Lecture 2: Decays and Reactions

Jan Conrad

Summary of last lecture

- Unlike particle physics, nuclear physics considers multi-body, composite objects → phenomenolgical modelling → many applications.
- Typical sizes of nuclei: ~fm,

  Typical energies ~MeV,

  Typical densities: 10^{14}g/cm^3

Summary of last lecture

- The charge distribution (and size) of a nucleus can be experimentally determined by measuring the angular distirbution of electron scattering
- The mass distribution can be determined by scattering of α particles (the original Rutherford scattering experiments)

$$\boxed{R = r_0 A^{1/3}}$$

$r_0 \sim 1,2$ fm

Summary of last lecture

$$\boxed{B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta\,(A,Z)}$$

$a_v = 15.6$ MeV
$a_s = 17.2$ MeV
$a_a = 23.3$ MeV
$a_C = 0.70$ MeV

Note (different notation used by MANY authors):

- change in notation for assymetry term (x 4 for the coefficient)
- $\delta = f(A) = a_5 A^{-\frac{1}{2}}$

Open points

- Assymetry term vs. pairing term.
- Short range of nuclear force versus QCD.

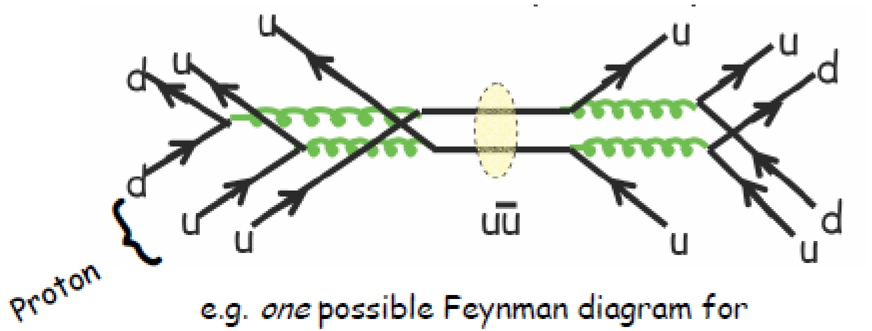
Assymetry term vs. Pairing term

- Assymetry term: N==Z.
  - Reason: Pauli-principle→symmetry in the strong force (protons and neutrons respond the same) → ”Iso-spin pairs”

- Pairing term: N even, Z even:
  - Reason: Pauli principle → protons come in pairs of opposite spin, neutrons come in pairs of opposite spin → unpaired proton, neutron → reduced binding energy

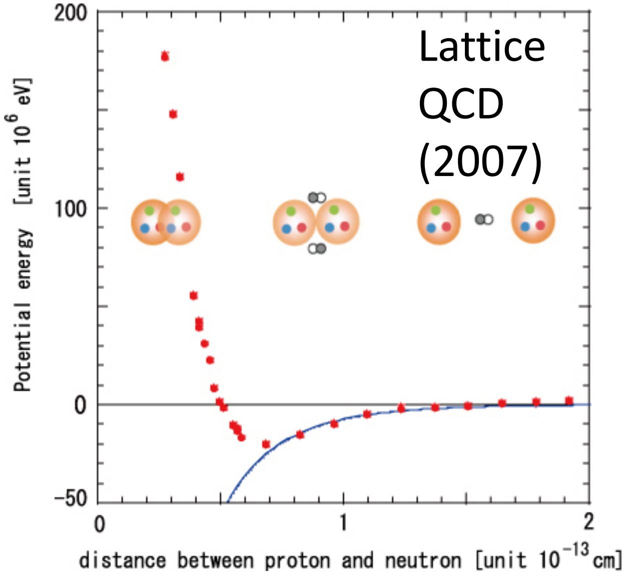
Nuclear force vs. QCD

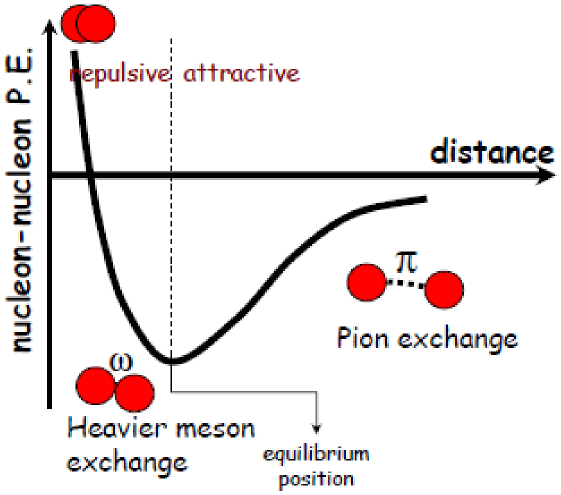
- Nuclear force ≠ QCD



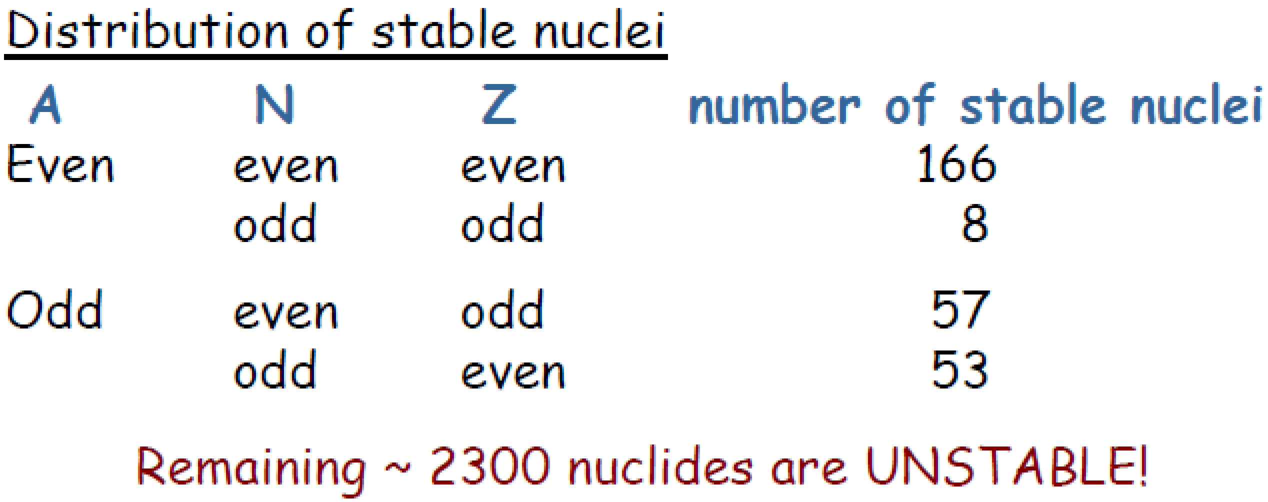
e.g. *one* possible Feynman diagram for proton - proton scattering at the quark level

- Nuclear force: adequately described by meson exchange





Nuclear stability



Distribution of stable nuclei

A	N	Z	number of stable nuclei
Even	even	even	166
	odd	odd	8
Odd	even	odd	57
	odd	even	53

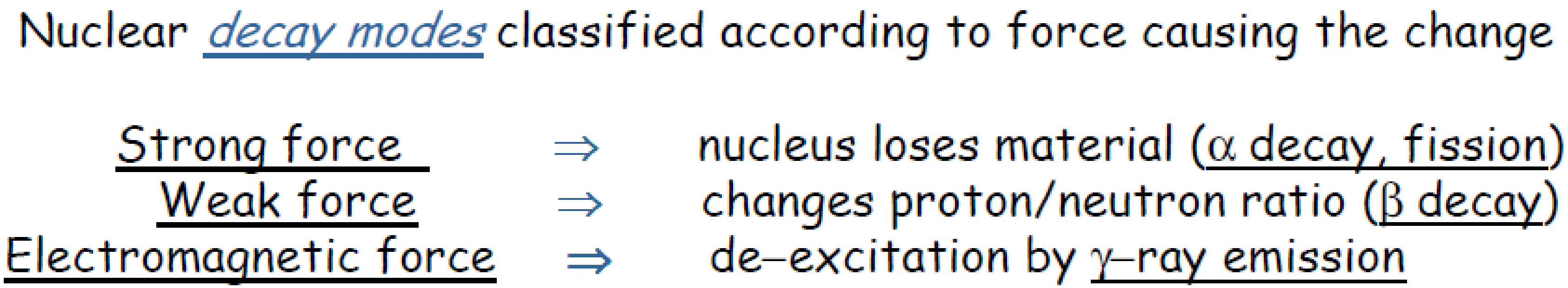
Remaining ~ 2300 nuclides are UNSTABLE!

What are the determinants of nuclear stability?

How do nuclei decay?

How do nuclei interact?

