Lecture 2

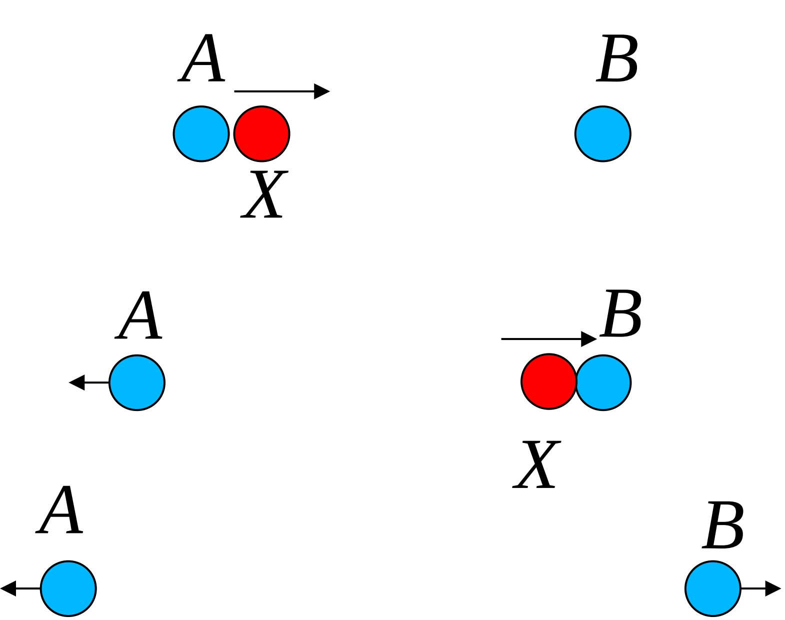
Outline

- Conceptual overview
- Scattering (theory)
  - Identify the observables that can be predicted
- Scattering (experiment)
  - Ensure that the observables which are measured are those which can be predicted !
- Decays

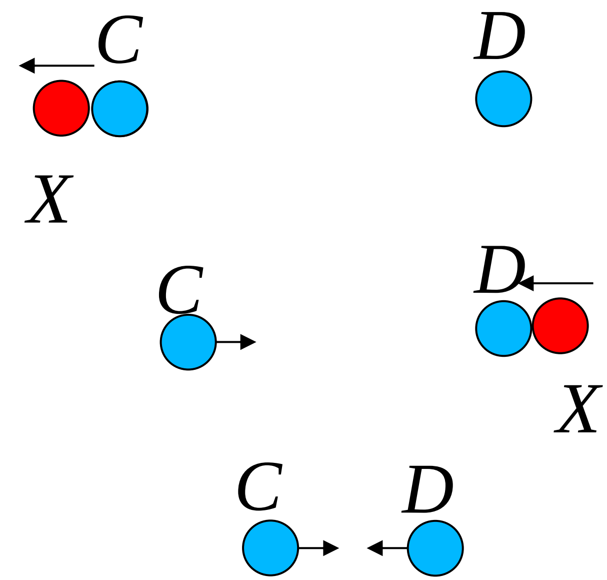
How particles interact

Simple cartoon picture.

Particles interact via the exchange of a force carrying particle (propagator)



Repulsion



Attraction

Particle A emits particle X which is absorbed by particle B.

Particle C emits particle X

Heisenberg's uncertainty principle $\Delta x \Delta p \sim \hbar$.

The position of particle X is not determined (Δx)

The "quantum path" between the start and end points is not like a classical path.

The reaction can take place as per the diagram.

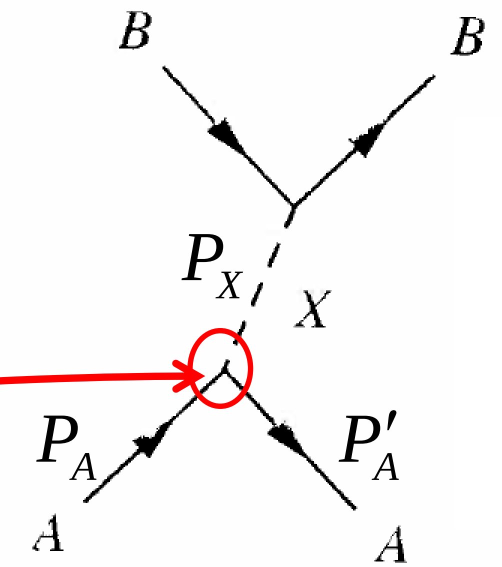
Particles C and D attract each other.

Generic particle exhcnage

A high energy particle B scatters off a static particle A via the exchange of particle X.

Particles A and B real (can be observed).

