Nuclear physics

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Office hour: Wednesday 13-14, tell me in advance if you are coming

Material

- B.R. Martin: Nuclear and Particle Physics
- J. Lilley: Nuclear Physics – Principles and Applications (Wiley, 2001)
- Some lecture notes by David Watts.
- Later on some slides from our local ”neutron star expert”, Stephan Rosswog

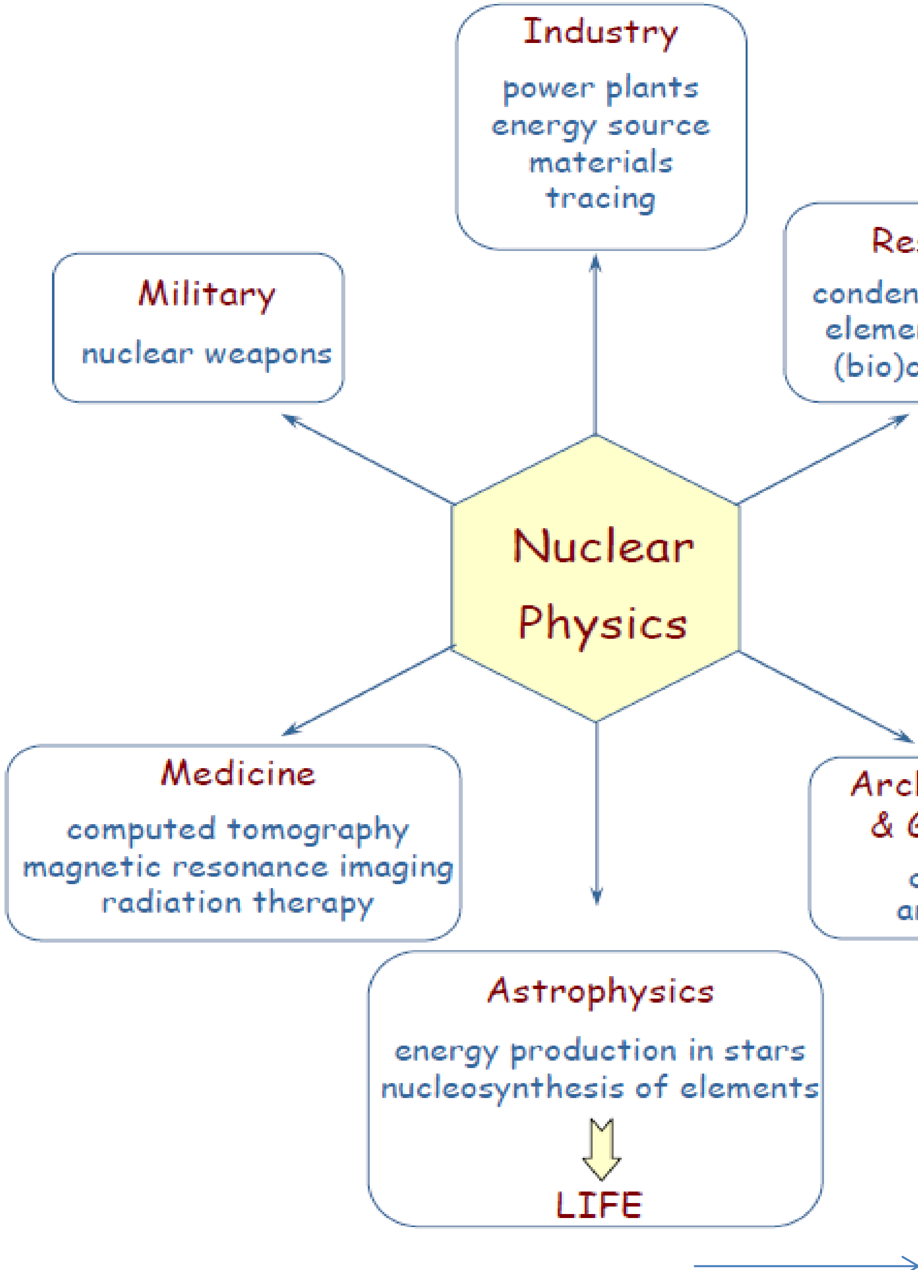
Practical issues

- Hand-in exercises will be distributed **October 2**, deadline will be **October 9**
- Nuclear physics raises a few questions related to research ethics, the discussion of which therefore will be part of the hand-in exercise.
- These will be also discussed in a seminar **October 9.**