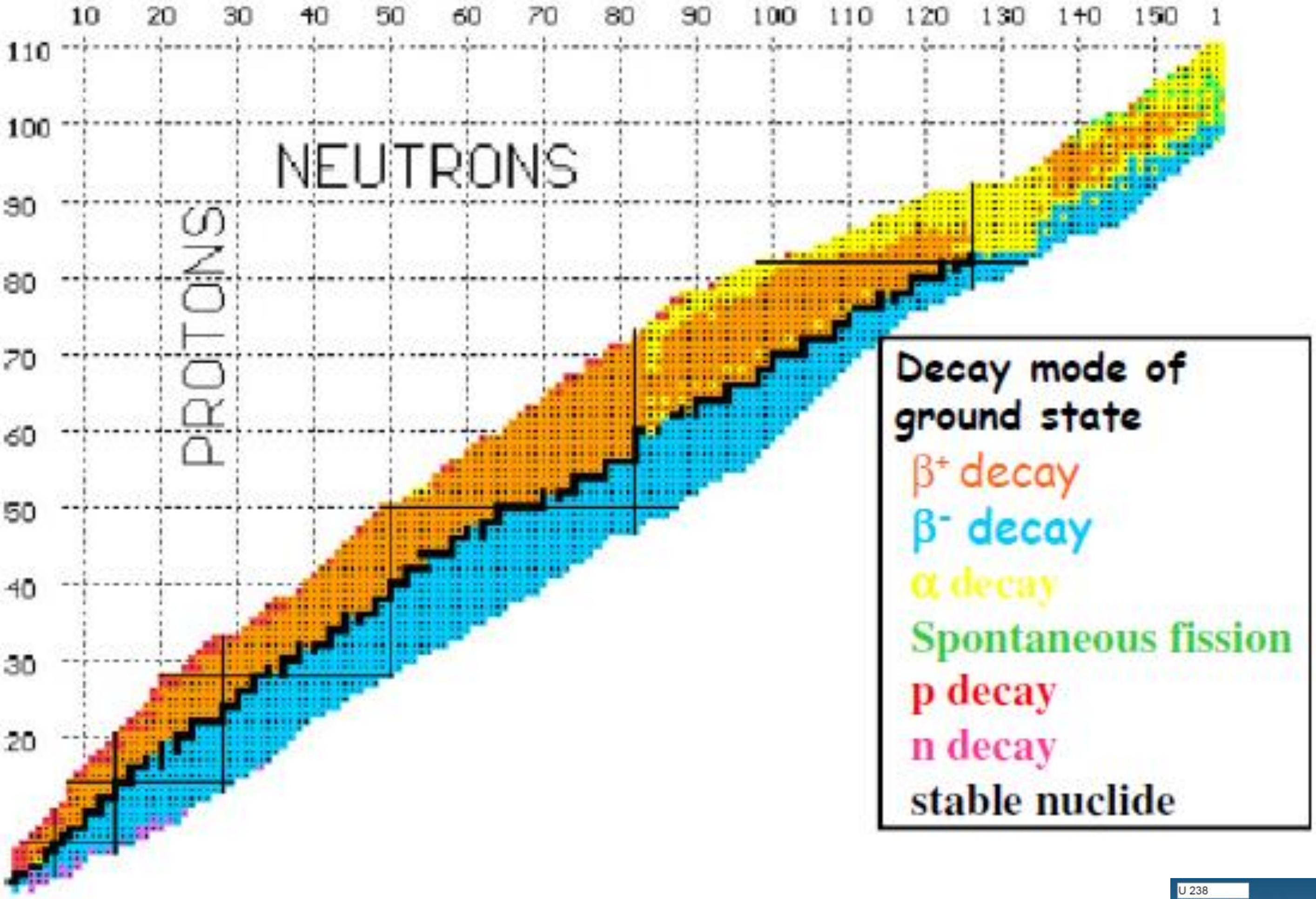
Nuclear decay modes



Nuclear *decay modes* classified according to force causing the change

Strong force ⇒ nucleus loses material (α decay, fission)
Weak force ⇒ changes proton/neutron ratio (β decay)
Electromagnetic force ⇒ de–excitation by γ–ray emission

(can you relate this to lifetime)?


10
20
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140
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1
110
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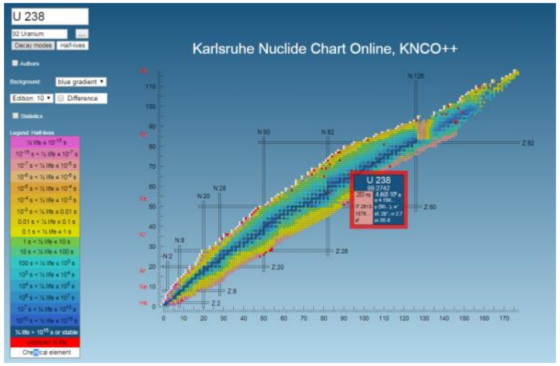
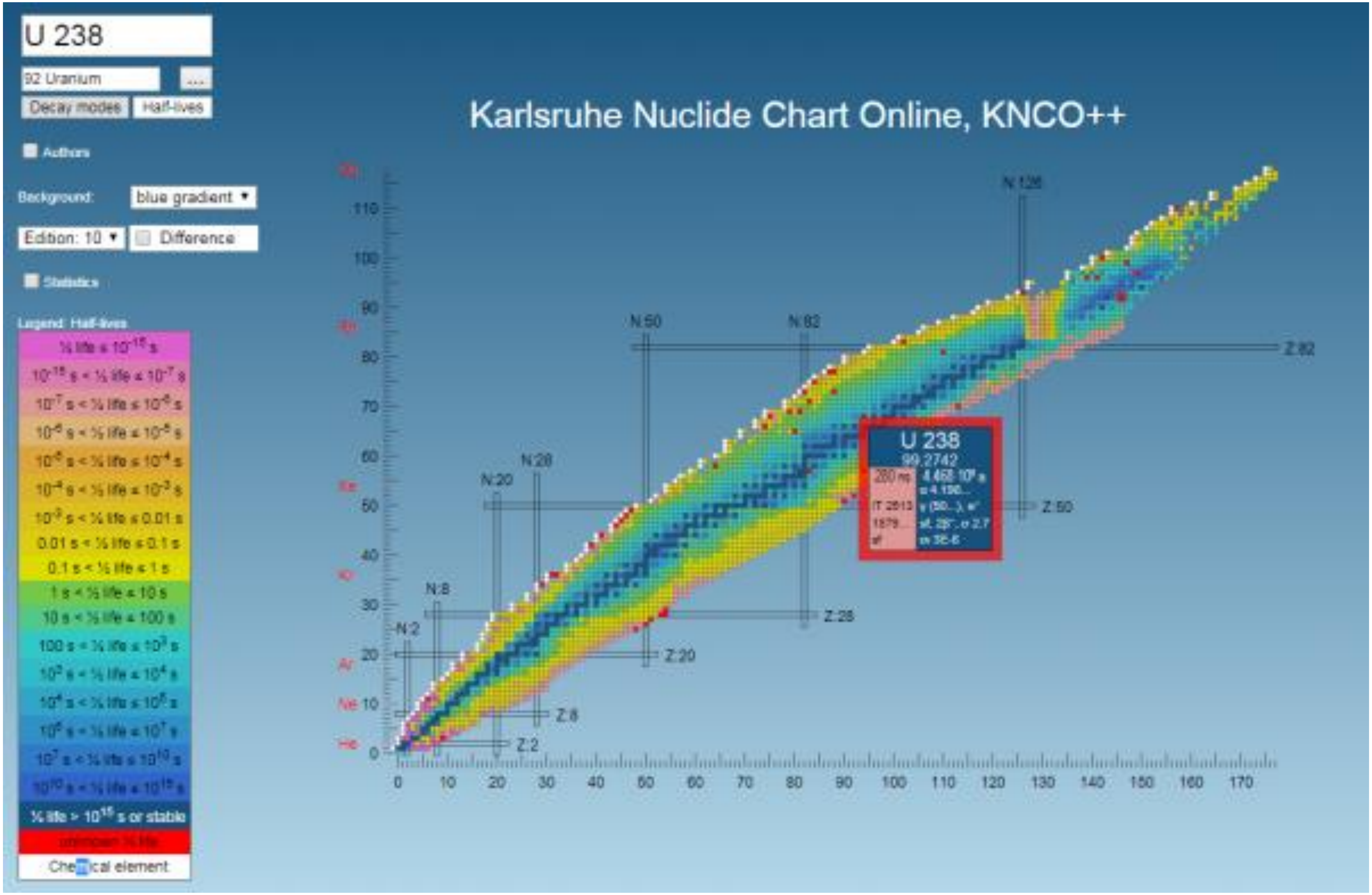
70
60
50
40
30
20
NEUTRONS
PROTONS
Decay mode of ground state
β+ decay
β- decay
α decay
Spontaneous fission
p decay
n decay
stable nuclide


U 238
92 Uranium
Decay modes
Half-lives
Karlsruhe Nuclide Chart Online, KNCO++
U 238



Karlsruhe Nuclide Chart Online, KNCO++

U 238

92 Uranium

Background: blue gradient

Edition: 10

☐ Difference

☐ Authors

☐ Statistics

Legend: Half-lives

- ½ life ≤ 10⁻¹⁵ s
- 10⁻¹⁵ s < ½ life ≤ 10⁻⁷ s
- 10⁻⁷ s < ½ life ≤ 10⁻⁶ s
- 10⁻⁶ s < ½ life ≤ 10⁻⁵ s
- 10⁻⁵ s < ½ life ≤ 10⁻⁴ s
- 10⁻⁴ s < ½ life ≤ 10⁻³ s
- 10⁻³ s < ½ life ≤ 0.01 s
- 0.01 s < ½ life ≤ 0.1 s
- 0.1 s < ½ life ≤ 1 s
- 1 s < ½ life ≤ 10 s
- 10 s < ½ life ≤ 100 s
- 100 s < ½ life ≤ 10³ s
- 10³ s < ½ life ≤ 10⁴ s
- 10⁴ s < ½ life ≤ 10⁵ s
- 10⁵ s < ½ life ≤ 10⁷ s
- 10⁷ s < ½ life ≤ 10¹⁰ s
- 10¹⁰ s < ½ life ≤ 10¹⁵ s
- ½ life > 10¹⁵ s or stable
- unknown ½ life
- Chemical element

Q-value

- Q- value is defined as the energy released in a nuclear reaction, e. g. αdecay:

$${}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{X}'_{N-2} + \alpha$$

$$Q = (m_{\mathrm{X}} - m_{\mathrm{X}'} - m_{\alpha})c^2$$

- Allowed decays will have positive Q-values.

