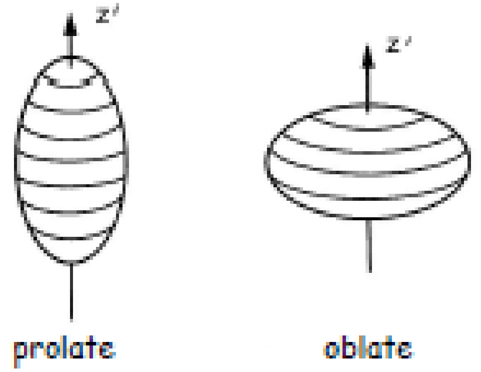


Single quanta of vibration called a phonon

A quadropole phonon carries $J^{\pi} = 2^{+}$



Rotation

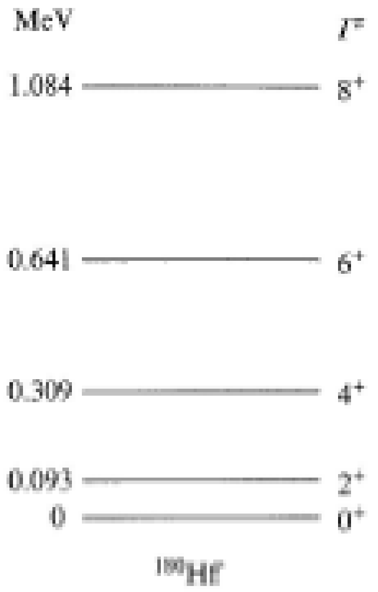


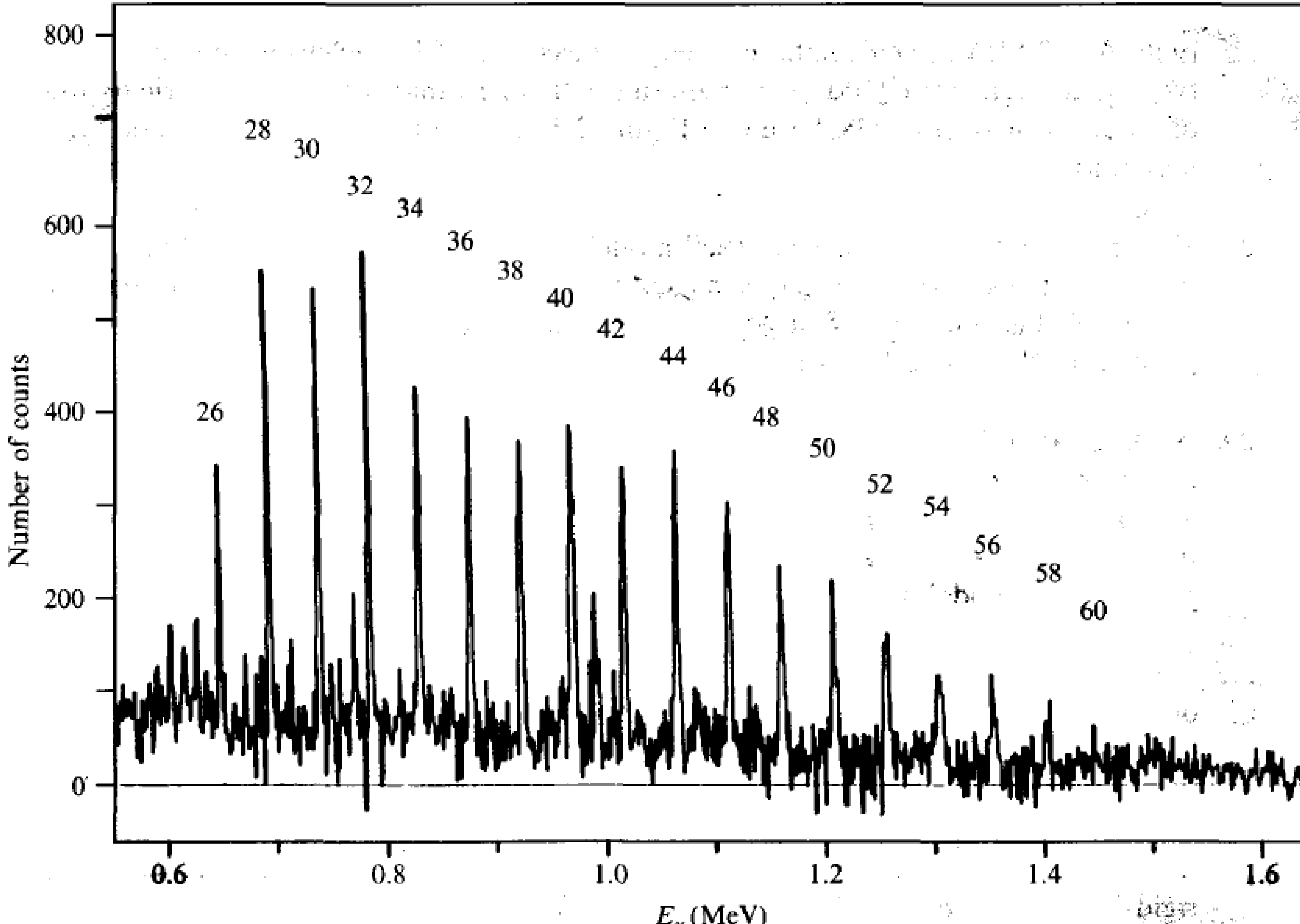
Rotational state only possible
for non spherical potential

spherical symmetry has no preferred direction in space
→ a rotation would not lead to any observable change