Letcure 1: Nuclear shapes and sizes, the Bethe-Weizsäcker formula

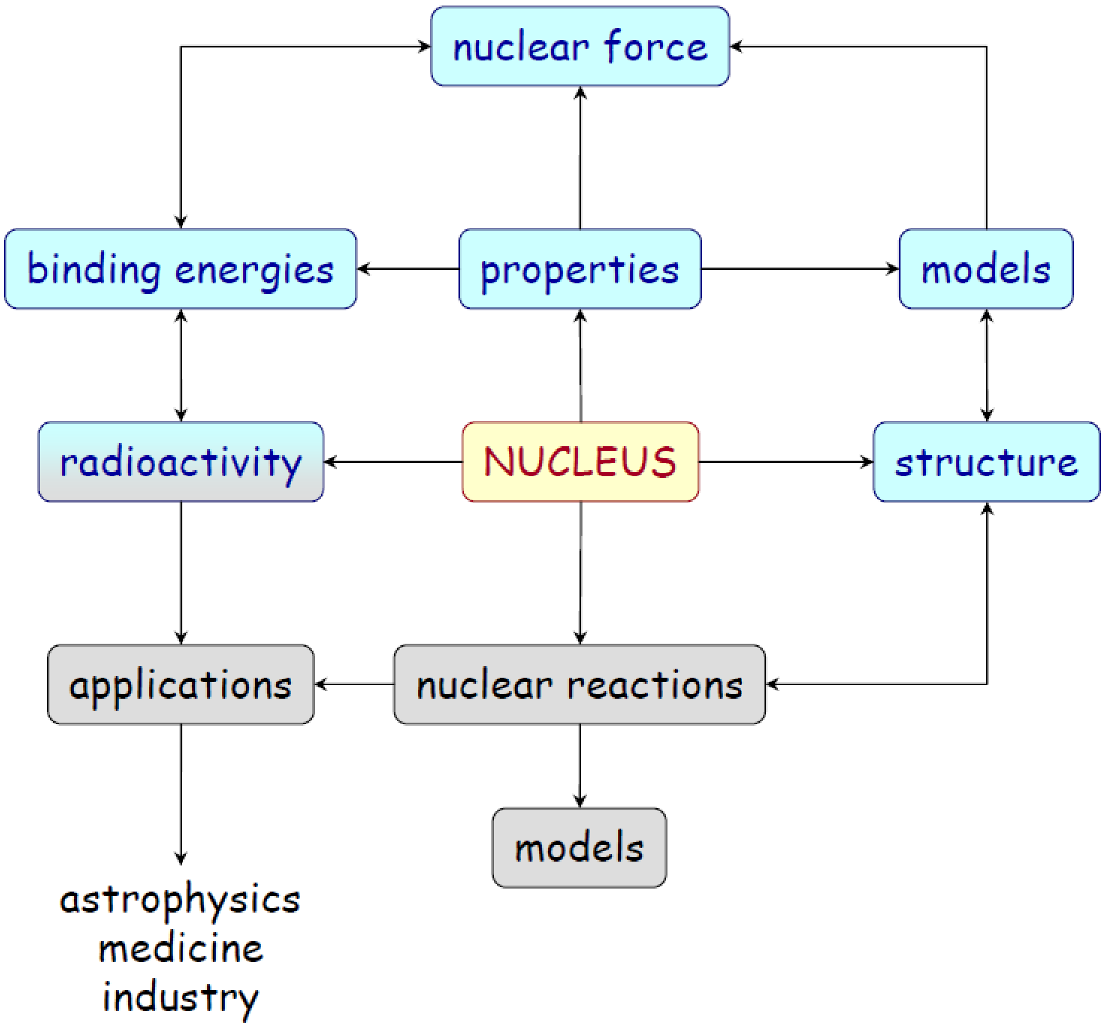
Nuclear- and Particle Physics

- Particle Physics: Two-body, or few body, mainly point-like
- Nuclear physics: Many-body
- In principle, it should be possible to calculate nuclear properties from Quantum Chromo Dynamics (strong force dominant in nuclei). In practice: semi-empirical and phenomenological models needed.

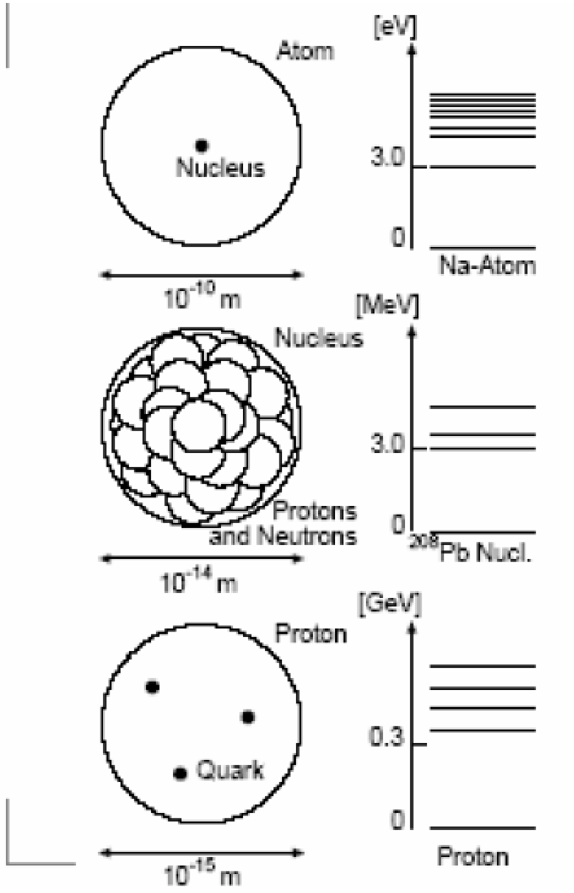


In house research:
searches for Dark Matter!

Course overview



Scales



Typical energy scale in nuclei (MeV) is much higher than in atomic case (eV)

Nuclei are dense objects: $1cm^3$ has mass ~ $2.3x10^{11}$ kg (equivalent to 630 empire state buildings!!)

which quarks make the proton? Exicted state of proton?
1e-10 , what kind of radiation?

Nuclear properties

Z – atomic number = the number of protons,

N – neutron number = the number of neutrons,

A – mass number = the number of nucleons, so that $A = Z + N$.

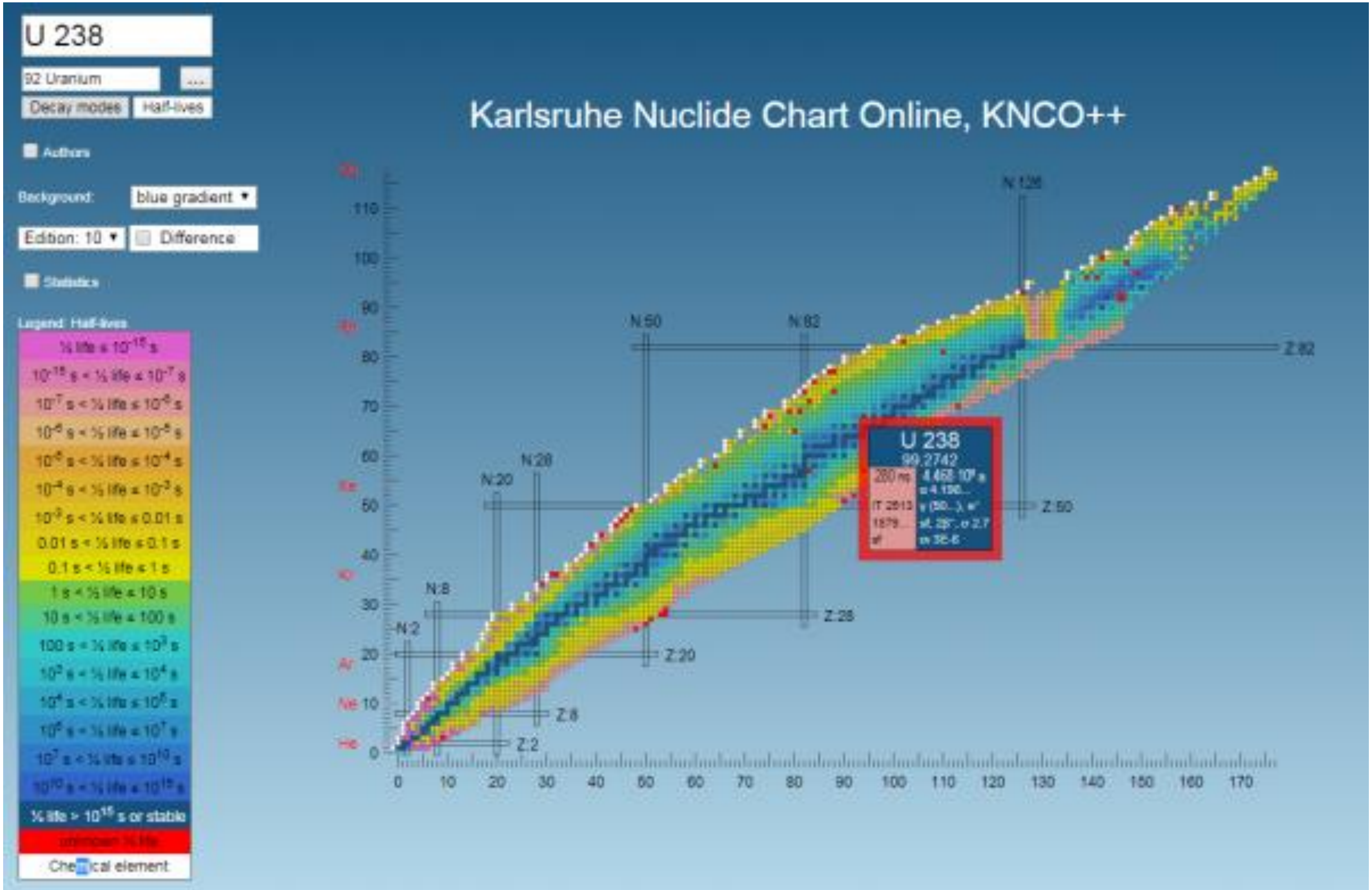
$$ {}^{A}_{Z}X_{N} $$

nuclides with the same mass number are called *isobars*,

nuclides with the same atomic number are called *isotopes*,

nuclides with the same neutron number are called *isotones*.