Radioactive decay

- Activity:

$$\mathscr{A} = -\mathrm{d}N/\mathrm{d}t = \lambda N,$$

λ: decay constant (stays constant in time)
Units: 1 Becquerel (Bq) = 1 decay/second , 1 Curie (Ci) = 3.7×10^{10} Bq

- Time dependence (exponential decay law):

$$\mathscr{A}(t) = \lambda N_0 \exp(-\lambda t),$$

Lifetime and half-life

Mean lifetime:

$$\tau \equiv \frac{\int t \, \mathrm{d}N(t)}{\int \mathrm{d}N(t)} = \frac{\int\limits_0^\infty t \exp[-\lambda t] \, \mathrm{d}t}{\int\limits_0^\infty \exp[-\lambda t] \, \mathrm{d}t} = \frac{1}{\lambda}.$$

Half-life:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2.$$

Example: decay chain

$$
\begin{aligned}
&{}^{79}_{38}\mathrm{Sr} \rightarrow {}^{79}_{37}\mathrm{Rb} + e^{+} + \nu_e && (2.25\ \mathrm{min}) \\
&\qquad\qquad \hookrightarrow {}^{79}_{36}\mathrm{Kr} + e^{+} + \nu_e && (22.9\ \mathrm{min}) \\
&\qquad\qquad\qquad\qquad \hookrightarrow {}^{79}_{35}\mathrm{Br} + e^{+} + \nu_e && (35.04\ \mathrm{hours})
\end{aligned}
$$

http://periodictable.com/Isotopes/035.79/index.p.full.html

Did the binding energy increase or decrease?

Li Be Na Mg K Ca Rb Sr Cs Ba La Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb Lu Hf Ta W Re Os Ir Pt Au Hg Tl Pb Bi Po At Rn
Fr Ra Ac Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No Lr Rf Db Sg Bh Hs Mt Ds Rg Cn Nh Fl Mc Lv Ts Og
B C N O F Ne Al Si P S Cl Ar Sc Ti V Cr Mn Fe Co Ni Cu Zn Ga Ge As Se Br Kr Y Zr Nb Mo Tc Ru Rh Pd Ag Cd In Sn Sb Te I Xe

Bromine Main Page
Bromine Pictures Page
Bromine Technical Data

Black White Gray

Isotopes of Bromine (click to see decay chain):

^{67}Br ^{68}Br ^{69}Br ^{70}Br ^{71}Br ^{72}Br ^{73}Br ^{74}Br ^{75}Br ^{76}Br ^{77}Br ^{78}Br ^{79}Br ^{80}Br ^{81}Br ^{82}Br ^{83}Br ^{84}Br ^{85}Br ^{86}Br ^{87}Br ^{88}Br ^{89}Br ^{90}Br ^{91}Br ^{92}Br ^{93}Br ^{94}Br ^{95}Br ^{96}Br ^{97}Br

^{79}Br

Half-life Stable

Boson, 35p 44n

Spin 3/2 Parity -1

Atomic Weight 78.918337087
Abundance 50.69%
Mass Excess -76.068514MeV
Binding Energy 8.6876MeV
Magnetic Moment 2.1064 μ
Quadrupole Moment 0.331

This isotope is stable and thus has no decay products, so instead we show decay chains that lead down to it.

Show Isotope Symbols.
Show Decay Image Smaller.

Click any isotope in diagram to see its data.

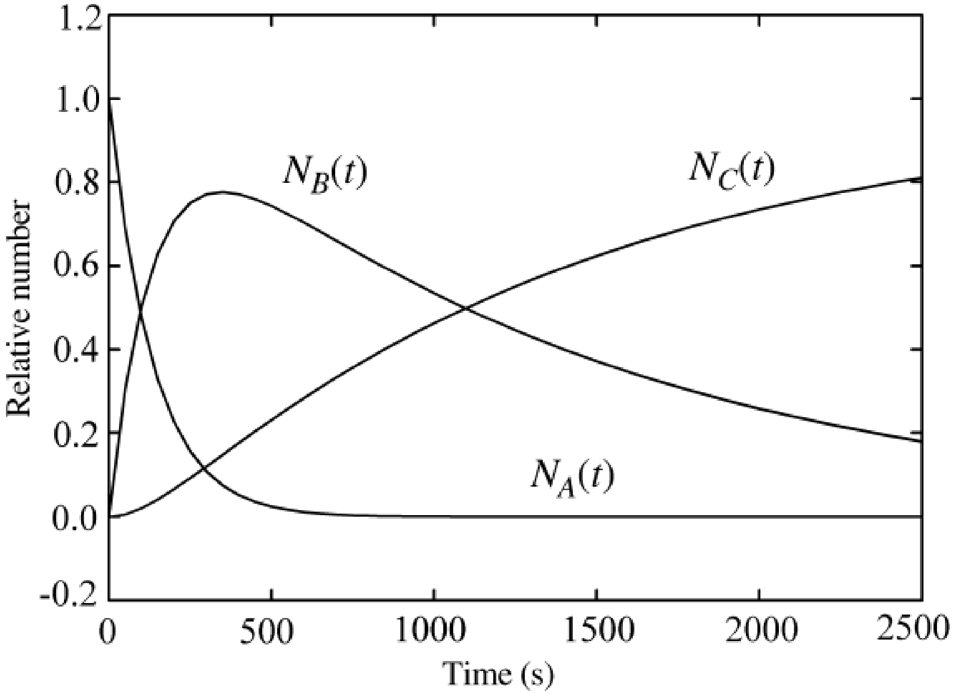
Decay chains

$$N_A(t) = N_A(0)\exp(-\lambda_A t),$$

$$\mathrm{d}N_B(t)/\mathrm{d}t = -\lambda_B N_B + \lambda_A N_A.$$

$$N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} N_A(0)[\exp(-\lambda_A t) - \exp(-\lambda_B t)].$$

……0



Beta-decay

$$n \longrightarrow p + e^{-} + \bar{\nu}_e;$$

Beta decay

$$p \longrightarrow n + e^{+} + \nu_e,$$

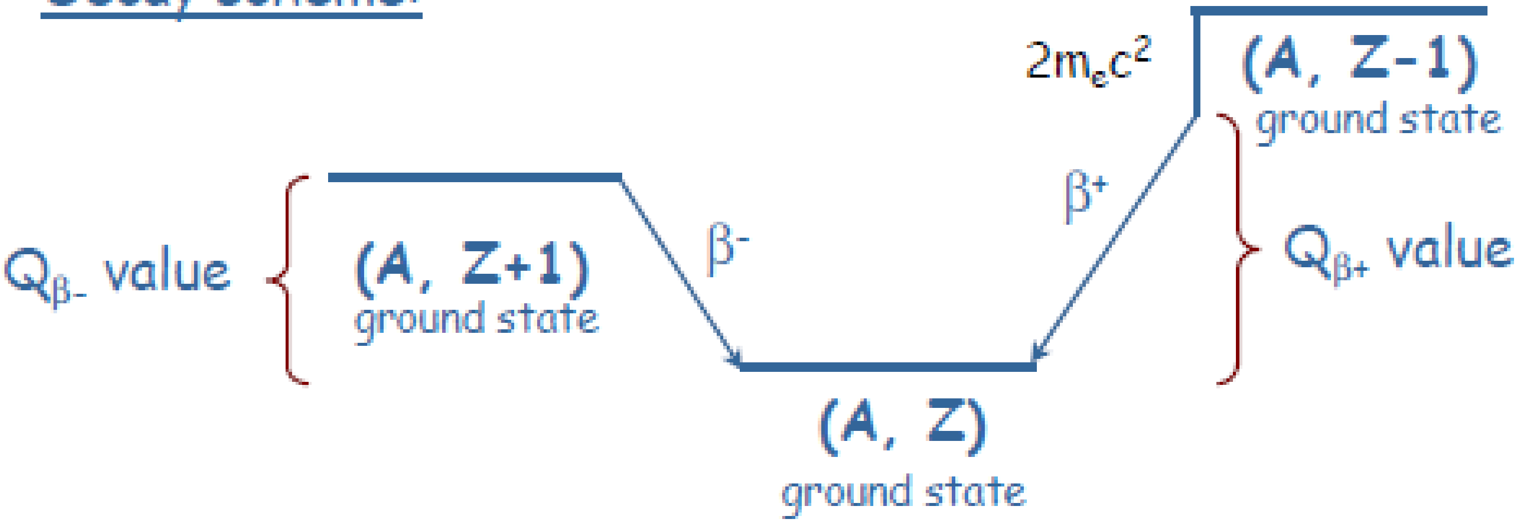
Positron emission

$$e^{-} + {}^{111}_{51}\mathrm{Sb} \longrightarrow {}^{111}_{50}\mathrm{Sn} + \nu_e$$

Electron capture

Beta-decay scheme

Decay scheme:



We will get to this later (”Fermi-theory of beta decay”).