Particle X is virtual (only internal)



Lower vertex

$$P_A = \left(M_A c\ ,0\right) \quad P_A' = \left(\frac{E_A'}{c}, \vec{p}_A'\right) \quad P_X = \left(\frac{E_X}{c}, \vec{p}_A'\right)$$

$$P_A' \bullet P_A' = \frac{E'^2_A}{c^2} - p'^2_A = M_A^2 c^2 \quad \Rightarrow \quad E_A' = \sqrt{M_A^2 c^4 + p_A'^2 c^2}$$

$$P_X \bullet P_X = \frac{E_X^2}{c^2} - p'^2_A = M_X^2 c^2 \quad \Rightarrow \quad E_X = \sqrt{M_X^2 c^4 + p_A'^2 c^2}$$

Generic particle exchange

Energy conservation violation at vertex:

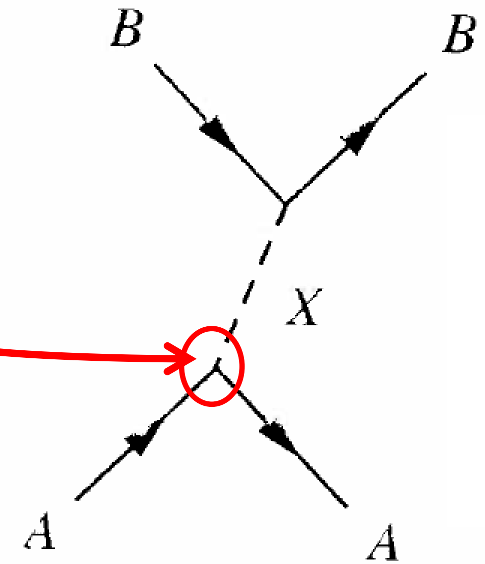
$$\Delta E = E_X - E'_A + M_A c^2 = \sqrt{M_X^2 c^4 + p_A^2 c^2} - \sqrt{M_A^2 c^4 + p_A'^2 c^2} + M_A c^2$$

$$\Delta E_{p'_A \to 0} = M_X c^2$$

$$\Delta E_{p'_A \to \infty} = M_A c^2$$

$$\Rightarrow \ \Delta E \geq 0$$

Energy is not conserved at the vertex !



Heisenberg's uncertainty principle allows

apparent energy violation ΔE for a short time interval $\Delta \tau$: $\Delta E \Delta \tau \sim \hbar$

$\Rightarrow$ Particle X speed $\sim c$

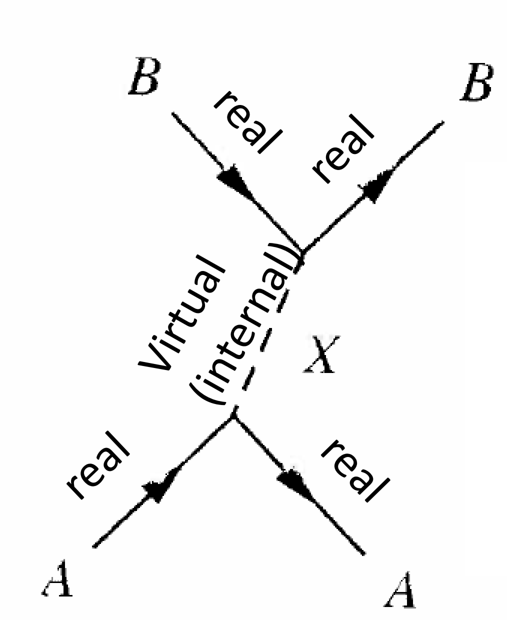
$\Rightarrow$ Particle X distance travelled $R \sim c\Delta\tau$

$\Rightarrow$ Range of force $R \sim \dfrac{\hbar}{M_X c}$

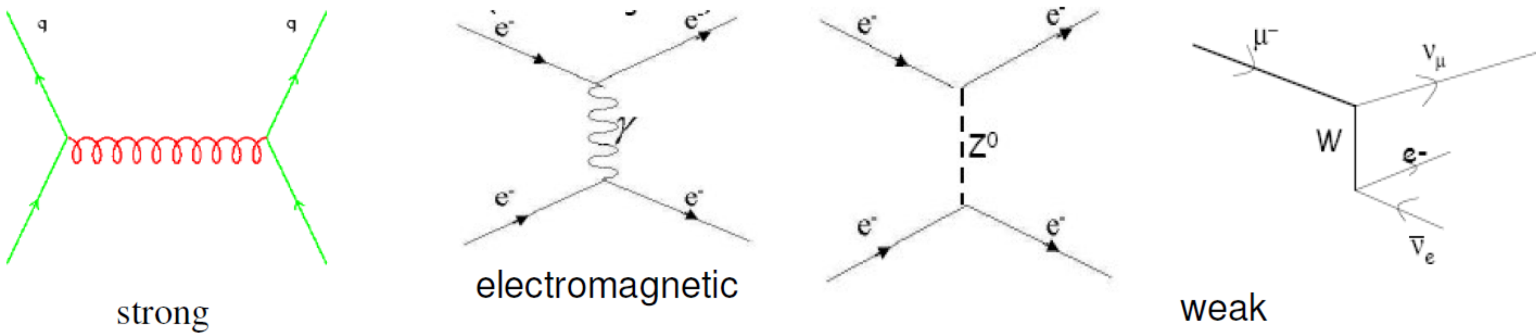
Virtual and real particles

Virtual particles can go "off mass shell" i.e. possess a different mass to that given in the books!