Nuclide chart:



What does it show?

Masses and binding energy

Mass:

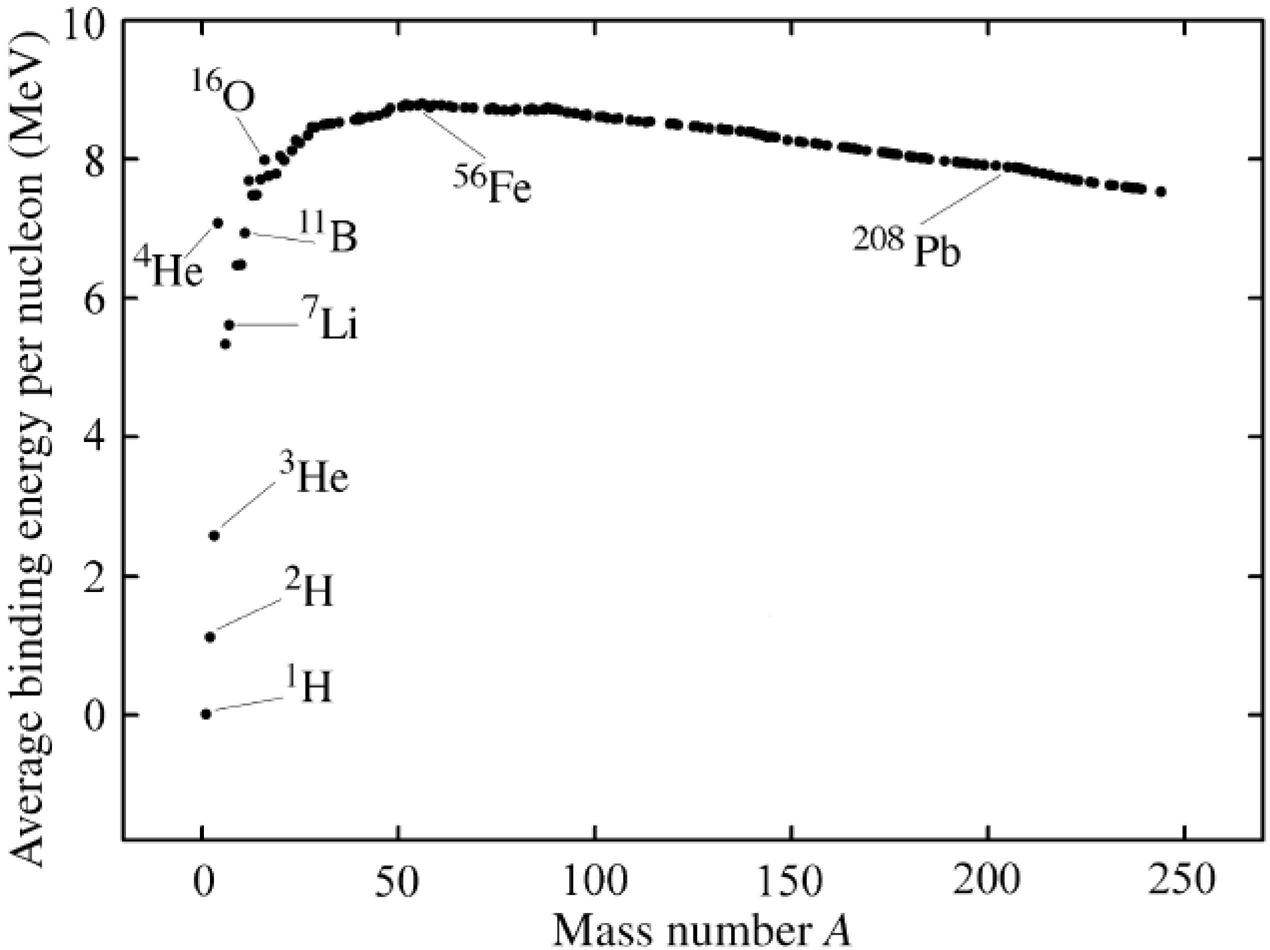
$$M(Z,A) < Z\,(M_p + m_e) + N\,M_n\,.$$

Mass deficit:

$$\Delta M(Z,A) \equiv M(Z,A) - Z\,(M_p + m_e) - N\,M_n$$

Binding energy:

$$-\Delta M c^2$$

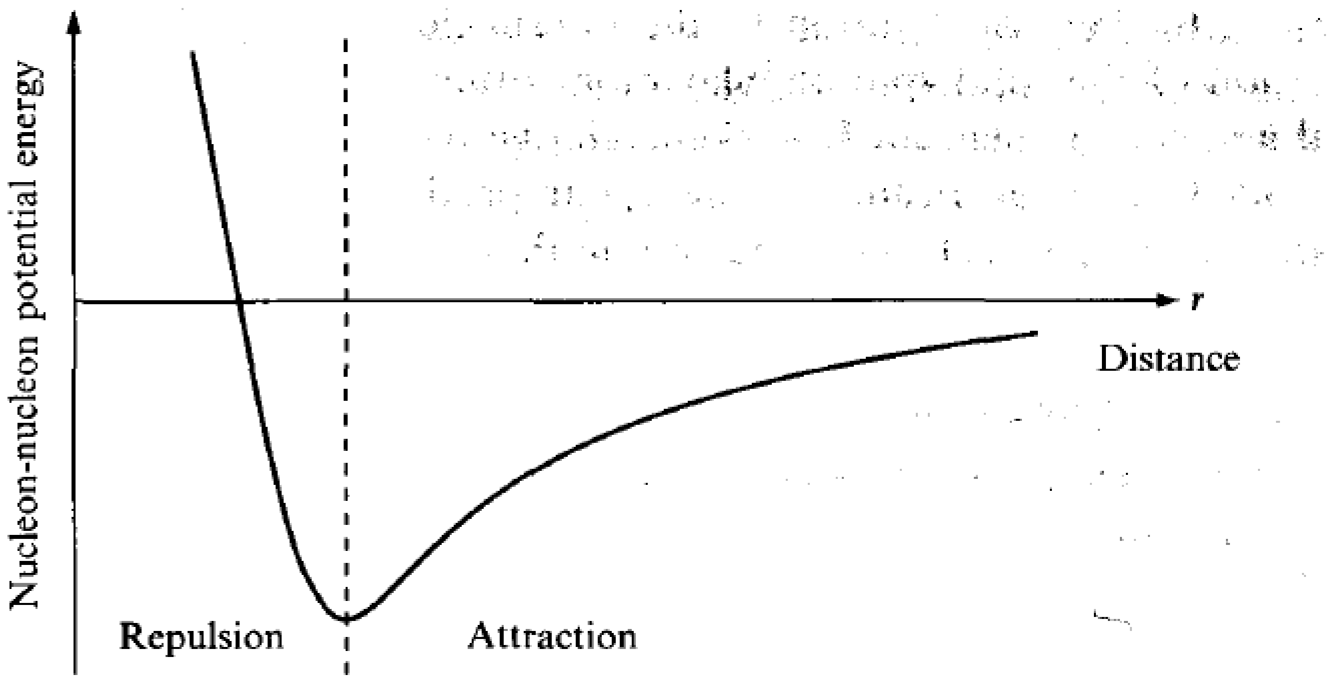


Martin, p 34: mass spectrometer

We will spend a signficant part of our lectures on understanding this chart.