Q-values of β+/β- decays

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X + {}_{-1}e + \nu$$

Mass defect:

$$\Delta m = [M_X - Z m_e] - [(M_Y - (Z-1) m_e) + m_e]$$

$$= M_X - M_Y - 2 m_e$$

$$Q = (M_X - M_Y - 2 m_e) c^2$$

Exercise: repeat this for β-decay

Mass parabolas

$$M(A,Z)c^2 = aA + bZ + cZ^2 \pm \delta\,(A,Z)$$

$$\Downarrow$$

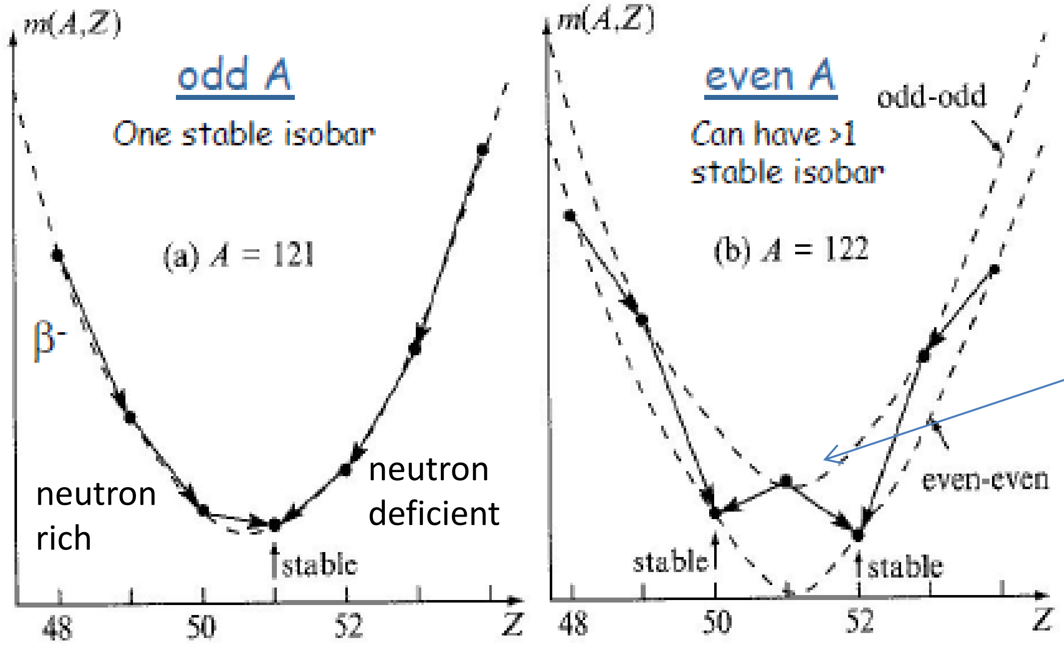
quadratic function in Z ⇒ PARABOLA

odd A ⇒ $\delta = 0$ ⇒ one parabola only
even A ⇒ $\pm\,\delta$ ⇒ two parabolae

Minimum stable isobar $\Rightarrow \left.\frac{\partial m}{\partial Z}\right|_{A=\text{const.}} = 0$

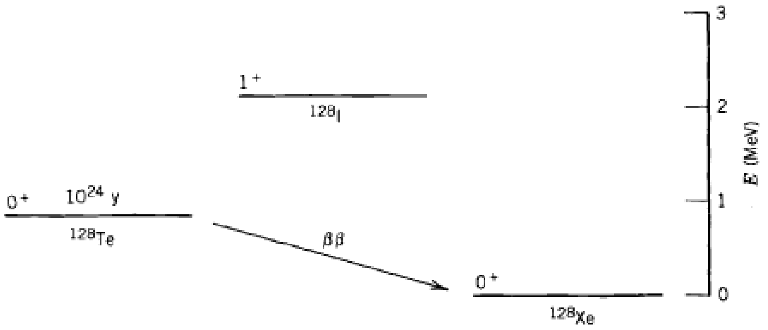
$$\Downarrow$$

$$Z_{stable} = \frac{A}{1.972 + 0.015A^{2/3}}$$

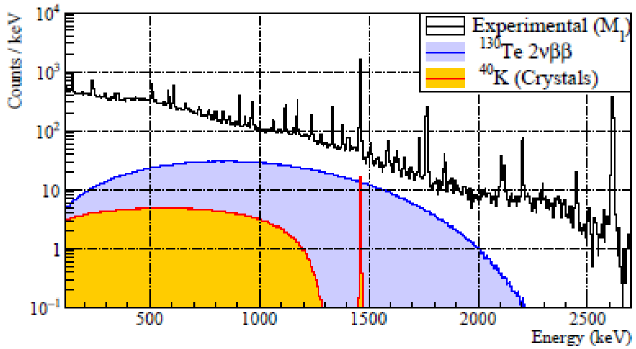


Double β decay

Double β decay



Measurement of the Two-Neutrino Double Beta Decay Half-life of 130Te with the CUORE-0 Experiment, Cuore Collaboration,
Eur. Phys. J. C (2017) 77: 13

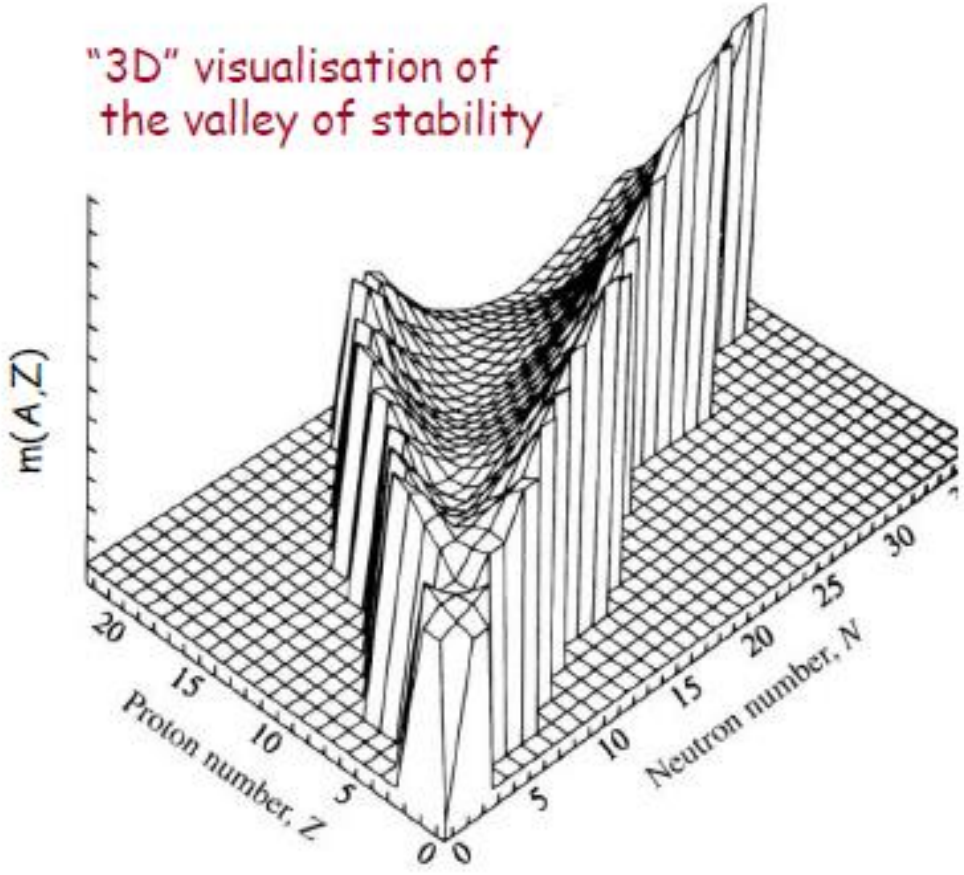


$$T^{2\nu}_{1/2} = [8.2 \pm 0.2\ (\text{stat.}) \pm 0.6\ (\text{syst.})] \times 10^{20}\ \text{y.}$$

Go to chart

- There is a large experimental effort to search for the *neutrino-less double β decay* which would have profound consequences for neutrino physics (neutrino mass).