The apparent energy conservation violation can be understood in this way.



The fundamental forces



Interaction	Relative strength	Range	Exchange	Mass (GeV)	Charge	Spin
Strong	1	Short (≈ fm)	Gluon	*0*	*0*	*1*
Electromagnetic	1/137	Long *(1/r²)*	Photon	*0*	*0*	*1*
Weak	10^{-9}	Short (≈ 10^3 fm)	*$W^+ W^-, Z$*	*80.4, 80.4, 91.2*	*+e, -e, 0*	*1*
Gravitational	10^{-38}	Long *(1/r²)*	Graviton ?	*0*	*0*	*2*

No quantum field theory yet for gravity

The weak force has a limited range and reduced strength owing to the masses of the exchange particles (W^{+-}, Z)

[The strong force with a massless gluon "should" have a limitless range but the gluons interact in a special way (next lectures).]

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- Scattering (theory)
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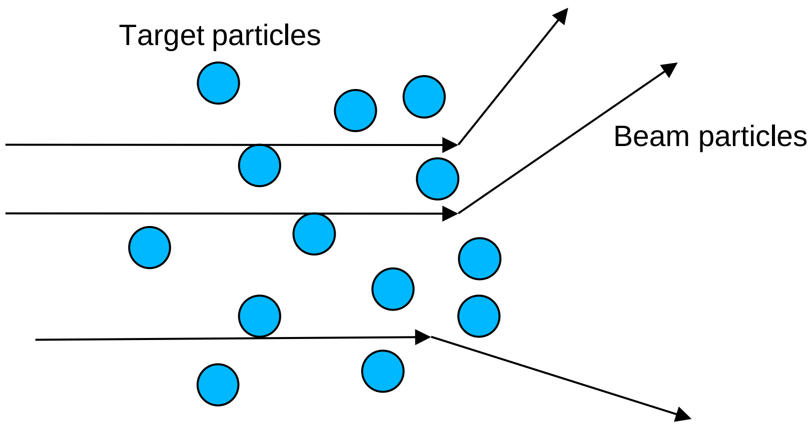
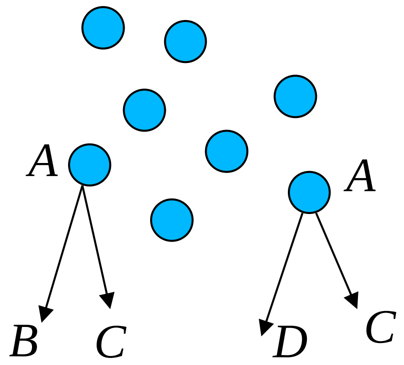
  - Ensure that the observables which are measured are those which can be predicted !
- Decays

How do we experimentally observe forces ?



Scattering



Decay

Scatter : beam particles off target particles.

Decay : observe decay rates and decay particles.

Need a theoretical formalism to calculate this.

Scattering Interaction rates

Quantum mechanics and perturbation theory: Fermi's golden rule:

$$W = \frac{2\pi}{\hbar}\left|\mathcal{M}\right|^2 \rho\left(E^{'}\right)$$

$\mathcal{M} =$ matrix element/probability amplitude

$\mathcal{M}$ must be calculated, eg, Feynman diagrams.

$\rho\left(E^{'}\right) =$ phase space factor.

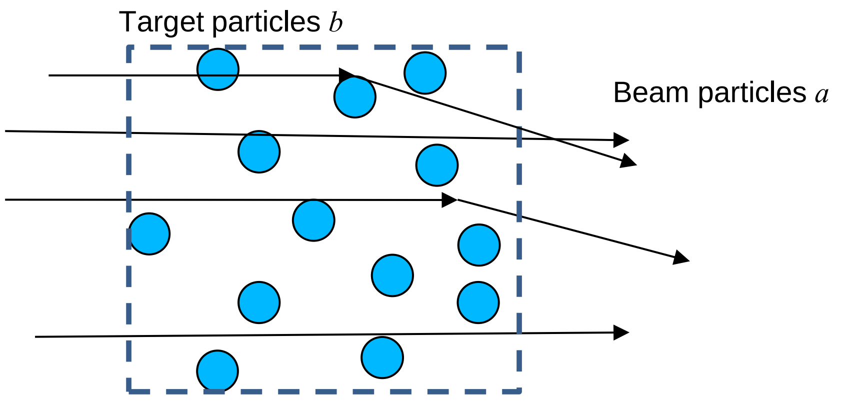
Consider set of beam particles scattering off target particles in a volume.