How could you produce energy here?

Nuclear force



~ .5 fm

+ spin dependent

Nuclear shapes and sizes

- Charge distribution

  use electrons as probe → point like particles, experience electromagnetic interaction only and not strong (nuclear) force,

- Mass distribution

  use hadrons as probe → strong force → α-particle

What do we define as nuclear size?

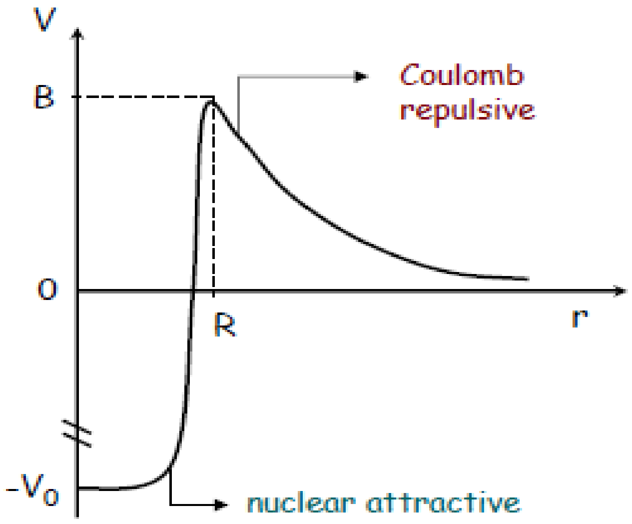
Consider the following:

- the nucleus has a net positive charge Ze (Z protons)
- take into account Coulomb + nuclear force

Coulomb → extends to ∞ as $1/R^2$

nuclear force → has short ($\sim 10^{-15}$ m) range

Resulting potential



Define:
barrier height B at a distance from centre R:

$$B = \frac{Zze^2}{R}$$

for incident charge ze

R = POTENTIAL RADIUS ⇒ related to range of nuclear force

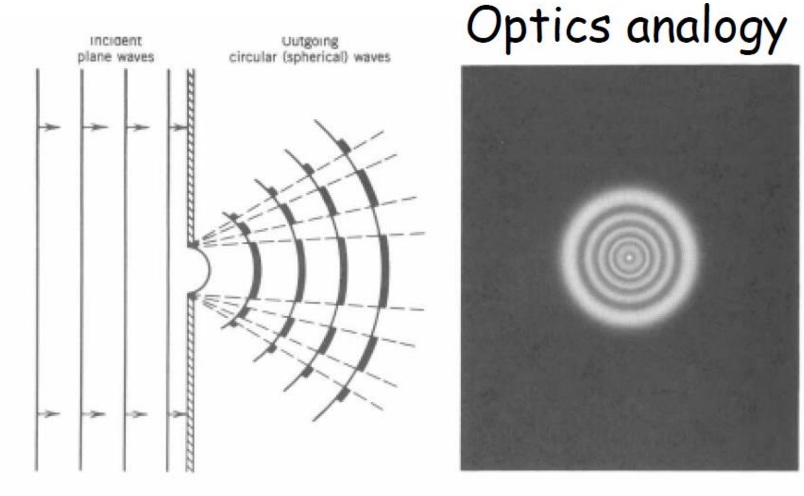
potential radius > charge (or mass) radius

CHARGE RADIUS ⇒ related to charge distribution

Electron scattering

Consider angular distribution:

Optics analogy



$E \sim pc$

$p = h/\lambda$ $\Rightarrow$ $E = \frac{\hbar c}{\lambda} = \frac{197.3 \text{ MeV fm}}{0.5 \text{ fm}} \sim 400 \text{ MeV}$

assume nucleus behaves as a circular disk

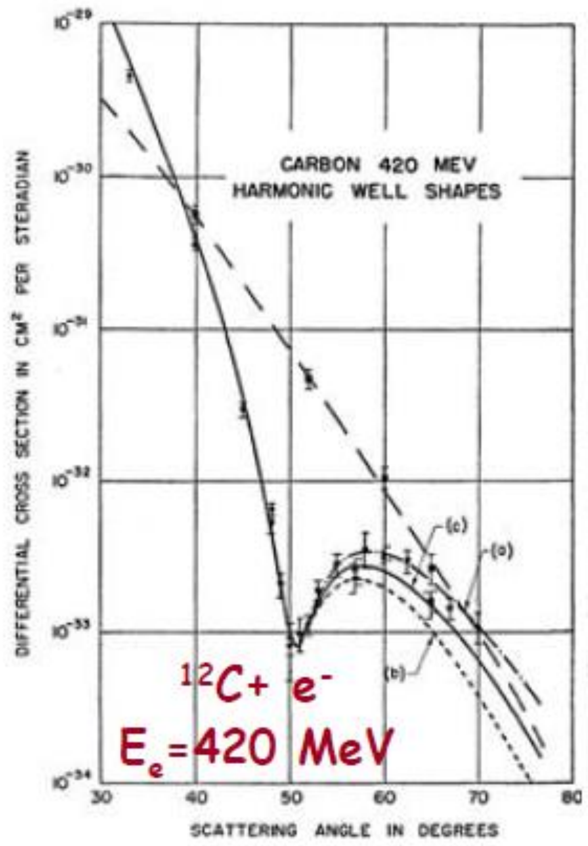
⇓

1st minimum at θ :

$$\sin\theta = \frac{1.22\lambda}{2R}$$

R = CHARGE RADIUS

Electron scattering



E = 420 MeV ~ pc

$\lambda = h/p \sim 3$ fm

$\Theta = \sin^{-1}(1.22\lambda/d) = 50^o$ for ^{12}C

$R(^{12}C) = 2.5$ fm

Formula: 2.74 fm

In real life, the nucleus will not be a disk.

Rutherford cross-section and charge distribution.

p 27 in Martin
Tutorial today.

Spin0 point-like probe
charged point-like target

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2 \sin^4(\theta/2)},$$

Including spin of electron:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \left[1 - \beta^2 \sin^2(\theta/2)\right],$$

Including non-point like
nature of the target:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{expt}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left|F(\mathbf{q}^2)\right|^2. \qquad \mathbf{q} \equiv \mathbf{p} - \mathbf{p}'$$

Fourier transform of charge
distribution:

$$F(\mathbf{q}^2) = \frac{4\pi\hbar}{Zeq} \int_0^\infty r\rho(r) \sin\left(\frac{qr}{\hbar}\right) \mathrm{d}r,$$