The valley of stability

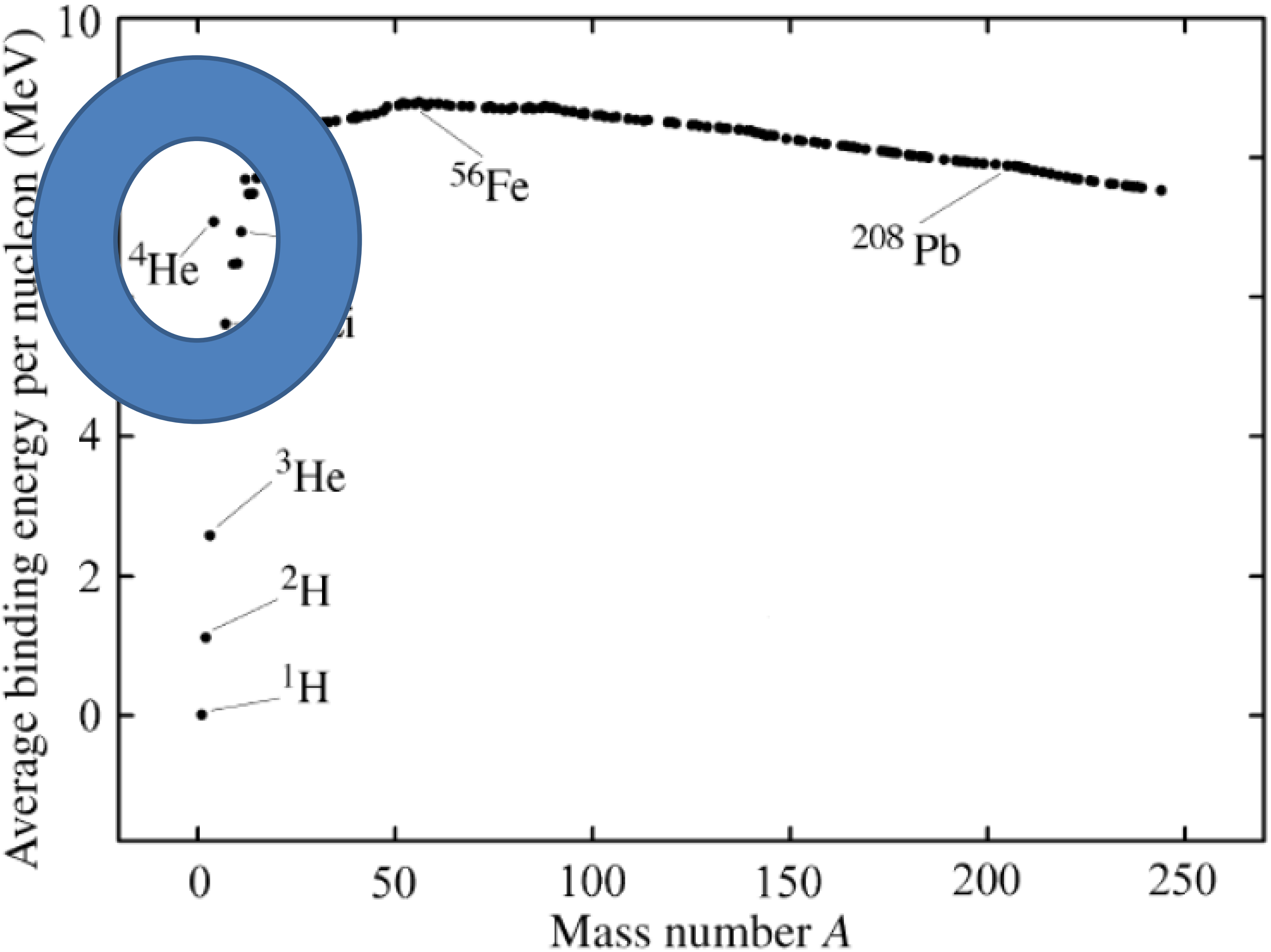


α-decay

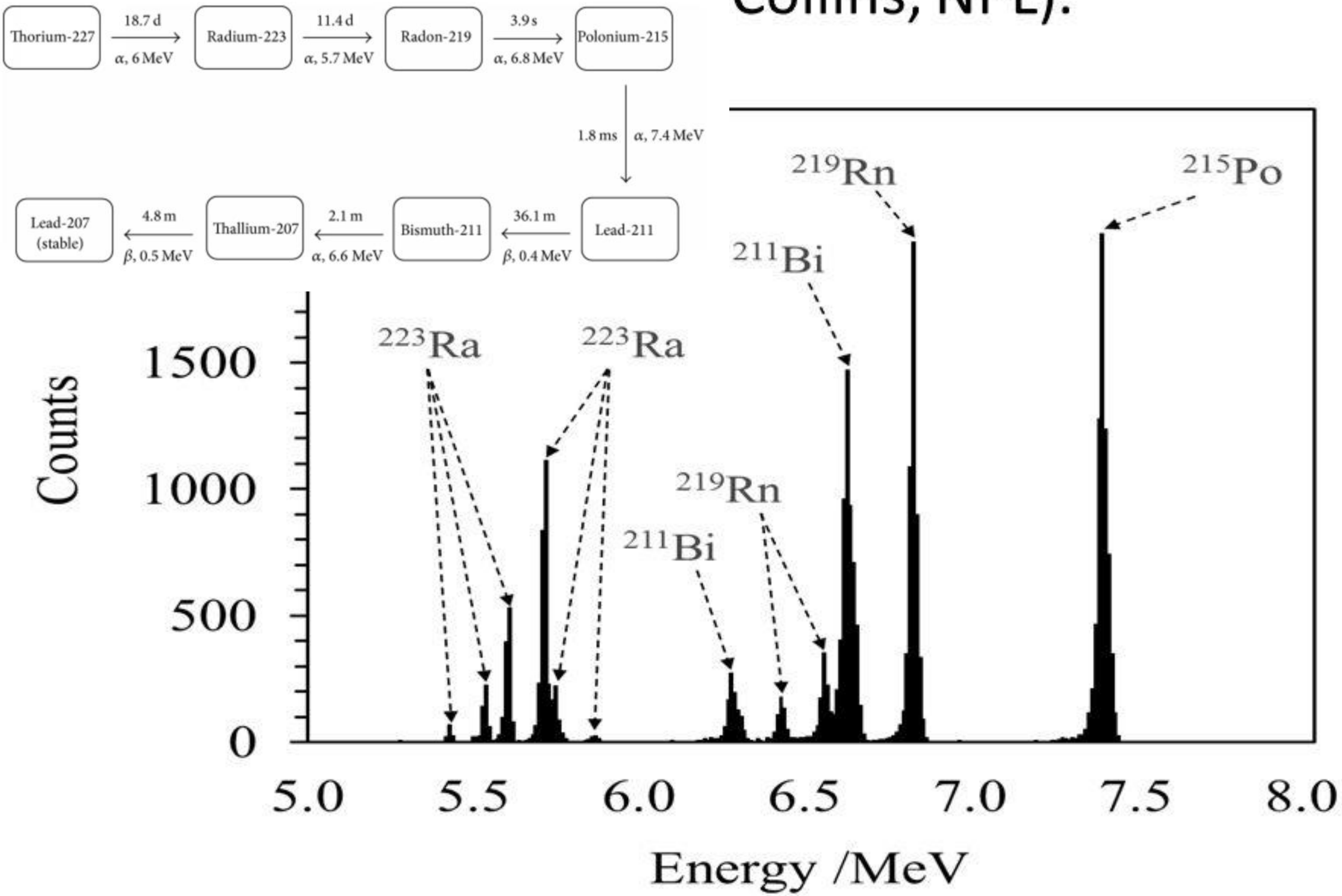
- Alpha decay is a special type of fission that results in the emission of a Helium nucleus

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha $$

- This decay is most prevalent among nuclei with $82 < Z < 92$, $A >$ roughly 150. This can be derived by the B-W formula (approx. $N=Z$)


Average binding energy per nucleon (MeV)
10
4
2
0
0
50
100
150
200
250
Mass number A
$^{56}\mathrm{Fe}$
$^{208}\mathrm{Pb}$
$^{4}\mathrm{He}$
$^{3}\mathrm{He}$
$^{2}\mathrm{H}$
$^{1}\mathrm{H}$

Alpha decay spectrum of ^{223}Ra and daughters Collins, NPL).

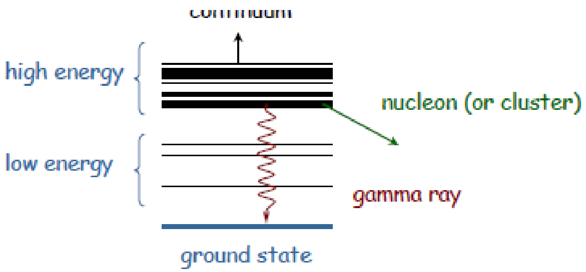


Thorium-227 → 18.7 d, α, 6 MeV → Radium-223
Radium-223 → 11.4 d, α, 5.7 MeV → Radon-219
Radon-219 → 3.9 s, α, 6.8 MeV → Polonium-215
Polonium-215 → 1.8 ms, α, 7.4 MeV → Lead-211
Lead-211 → 36.1 m, β, 0.4 MeV → Bismuth-211
Bismuth-211 → 2.1 m, α, 6.6 MeV → Thallium-207
Thallium-207 → 4.8 m, β, 0.5 MeV → Lead-207 (stable)

Gamma decay

- β-decay happens in nuclei because the nucleus is not in the energetically most favored state.
- β- and α-decays lead to excited nuclear states
- De-excitation can lead to gamma-ray emission (keV-MeV energies) → certain selection rules that can be understood within the shell model

→ ch. 7



What degrees of freedom could you speculate?