$W =$ reaction rate per beam and target particle.

$$= \frac{R}{N_a N_b}$$

$N_a =$ No. a particles in scattering volume

$N_a =$ No. b particles in scattering volume



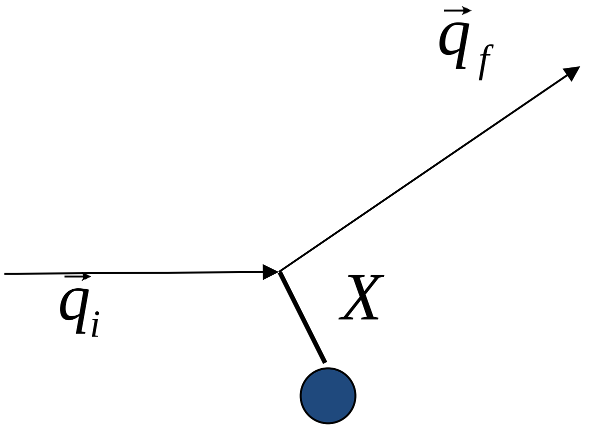
Scattering volume

Amplitude

Ingredient in calculation of reaction rates.

Amplitude for single particle exchange for a particle
with momentum $\vec{q}_i$ to be scattered to a final state with momentum $\vec{q}_f$.

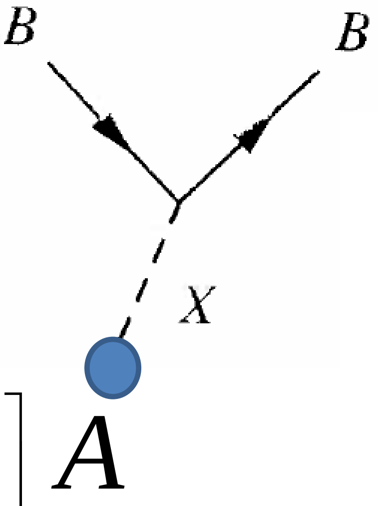
$$\mathcal{M}\left(\vec{q}\right)=\int d^3\vec{r}\; e^{i\frac{\vec{q}_f\bullet\vec{r}}{\hbar}}\, V(\vec{r})\; e^{-i\frac{\vec{q}_i\bullet\vec{r}}{\hbar}} = \int d^3\vec{r}\; e^{-\frac{i\vec{q}\bullet\vec{r}}{\hbar}}\, V(\vec{r}) \qquad \vec{q}=\vec{q}_f-\vec{q}_i$$



Require a potential $V(\vec{r})$ for a force.

Yukawa potential

Potential energy for a particle B scattering off a static particle A:



$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-\frac{r}{R}}}{r} \quad \text{(Yukawa potential)}$$

R=range, $g \equiv$ coupling $\equiv$"charge" on A and B.

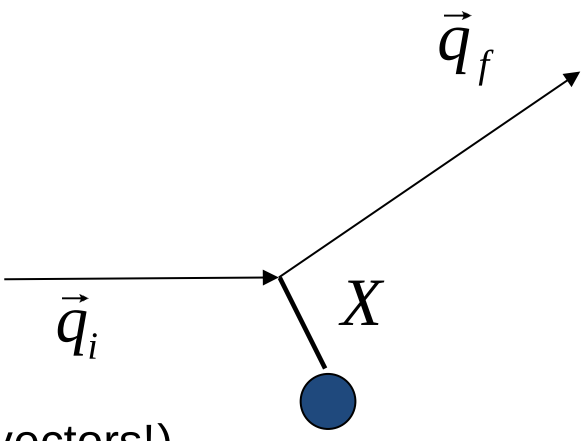
Eg EM force considering an electron scattering off a proton

$$g_{em} = \frac{q}{\sqrt{\varepsilon_0}} \qquad V(r) = -\frac{q^2 e^{-\frac{r}{R}}}{4\pi\varepsilon_0 r}$$

Limit $R \to \infty \quad V(r) \to -\dfrac{q^2}{4\pi\varepsilon_0 r}$ (Coulomb energy)

Amplitude

$$\mathcal{M}(\vec{q}) = \int d^3\vec{r}\; e^{i\frac{\vec{q}_f \bullet \vec{r}}{\hbar}}\, V(\vec{r})\; e^{-i\frac{\vec{q}_i \bullet \vec{r}}{\hbar}} = \int d^3\vec{r}\; e^{-\frac{i\vec{q}\bullet\vec{r}}{\hbar}}\, V(\vec{r}) \qquad \vec{q} = \vec{q}_f - \vec{q}_i$$



$$\Rightarrow \mathcal{M}(\vec{q}) = \frac{-g^2\hbar^2}{|\vec{q}|^2 + M_X^2 c^2}$$

Full relativistic treatment:

$$\mathcal{M}(q') = \frac{g^2\hbar^2}{q'^2 - M_X^2 c^2} \qquad q'^2 = (E_f - E_i)^2 - \left(\vec{q}_f - \vec{q}_i\right)^2 c^2$$ (Four vectors!)