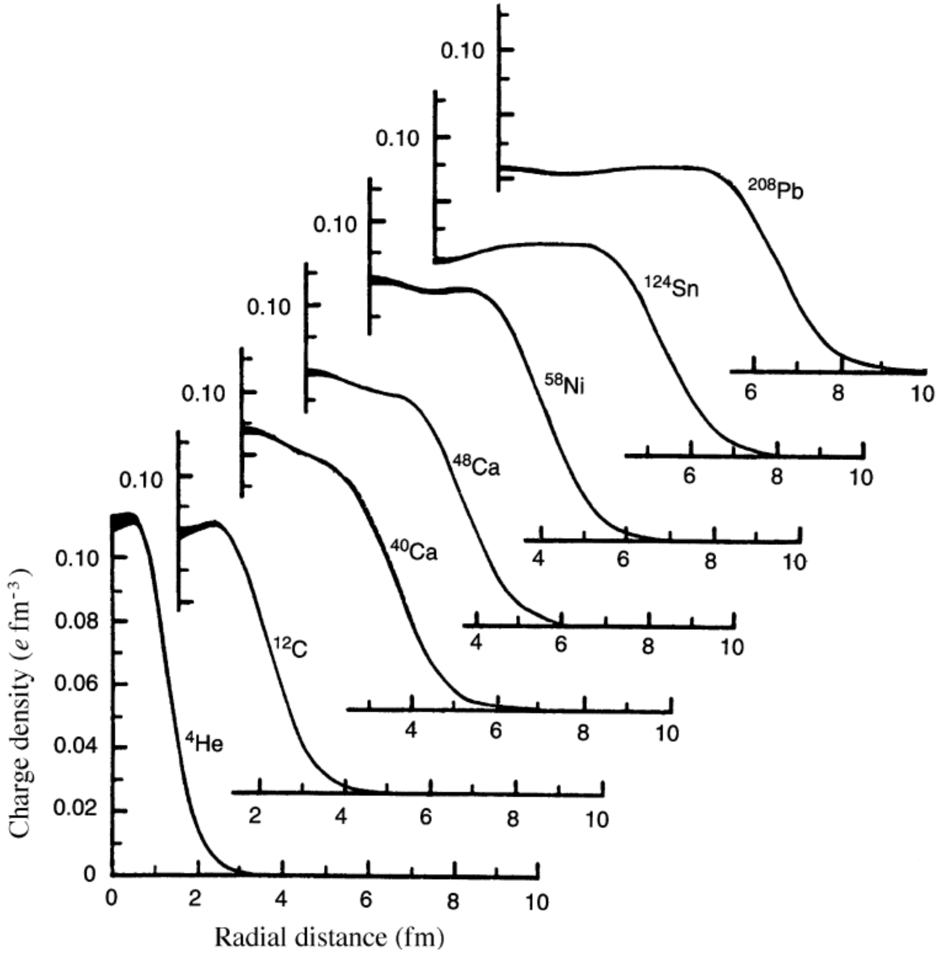
Example

$$\rho(r) = \text{constant}, \quad r \leq a$$
$$= 0 \quad r > a$$

$$F(\mathbf{q}^2) = 3[\sin(b) - b\cos(b)]b^{-3},$$

$$b \equiv qa/\hbar.$$

Nuclear sizes



$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r - R}{a}\right)}$$

R = Radius at half density

a = diffuseness parameter

ρ_0 ~ central density

nuclei ~ const. density

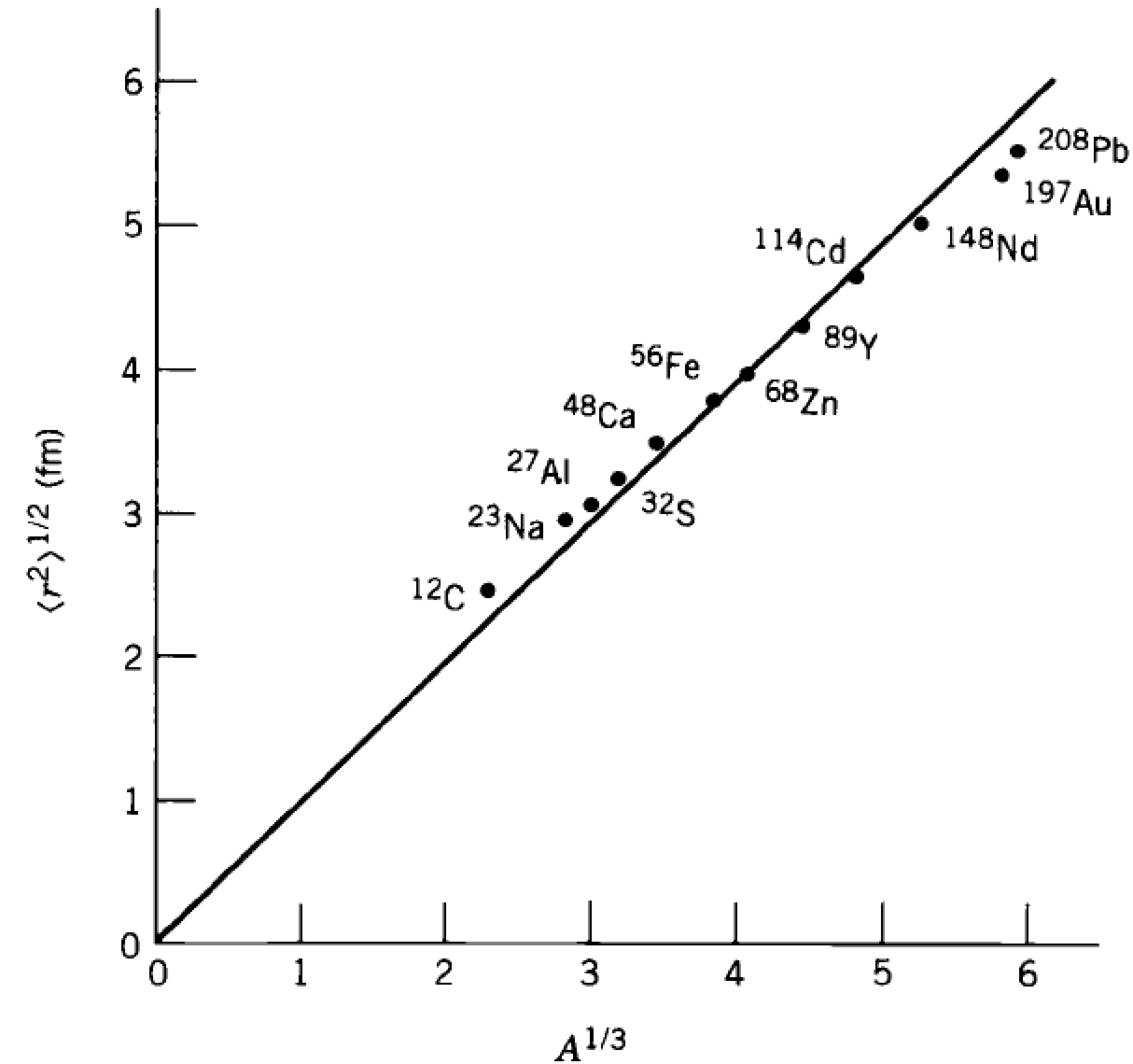
$M = V \times \rho$

$A = 4/3 \pi R^3 \rho$

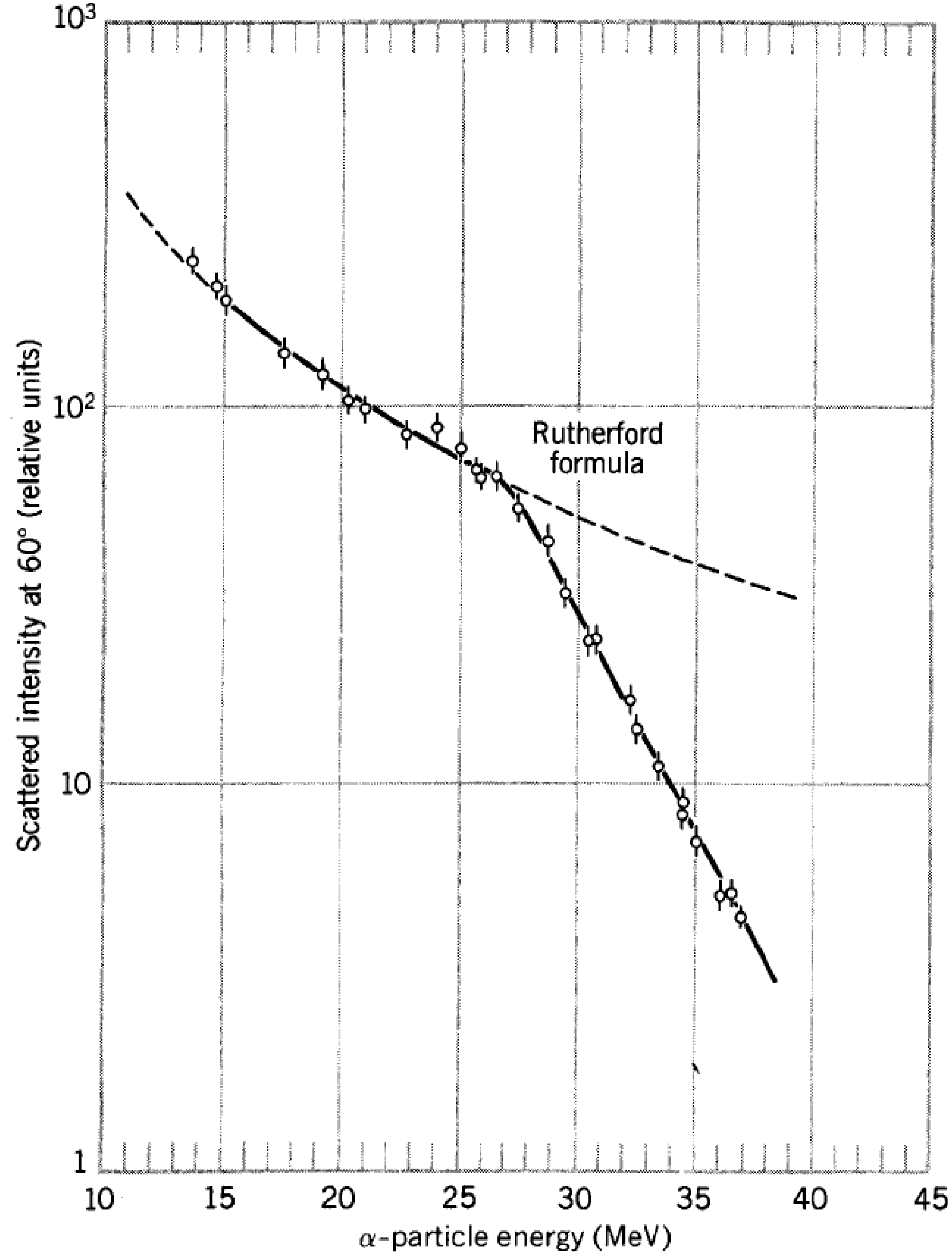
$$R = r_0 A^{1/3}$$

r_0 ~ 1,2 fm