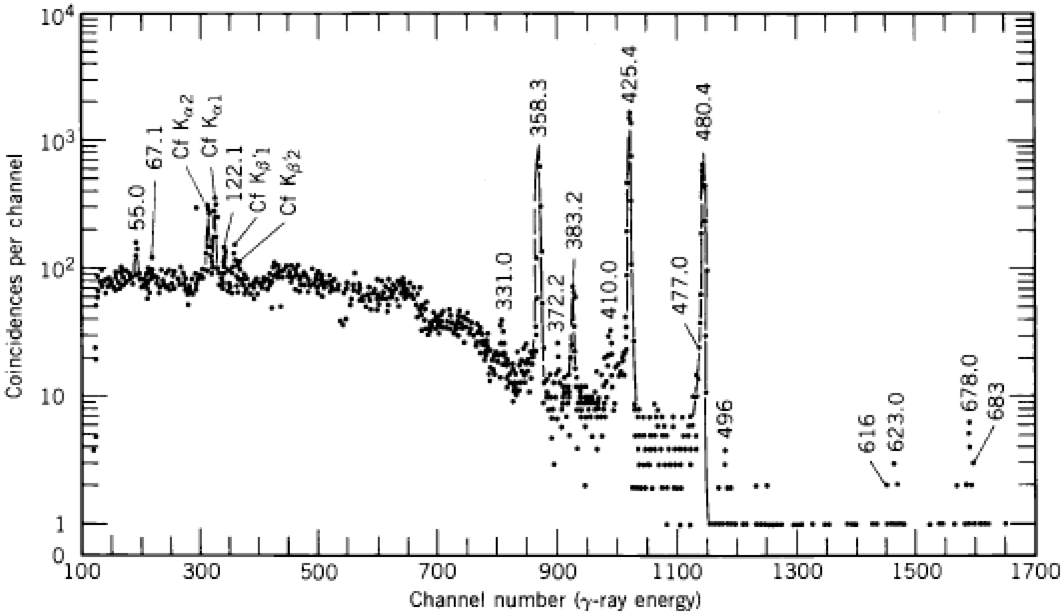
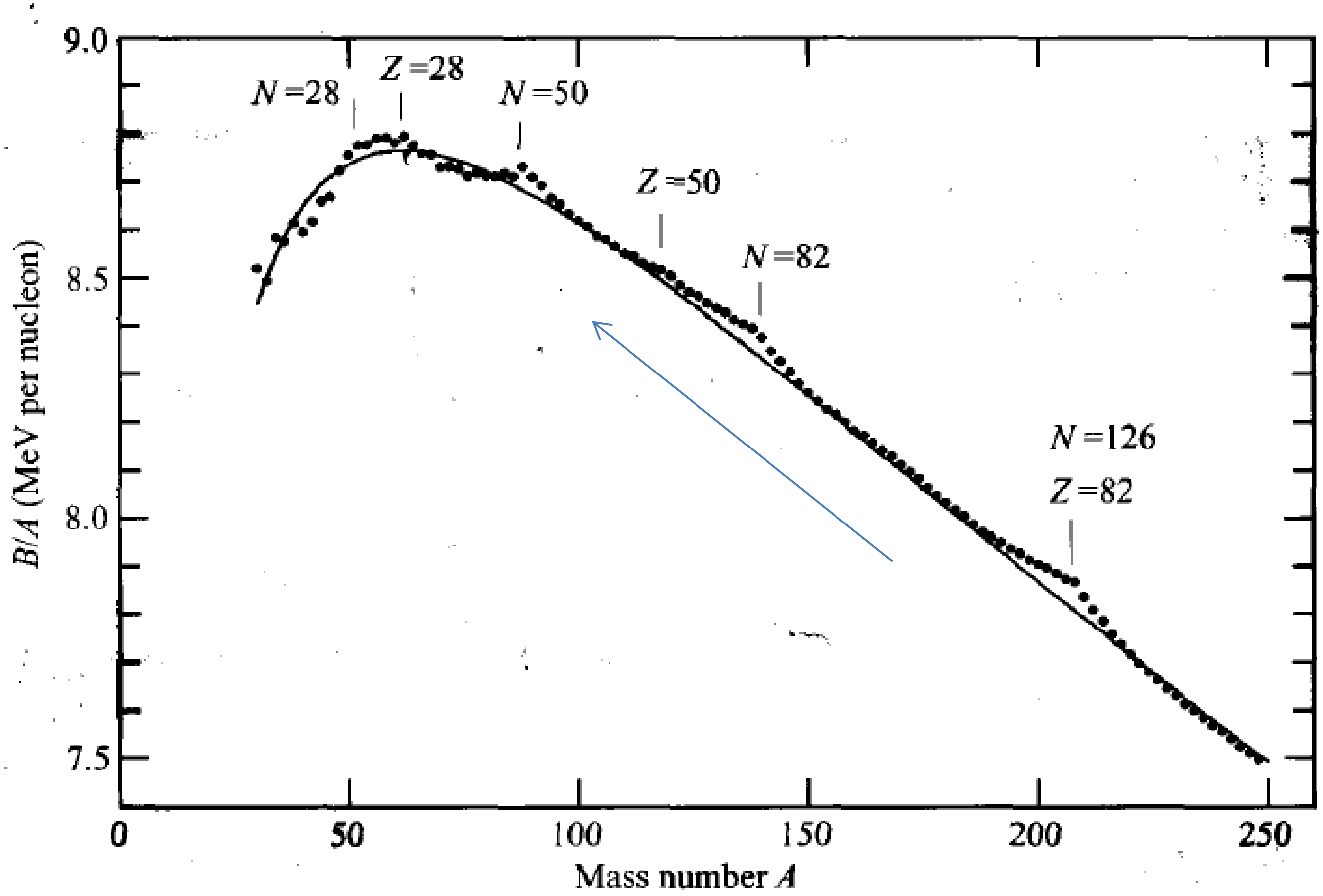


Figure 8.12 γ-ray spectrum of ^{251}Fm in coincidence with all α decays in the range 6.0 to 7.7 MeV. The spectrum was obtained with a Ge(Li) detector.

Fission



Consider $^{238}U \rightarrow {}^{119}Pd + {}^{119}Pd$

Q= -238 * (7.6 MeV) + 2*119* (8.5 MeV) = 214 MeV

What are these energies here?

Is this a lot?

- Coal burning: $C+O_2 \rightarrow CO_2$
  - → 10^5 J/g
- 214 MeV *N_A/238g/mol * 10^{-13} J/MeV
  - →10^{11} J/g

Clearly, nuclear fission seems to be an efficient source of energy

Spontaneous fission

- Break-up of nuclei without external action
- Daughters are of approximately equal mass (~difference of about 45 in mass number)
- Daughters usually β- decay into stable state.
- For even heavy nuclei, the half times are orders of magnitude longer than e.g. α decay
- Experimentally spontaneous fission is observed in heavy nuclei, can this be understood from the B-W formula? Not really.

Spontaneous fission

- Spontanoues fission seen as a result of an extreme deformation of the nucleus.
- Is there a situation where the spherical nucleus is not the energetically favoured situation (nucleus would then prefer to be in a non-spherical state) ??
- From the B-W formula this becomes a competition between the Coulomb and Surface terms.

Spontaneous fission

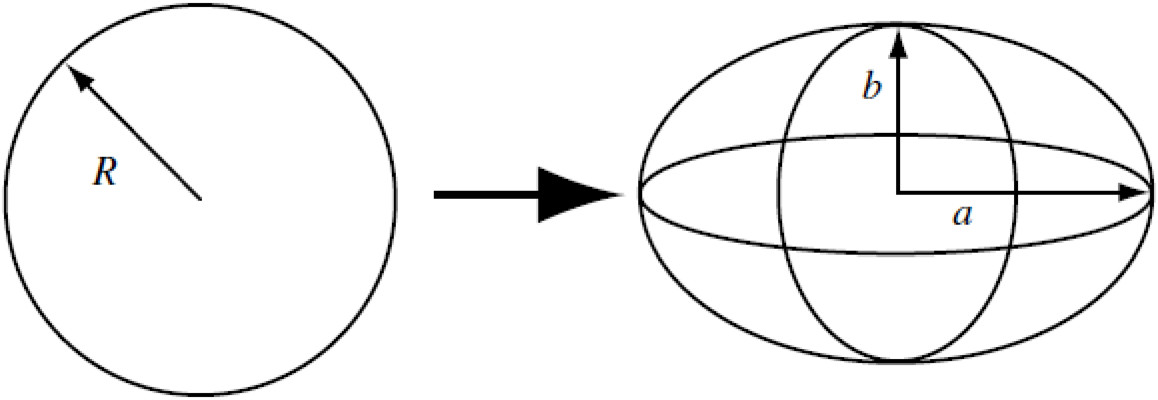


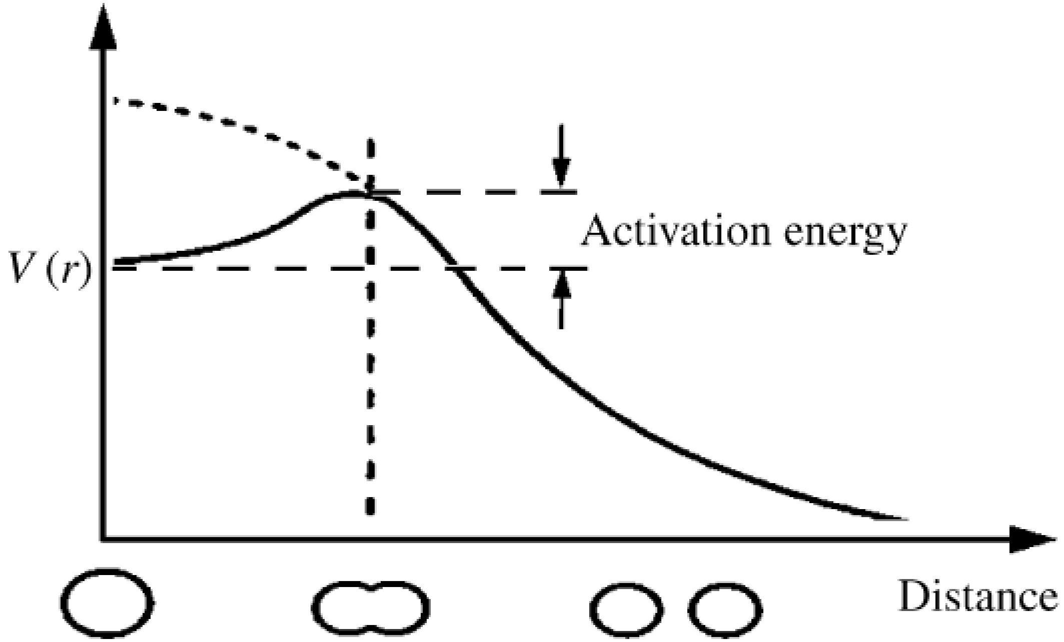
Figure 2.14 Deformation of a heavy nucleus