$\Rightarrow$ weak force is weak compared to the em force

$M_X \approx M_Z, M_W \approx 80$ GeV (weak) $M_X = 0$ GeV (em)

$\frac{g_{em}^2}{|q'|^2}$ (em) >> $\frac{g_W^2}{|q'|^2 - M_W^2 c^2}$ (weak) unless $|q'|^2 >> M_W^2 c^2$

Scattering theory

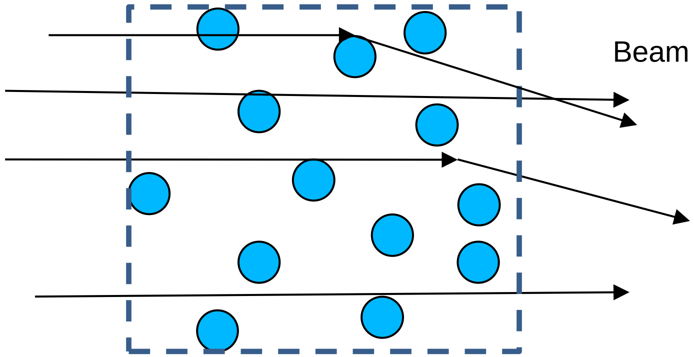
Consider set of beam particles a scattering off target particles b in a volume.

$W =$ reaction rate per beam and target particle.

$$= \frac{R}{N_a N_b} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho(E')$$

$N_a =$ No. a particles in scattering volume

$N_a =$ No. b particles in scattering volume



Scattering volume

Define $\sigma = \dfrac{\text{Number of interactions between } a \text{ and } b \text{ per unit time } \times \frac{1}{N_b}}{\phi_a} = \dfrac{R \times \frac{1}{N_b}}{\phi_a}$

$\phi_a =$ Number of a particles entering X per unit time per unit beam area (flux)

$$\Rightarrow \quad R = \sigma \phi_a N_B = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho(E') \; N_A N_B$$

$$\Rightarrow \quad \sigma = \frac{2\pi |\mathcal{M}|^2 \rho(E') \; N_A}{\hbar \phi_a}$$

The cross section

Quantum mechanics tells us that we can calculate a quantity which is related to the probability of a scatter between a and b

Cross-section: $\sigma = \dfrac{2\pi |\mathcal{M}|^2 \rho(E')\ N_A}{\hbar \phi_a}$

Is it an experimentally measurable observable ?

$\sigma = \dfrac{R \times \dfrac{1}{N_b}}{\phi_a} \Rightarrow R = \phi_a \times N_b \times \sigma$ = number of interactions per second

Define luminosity= $\mathcal{L} = \phi_a N_b$ (depends on the apparatus used).

$\Rightarrow\ R = \mathcal{L}\sigma$

We measure a reaction rate R. We know the luminosity $\mathcal{L} = \phi_a N_b$

$\Rightarrow$ We can measure $\sigma = \dfrac{R}{\mathcal{L}}$

Yes it is an experimentally measurable observable !

Integrated luminosity and total number of events

$R = \mathcal{L}\sigma$ gives rate of reactions.

An experiment can run over a time t.

$$\int_0^t R dt = \int_0^t \sigma \mathcal{L} dt$$

$$N = \int_0^t R dt = \text{Total number of interactions/"events"}$$

$$\mathcal{L}' = \int_0^t \mathcal{L} dt = \text{Integrated luminosity}$$

$$\Rightarrow N = \mathcal{L}'\sigma$$

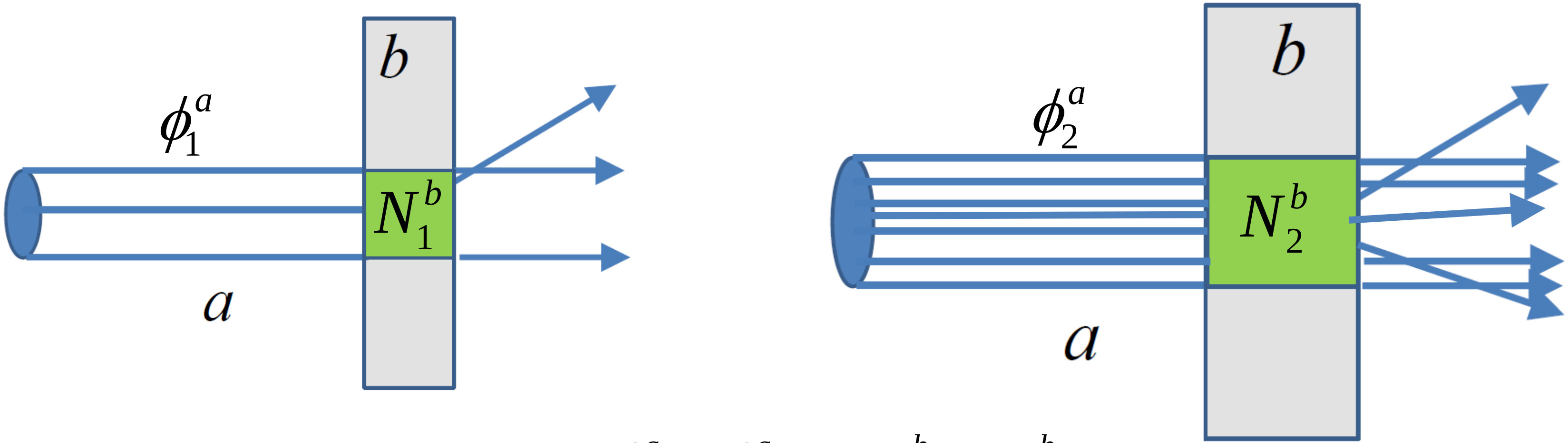
Questions

What are the dimensions of the cross section ?