This scaling implies that the volume per nucleon is roughly constant – like in an uncompressible fluid. This would not be the case if the strong force had infinite range



6
5
4
3
2
1
0
⟨r²⟩^1/2 (fm)
208Pb
197Au
148Nd
114Cd
89Y
56Fe
68Zn
48Ca
27Al
32S
23Na
12C
0
1
2
3
4
5
6
A^1/3



Rutherford formula
Scattered intensity at 60° (relative units)
α-particle energy (MeV)
10³
10²
10
1
10
15
20
25
30
35
40
45

Bethe-Weizsäcker Mass Formula

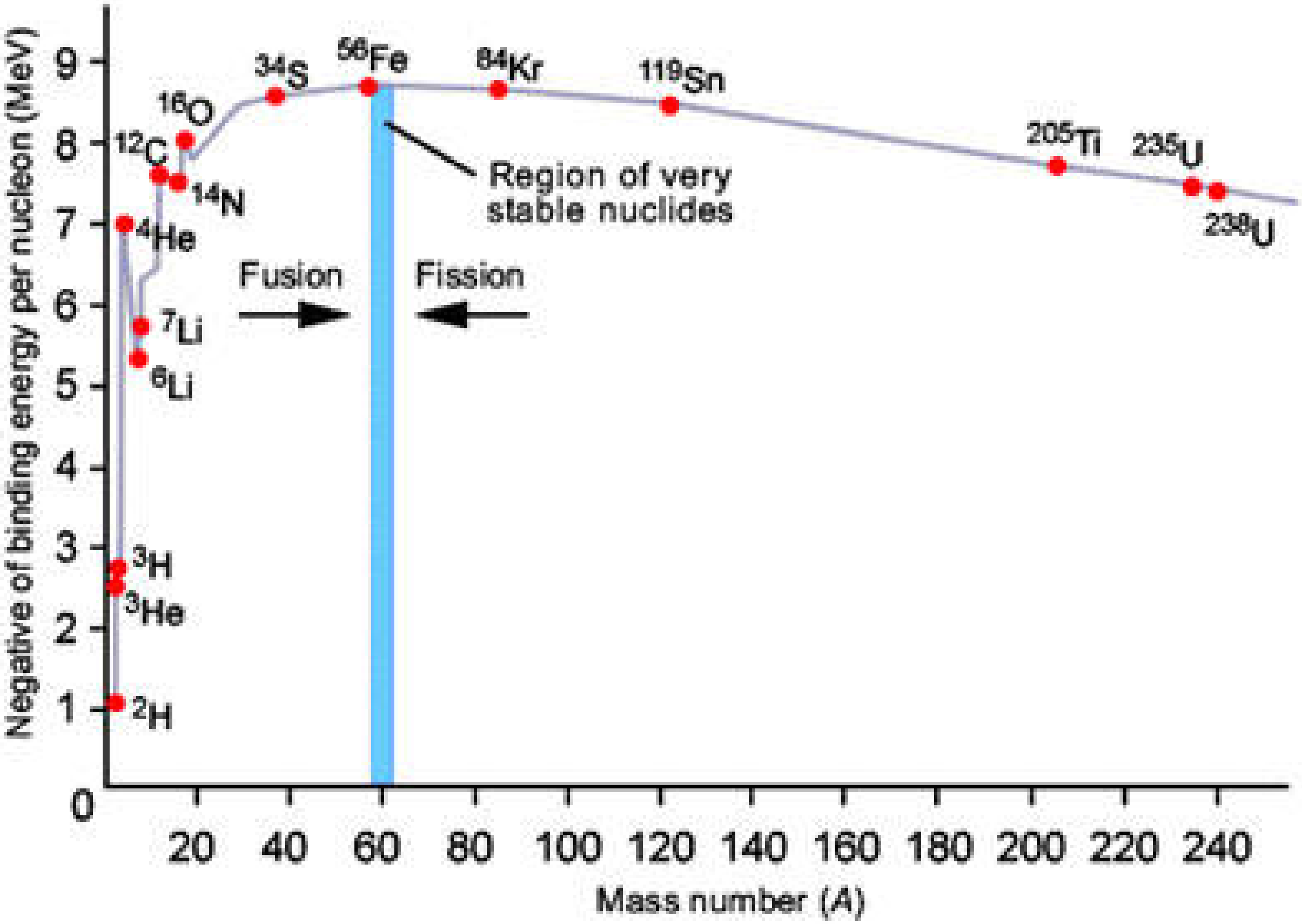
C.v. Weizsäcker: "Zur Theorie der Kernmassen" (1935)

- aka Semi-Empirical Mass Formula

Hans Bethe (1906-2005)
Los Alamos badge

Carl Friedrich von Weizsäcker (1912- 2007, rhs) Who is on the left?