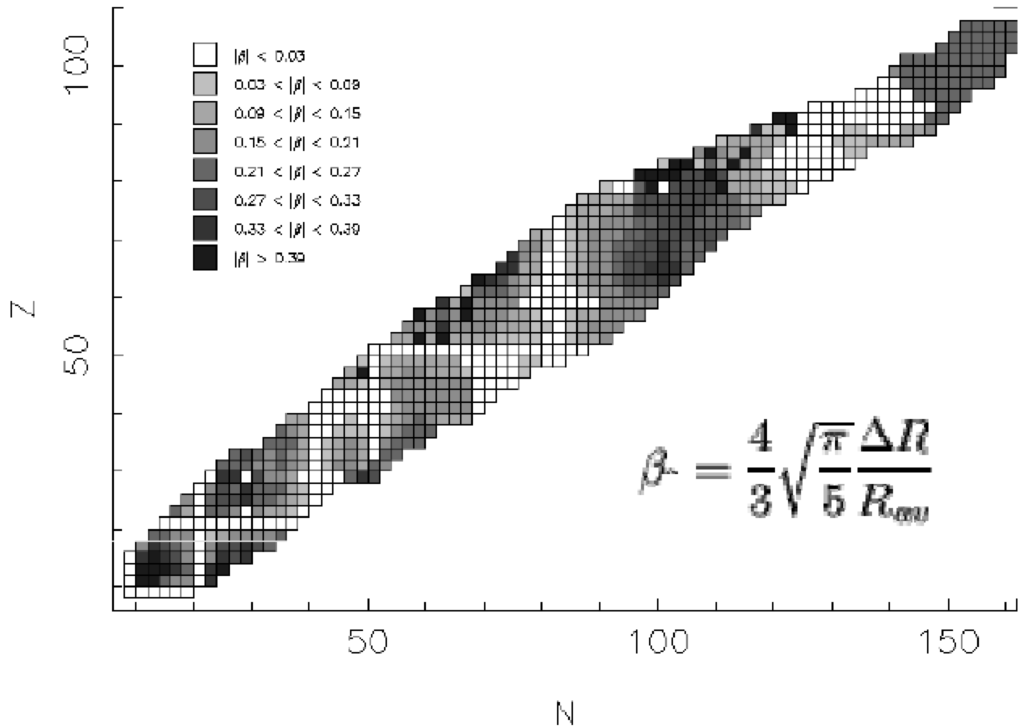
Rotational energy states

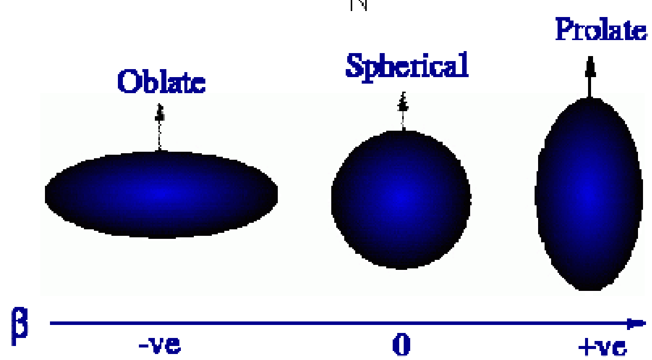
$$\boxed{E(J) = \frac{J(J+1)\hbar^2}{2I}}$$





Number of counts
800
600
400
200
0
28
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
60
26
0.6
0.8
1.0
1.2
1.4
1.6
E_γ (MeV)


|β| < 0.03
0.03 < |β| < 0.09
0.09 < |β| < 0.15
0.15 < |β| < 0.21
0.21 < |β| < 0.27
0.27 < |β| < 0.33
0.33 < |β| < 0.39
|β| > 0.39
$\beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}}$
Z
50
100
50
100
150
N


Oblate
Spherical
Prolate
β
-ve
0
+ve

Summary of today's lecture

- We have covered the basics of nuclear models (Fermi-Gas-model, Shell model, Collective model)
- The shell model was able to predict magic numbers after (a) Woods-Saxon and (b) spin-orbit coupling was introduced.
- Excited states → vibrational/rotational degrees of freedom.

- The shell model, including vibrational and rotational degrees of freedom is nowadays the most applied model.

[redacted]

Ab initio nuclear response functions for dark matter searches

D. Gazda, R. Catena, and C. Forssén
Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

We study the process of dark matter particles scattering off ^{3,4}He with nuclear wave functions computed using an *ab initio* many-body framework. We employ realistic nuclear interactions derived from chiral effective field theory at next-to-next-to-leading order (NNLO) and develop an *ab initio* scheme to compute a general set of different nuclear response functions. In particular, we then perform an accompanying uncertainty quantification on these quantities and study error propagation to physical observables. We find a rich structure of allowed nuclear responses with significant uncertainties for certain spin-dependent interactions. The approach and results that are presented here establish a new framework for nuclear structure calculations and uncertainty quantification in the context of direct and (certain) indirect searches for dark matter.

proach. Only very recently it was shown how to obtain residual effective valence-space interactions starting from the underlying microscopic internucleon interaction in a systematic, nonperturbative framework using *ab initio* methods [28–30]. However, it remains to be studied how theoretical model uncertainties can be quantified.