Does the cross section depend on the apparatus, eg beam area ?

Why bother with the cross section ? Surely we can calculate and measure the probability of an individual particle a interacting with an individual particle b ?

Two experiments observing a scattering off b



$$\phi_2^a > \phi_1^a \quad , \quad N_2^b > N_1^b$$

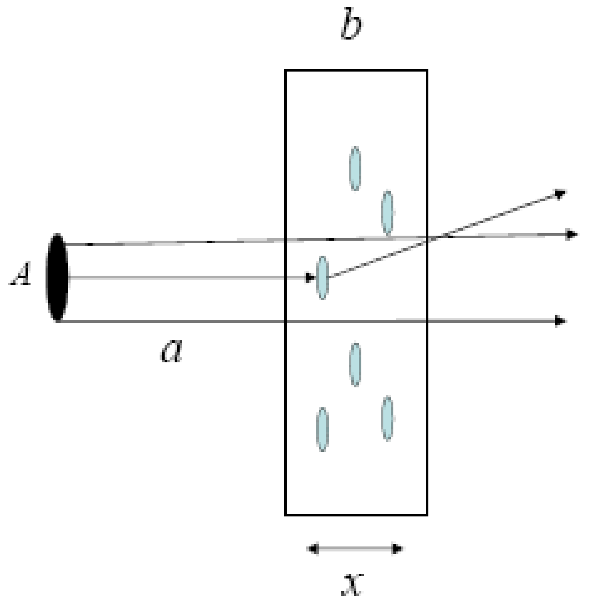
Luminosity:	$\mathcal{L}_1 = \phi_1^a N_1^b$	$\mathcal{L}_2 = \phi_2^a N_2^b$
Reaction rate:	R_1	R_2
Cross section:	$\sigma_1 = \frac{R_1}{\mathcal{L}_1}$	$\sigma_2 = \frac{R_2}{\mathcal{L}_2}$

Different observed reaction rates depending on luminosity of experiment:

$\mathcal{L}_2 > \mathcal{L}_1 \quad ; R_2 > R_1$

Cross section is specific to the individual particle reaction: $\sigma_1 = \sigma_2$

Visualising the cross section



Assume that each particle of type b (scattering centre) has a cross-sectional area σ. A beam particle must pass through the cross-sectional area to interact.

The bigger the area, the bigger the chance of an interaction !

Naive ! The EM force has infinite range.
"Works" in certain situations, eg some strong force reactions.

Cross section

In the previous example, the cross section was defined such that the particle a interacted in some way. It could have been scattered, annihilated - doesn't matter. Defined the total cross section: σ_{tot}.

When particles interact there are many different types of interactions possibe, each of which can be measured which contribute to σ_{tot}.

$Eg \quad e^{-}, p$

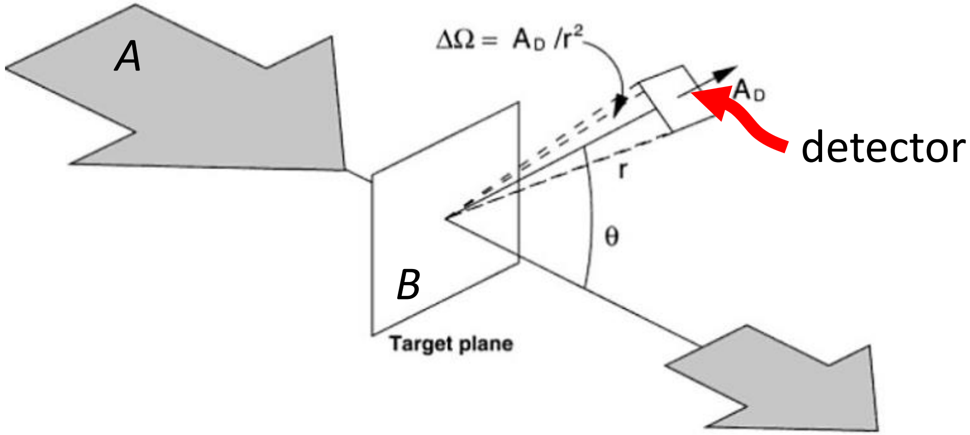
Elastic scattering $A + B \rightarrow A + B$ eg $e^{-} + p \rightarrow e^{-} + p \quad \sigma_1$

Inelastic scattering $A + B \rightarrow A + C + D$ eg $e^{-} + p \rightarrow e^{-} + n + \pi^{+} \quad \sigma_2$

$\sigma_{tot} = \sigma_1 + \sigma_2$

Can define cross section for particle scattering into a certain solid angle: $\dfrac{d\sigma}{d\Omega}$

Differential cross section



Study angular distributions of scattered particles:Eg $A+B \to A+X$ (X=anything)

Define a frame, eg B at rest E=energy of A.