Binding energy



Bethe-Weizsäcker formula

- The Bethe-Weizsäcker formula provides a formula for the mass of nuclei, inspired by the ”Liquid drop Model”

nucleus regarded as collection of neutrons and protons
forming a droplet of incompressible fluid

$$M(Z, A) = \sum_{i=0}^{5} f_i(Z, A).$$

Mass term

$$f_0(Z, A) = Z\,(M_p + m_e) + (A - Z)M_n.$$

Volume term

$$f_1(Z, A) = -a_1 A.$$

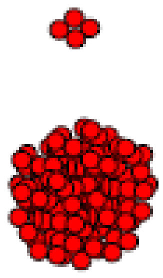
Volume energy: Each nucleon only feels interaction of close neighbours due to short range of nuclear force - *Gives a* positive binding energy which is roughly the same for each nucleon

N.B: ”-” sign → positive contribution to binding energy

Surface term

$$f_2(Z, A) = +a_2 A^{\frac{2}{3}},$$

Surface correction: Nucleons near surface of nucleus surrounded by fewer nucleons and will therefore experience less attractive potential energy than those inside the nucleus. Compensate with a reduction in binding energy proportional to number of nucleons in nuclear surface



Coulumb term

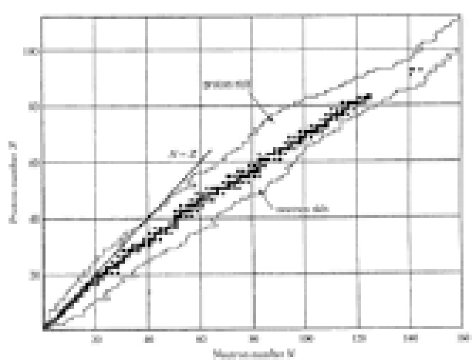
$$f_3(Z, A) = +a_3 \frac{Z(Z-1)}{A^{\frac{1}{3}}} \approx +a_3 \frac{Z^2}{A^{\frac{1}{3}}},$$

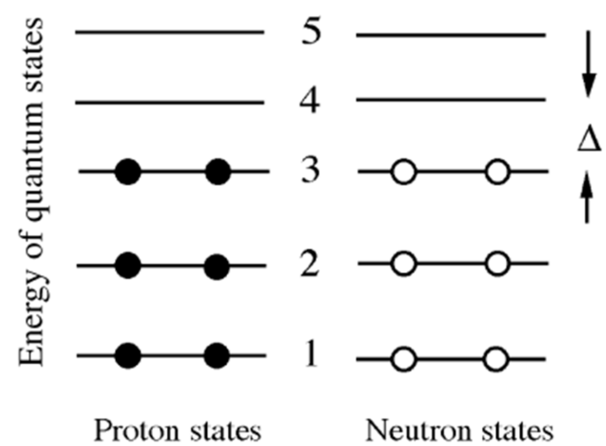
Coulomb energy: Nucleus has total charge Ze confined to a sphere of radius R. The resultant potential energy given by electrostatic theory $= \frac{3}{5} \frac{(Ze)^2}{4\pi\varepsilon_0 R}$

Assymetry term

$$f_4(Z, A) = +a_4 \frac{(Z - A/2)^2}{A}.$$

Symmetry term:
Stable light nuclei have N~Z (i.e. *A*~2Z). If *A* deviates from 2Z then binding energy is reduced.