$$\Delta E = (E_s + E_c) - (E_s + E_c)_{\mathrm{SEMF}} = \frac{\varepsilon^2}{5}\left(2a_s A^{\frac{2}{3}} - a_c Z^2 A^{-\frac{1}{3}}\right). \longrightarrow \frac{Z^2}{A} \geq \frac{2a_s}{a_c} \approx 49,$$

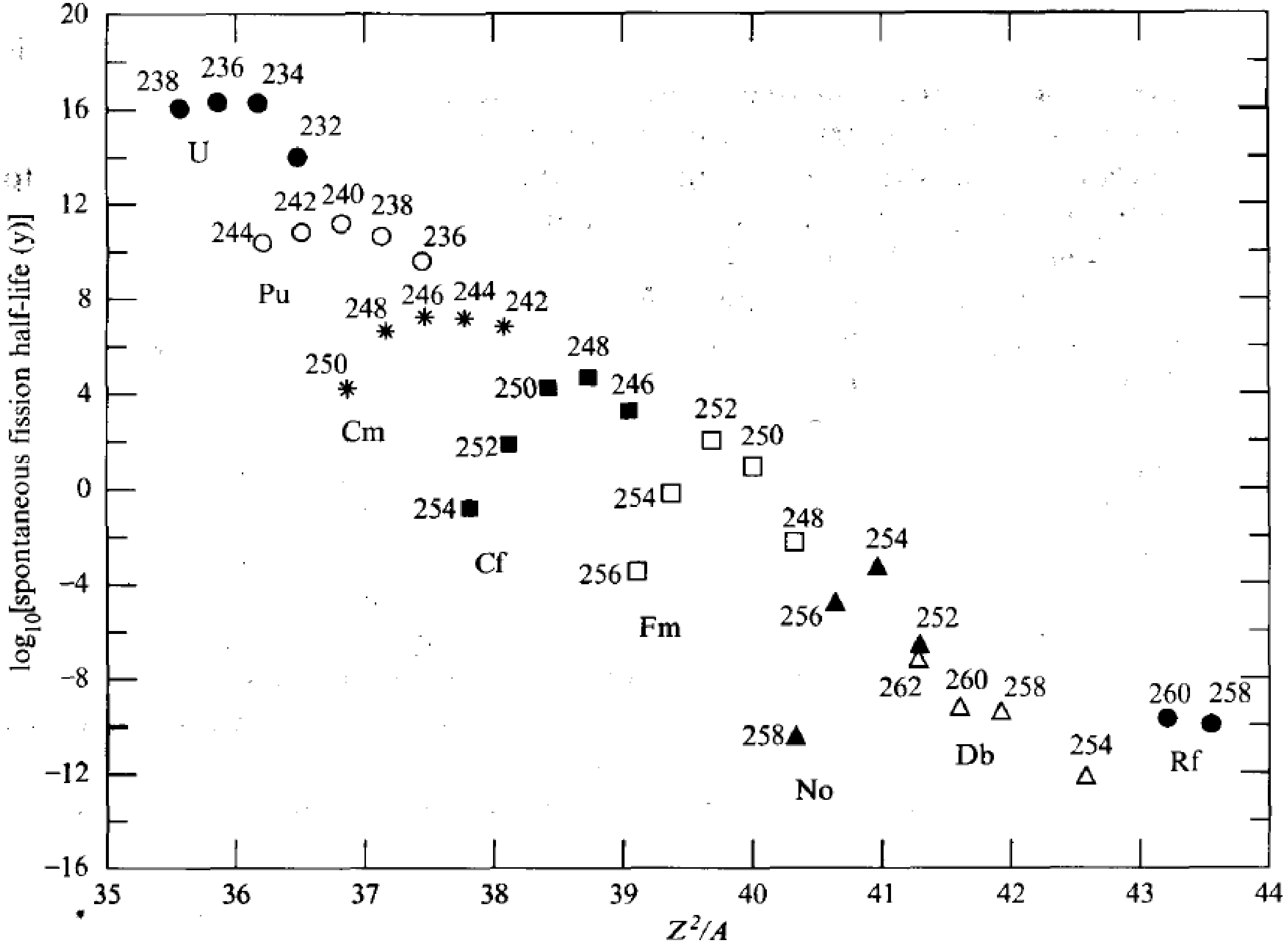
In reality heavy nuclei are non-spherical already in the ground state, which leads to some double counting.

Exercise: redo this exercise only considering split in 2

Spontaneous fission



Binding energy varies due to increase in surface energy and decrease in Coulumb energy
→ Coulumb barrier



$$\frac{Z^2}{A} \geq \frac{2a_s}{a_c} \approx 49,$$

We will talk about induced fission in the lecture about applications of nuclear physics.

Nuclear reactions

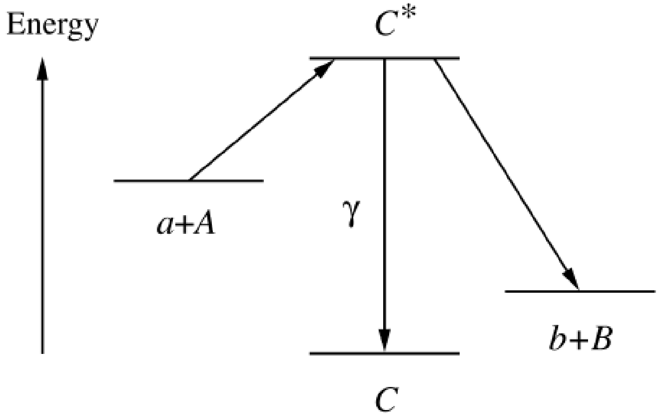
- Direct reactions (transit time ~interaction time (10^{-22}s)
  - elastic,inelastic scattering
  - Pick-up, stripping

$^{16}\mathrm{O}(p,d)^{15}\mathrm{O}$ $\qquad$ $p + {}^{16}\mathrm{O} \rightarrow d + {}^{15}\mathrm{O},$

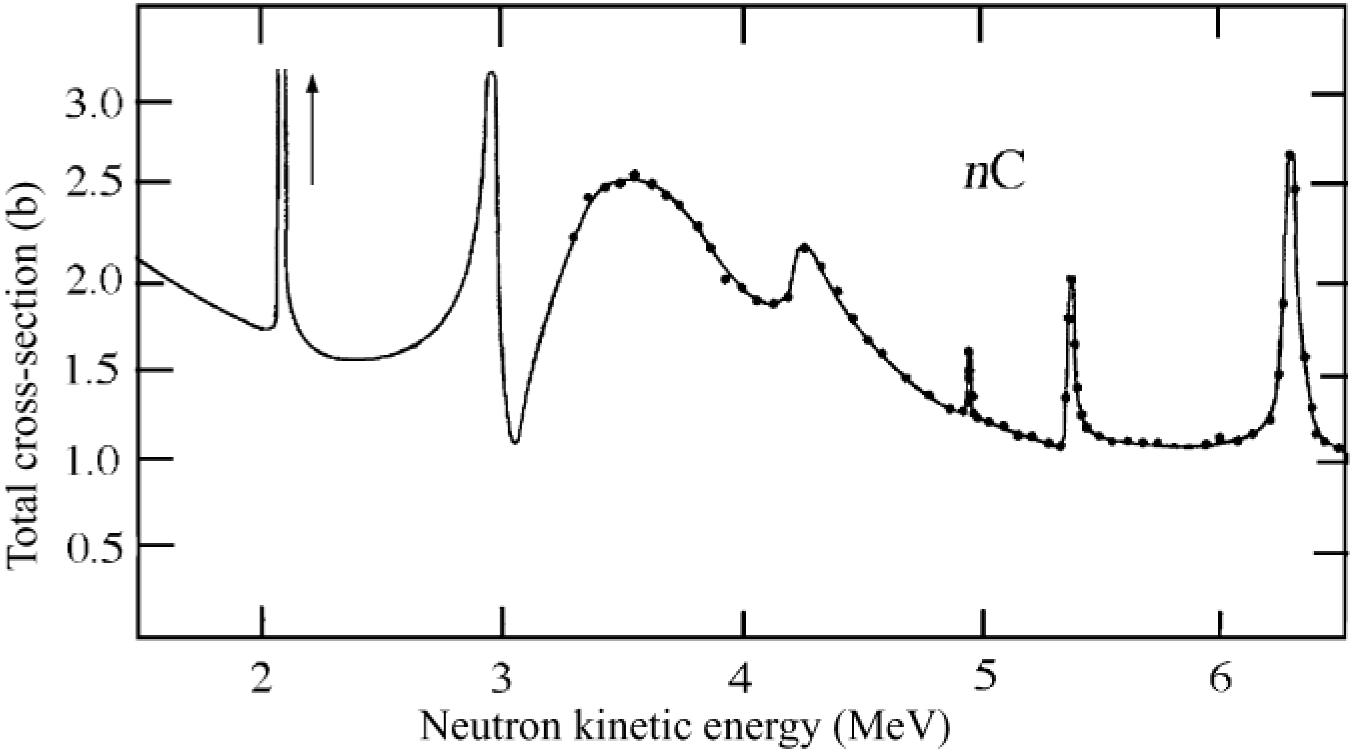
We already considered one of the direct reactions, which one?