Reaction rate R for processes leading to A scattered through solid angle

$$\Delta\Omega = \sin\theta d\theta d\phi = \frac{A_D}{r^2}$$

$$R(E,\theta,\Delta\Omega) = L\frac{d\sigma(E,\theta)}{d\Omega}\Delta\Omega$$

If scattered particle's energy E' can be measured:

$$\sigma(E) = \int_0^{E'_{\max}} \int_{4\pi} \frac{d^2\sigma(E,E',\theta)}{d\Omega dE'} d\Omega dE'$$

Cross section units

Units of barn.

Origin from nuclear physics :

an uranium nucleus is "as big as a barn".

1barn=1b=10^{-28}m^2 ; 1 millibarn = 1mb = 10^{-31}m^2

Typical cross section for beam energies of 10 GeV.

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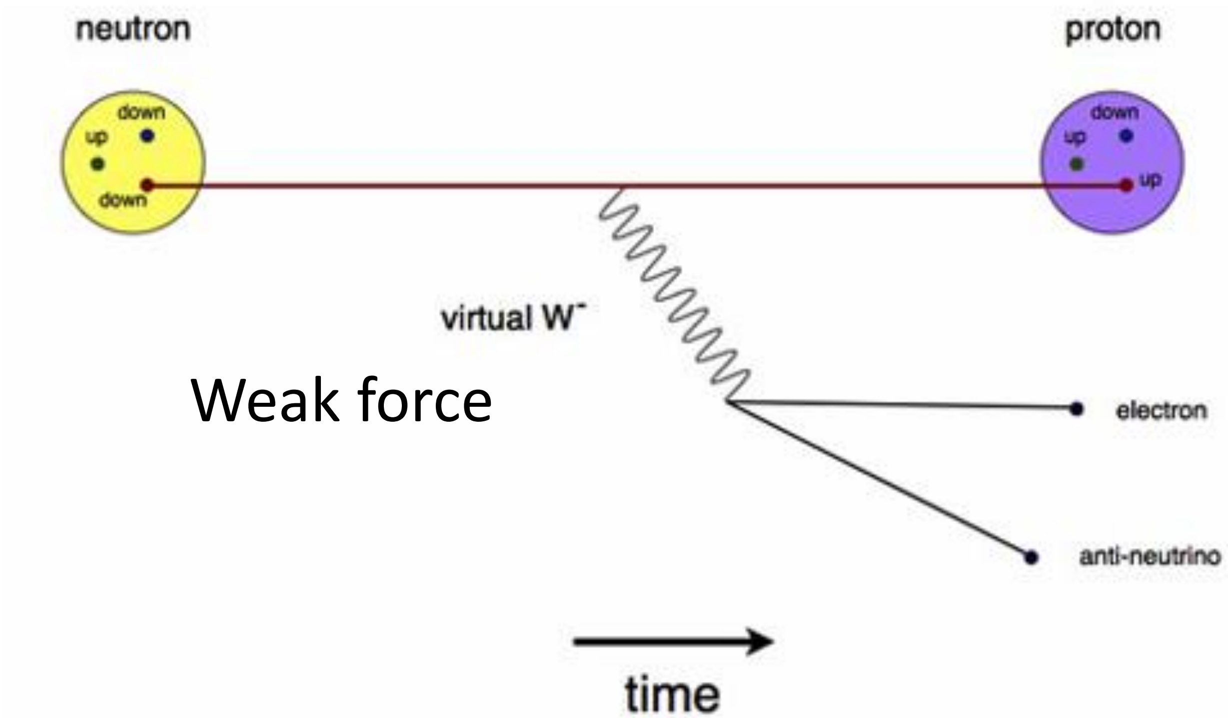
Decays



Weak force

As for scattering, decays mediated by virtual particles

Decay rates

Unstable particles: $W = 2\pi \left| \left| \mathcal{M} \right|^2 \right|^2 \rho\left(E'\right)$

$W = \Gamma = \frac{1}{\tau}$ = decay rate

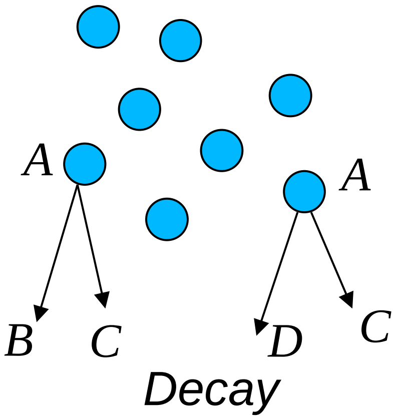
For different decay modes

Eg: $A \to B + C \ \left(\Gamma_1\right)$ or $A \to C + D \ \left(\Gamma_2\right)$...