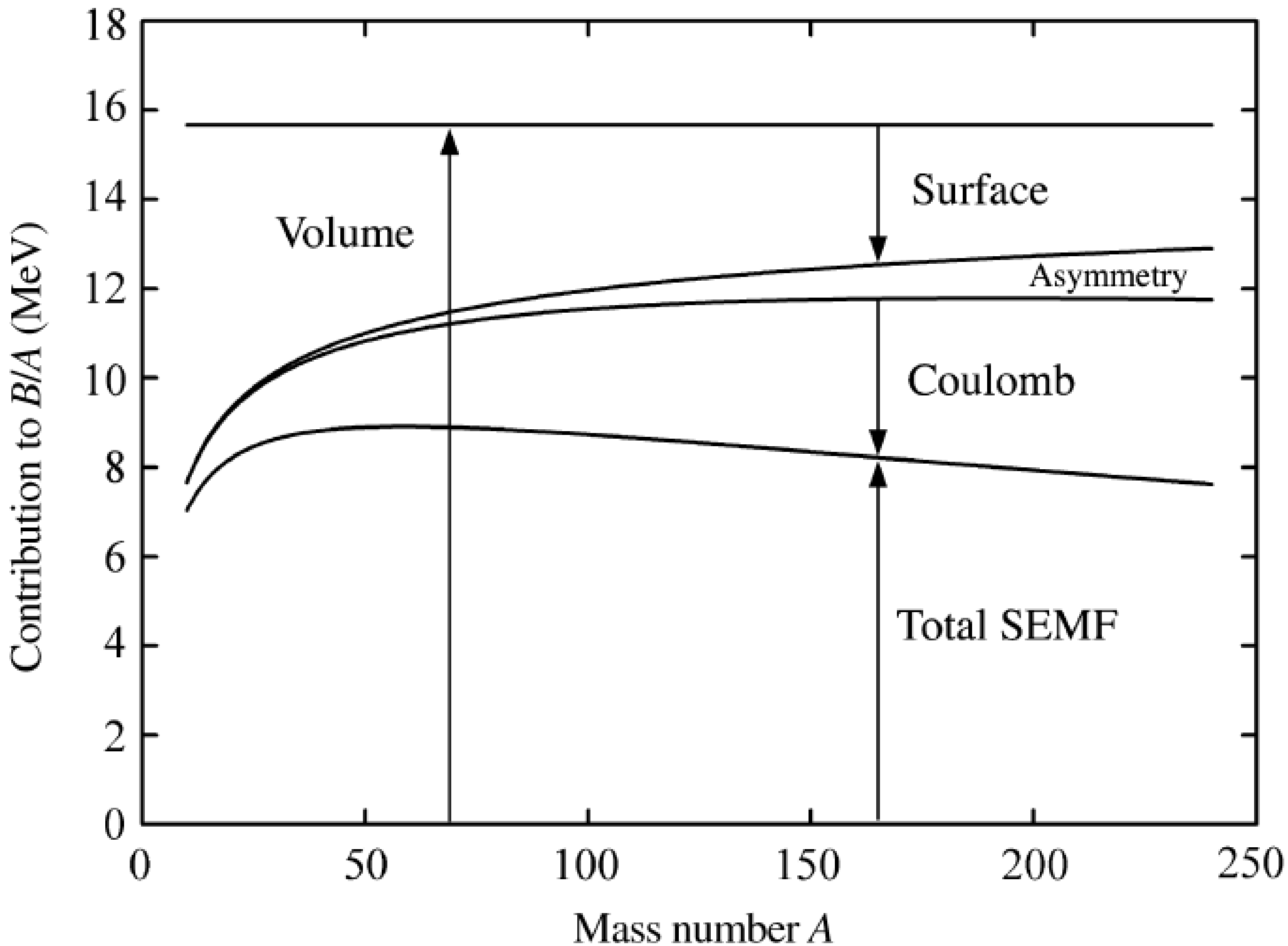
Pairing term

$$
\begin{aligned}
f_5(Z, A) &= -f(A), &&\text{if } Z \text{ even}, A - Z = N \text{ even} \\
f_5(Z, A) &= 0, &&\text{if } Z \text{ even}, A - Z = N \text{ odd; or}, Z \text{ odd}, A - Z = N \text{ even} \qquad (2.52) \\
f_5(Z, A) &= +f(A), &&\text{if } Z \text{ odd}, \ A - Z = N \text{ odd}
\end{aligned}
$$

Pairing term: Most stable nuclei have Z even and N even and therefore A even (even-even nuclei). Increases binding for even-even nuclei and reduces for odd-odd.

Nuclear species	Number observed [stable + longlived ($T_{1/2} > 10^9$y)]
Odd-odd	6 + 4 = 10
Odd-even	51 + 3 = 54
Even-odd	55 + 3 = 58
Even-even	166 + 11 = 177
	278 + 21 = 299

Contributions to the binding energy



Summary

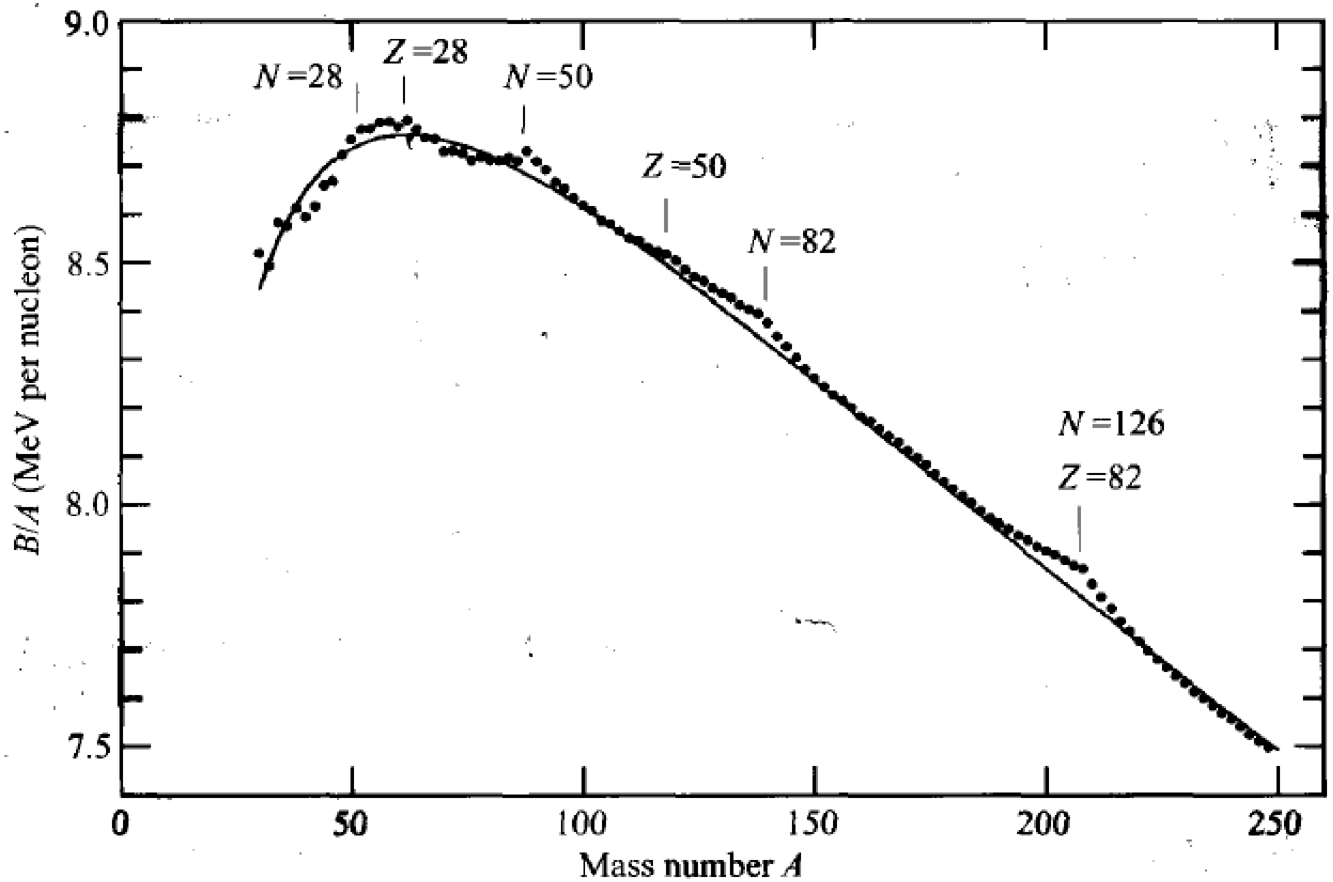
$$B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

a_v = 15.6 MeV
a_s = 17.2 MeV
a_a = 23.3 MeV
a_C = 0.70 MeV

Note (different notation used by MANY authors):

- change in notation for assymetry term (x 4 for the coefficient)
- $\delta = f(A) = a_5 A^{-\frac{1}{2}}$

BW formula and magic numbers



Summary of today's lecture

- Unlike particle physics, nuclear physics considers multi-body, composite objects → phenomenolgical modelling → many applications.
- Typical sizes of nuclei: ~fm,

  Typical energies ~MeV,

  Typical densities: 10^{14}g/cm^3

Summary of today's lecture

- The charge distribution (and size) of a nucleus can be experimentally determined by measuring the angular distirbution of electron scattering
- The mass distribution can be determined by scattering of α particles (the original Rutherford scattering experiments)

$$R = r_0 A^{1/3}$$

$r_0 \sim 1{,}2$ fm

Summary of today's lecture

- We started to talk about modelling the binding energy of nuclei.
- Nuclear models should be able to explain the (i) the dependence of binding energy on nuclear mass and (ii) nuclear reactions
- We discussed the Bethe-Weiszäcker formula, which is a parameterization of the binding energy as function of a with 4 free parameters.

Summary of today’s lecture

- In next lecture (”Decays and Reactions”) we will see how we can use the Bethe-Weizsäcker formula to predict the ”valley of stability” → mass parabolas.
- We saw also that the B-W formula gives a good description of the binding energy , but there are features that we do not understand: e.g. magic numbers, This will lead us to consider corrections to the B-W formula