Nuclear reactions

- Compound reactions
  - probe becomes loosely bound to the nucleus
  - energy of the probe gets distributed to all constituents, time-scale 10^{-15}-10^{-16}s



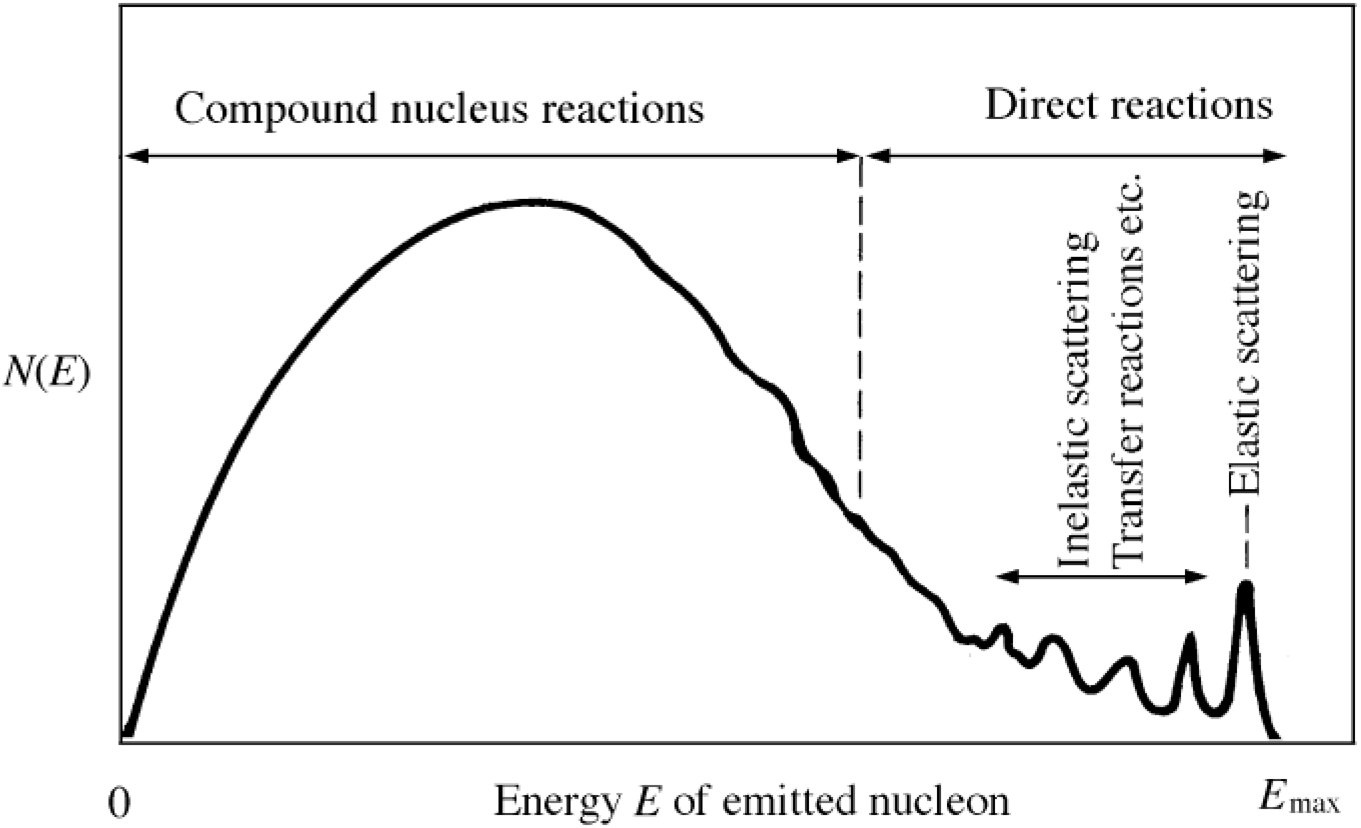
Cross-section, compound reactions



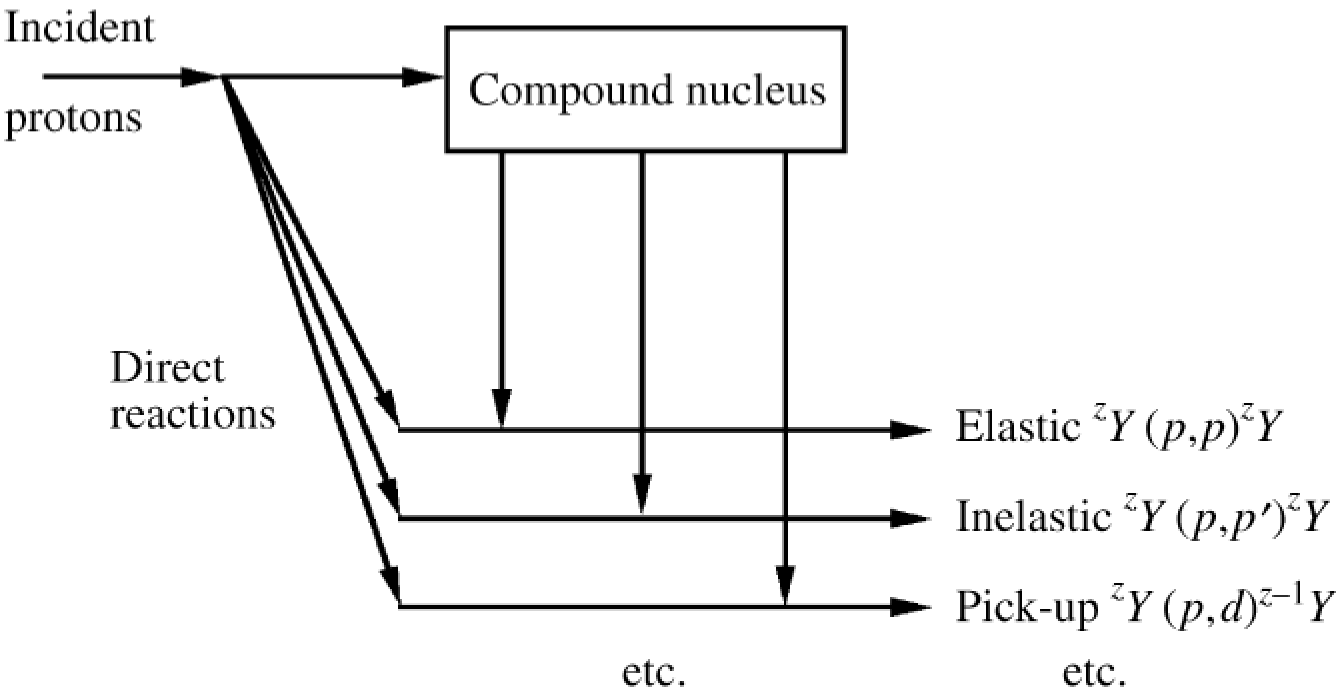
$$\Delta E \Delta t \geq \hbar.$$

Neutron- Uranium compound reactions will be important when considering induced fission.

Total cross-section (b) ?


Compound nucleus reactions
Direct reactions
Inelastic scattering
Transfer reactions etc.
Elastic scattering
N(E)
0
Energy E of emitted nucleon
E max

Compound nuclear reactions



Incident protons

Compound nucleus

Direct reactions

Elastic $^{z}Y\,(p,p)^{z}Y$

Inelastic $^{z}Y\,(p,p')^{z}Y$

Pick-up $^{z}Y\,(p,d)^{z-1}Y$

etc.

etc.

Summary of today's lecture

- We had an initial discussion of decays and reactions (we will revisit this subject in the next-to-next lecture)
- Introduced: half-life (decay-constant) and decay chains and Q-value
- Discussed initially: β decay, α decay, γ decay, spontaneous fission.

β-decay

- Is arguably the most important decay mode

- Using the B-W formula and β-decay we can derive the valley of β stability (← mass parabolas)

α-decay/Fission reactions

- α decay: prevalent to high mass nuclei, can be derived from B-W formula
- Spontaneous fission: can be derived from considering deviations of nuclei from spherical shape.
- Nuclear reactions: direct (like in particle physics, new particles can be found), capture (compound reactions).

Next lecture

Nuclear models.