Lifetime $\tau = \frac{1}{\Gamma_{tot}}$

$\Gamma_{tot} = \Gamma_1 + \Gamma_2 + \ldots$

Branching ratios $B_i = \frac{\Gamma_1}{\Gamma_{tot}}$



Decay via $N = N_0 e^{-\frac{t}{\tau}} = N = N_0 e^{-\Gamma_{tot} t}$

$N =$ number of undecayed particles at time t

$N_0 =$ number of undecayed particles at time $t = 0$

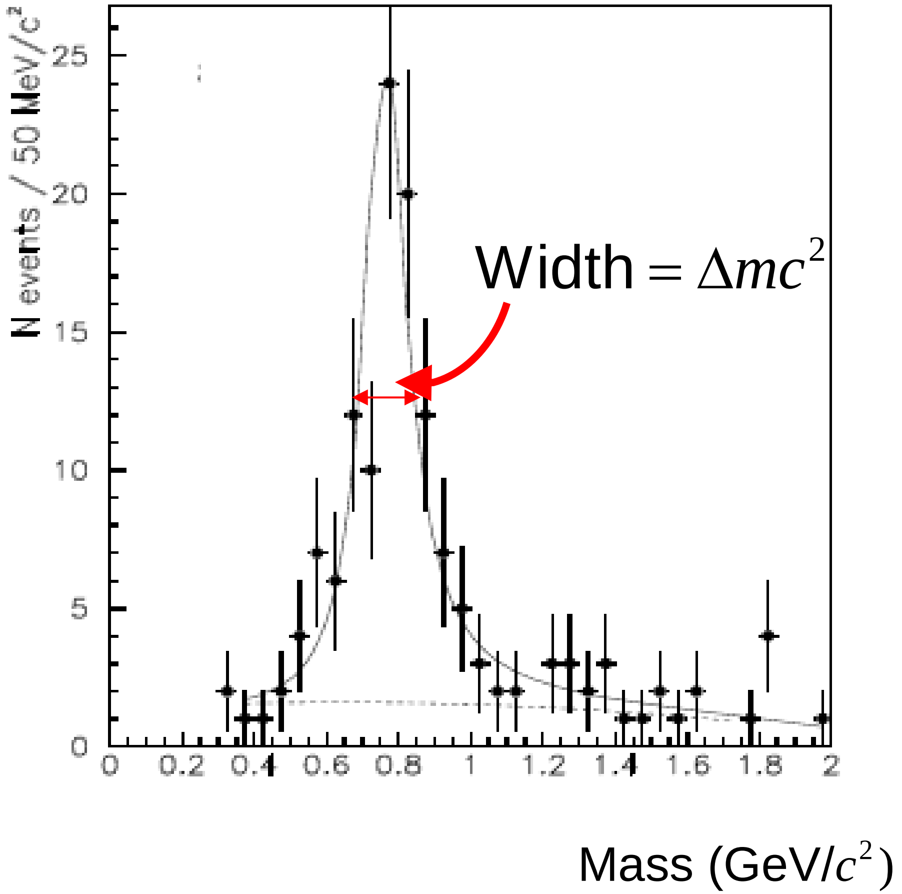
Resonances

$E = mc^2$

Experiment makes repeated measurement of the mass of a particle. Eg ρ particle.

Experimental resolution is negligible.

$\Rightarrow$ Width $= \Delta m \sim 200$ MeV/c^2



Width $\rightarrow$ Lifetime

$\Delta E \Delta \tau \sim \hbar$

The energy level (of an excited atom) or mass of an unstable particle is not determined !

Range of values of $\Delta E, \Delta m$:

$$\Delta E \Delta t \sim \hbar \quad \Rightarrow \quad \Delta m c^2 \Delta t \sim \hbar.$$

$\Delta t \sim$ Lifetime of excited state/particle.

Width of ρ : $\Delta m \sim 200$ MeV/c^2

$$\Rightarrow \text{ Lifetime } \tau \sim \frac{6.6 \times 10^{-34}}{2\pi \times 200 \times 10^{6} \times 1.602 \times 10^{-19}} \sim 10^{-24} - 10^{-23} \text{ s}$$

A reliable but very approximate method to estimate lifetimes.

Particle Mass

(a) Rest mass (in the tables) for real i.e. observable particles.
= on-shell mass
=average mass an experiment would
measure after many repeated measurements.

Baryon	Quark content	Charge	Mass
$N \begin{cases} p \\ n \end{cases}$	*uud* *udd*	+1 0	938.280 939.573

Even a perfect experiment would not return the same
value of mass for each measurement if the particle is not stable.

(b) A particle interacting virtually can go off mass-shell.
We never directly measure that mass.
You are at liberty to claim we don't really know if a certain particle mediated an interaction as a virtual state. Our best theories tell us that but virtual particles are not observables.

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  - Decay rates
  - Particle widths