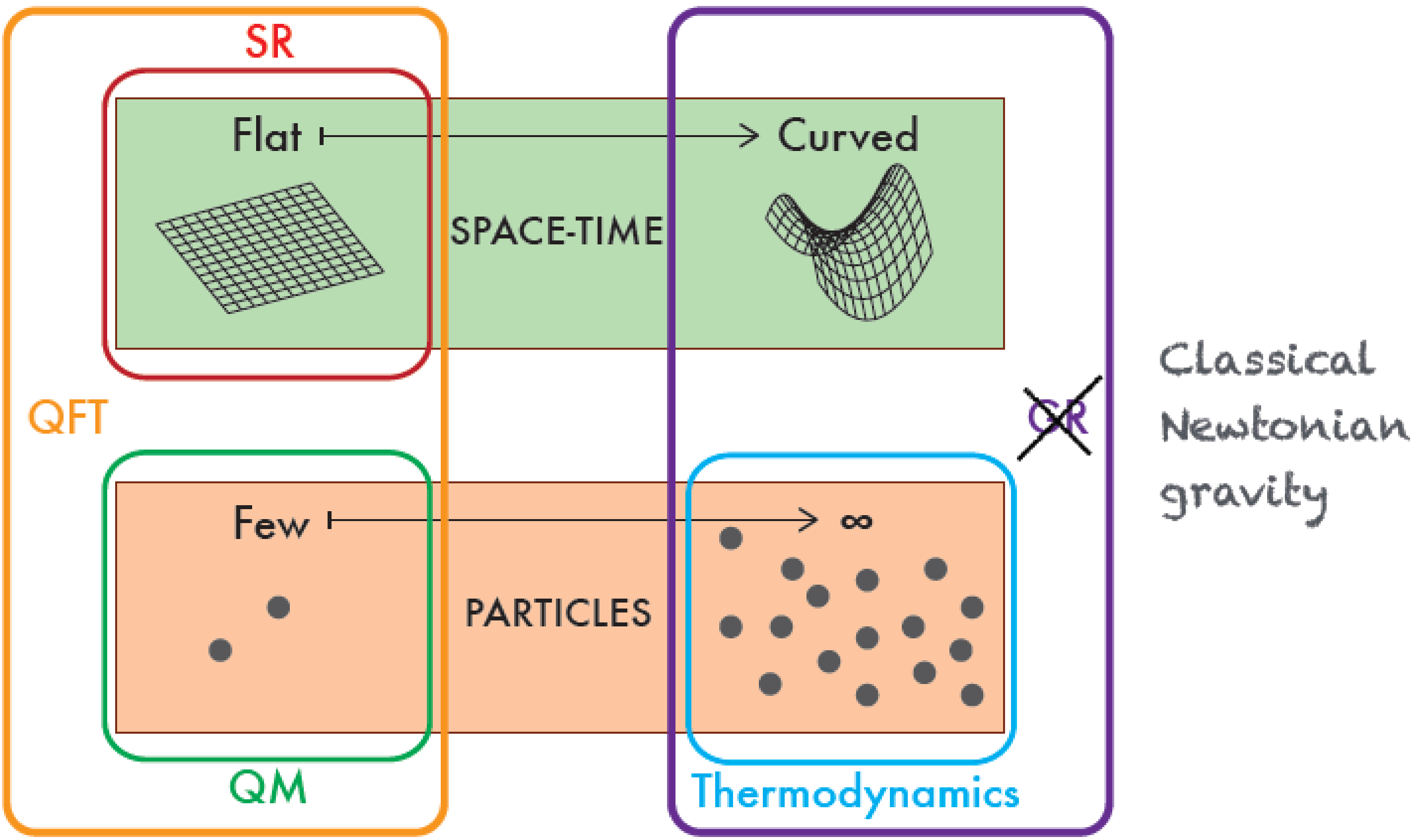
FK5024 - Nuclear & particle physics, astrophysics and cosmology

- Course information
- Review of special relativity
- Units

Theoretical background

Particle and nuclear physics rests on special relativity and quantum mechanics (review today SR)

Cosmology rests on general relativity and thermodynamics. Short-cuts used and GR avoided in this course.



From the small to the large

Object	Size (m)
Quark	$<10^{-18}$
Proton	10^{-15}
Nucleus	10^{-14}
Star	10^{9}
Galaxy	10^{21}
Galaxy cluster	10^{23}
The observable universe	10^{26}

During the course physical processes and phenomena spanning more than 44 orders of magnitude in size will be studied.

Course information

All relevant information to be found here:
http://staff.fysik.su.se/~lbe/FK5024/FK5024.htm

Topics :

- Particle physics
- Nuclear physics
- Astrophysics and cosmology

- Lectures on each topic
- 11 tutorials
- 2 seminars
- 3 sets of hand-in problems

Examination

Minimum requirements for passing:

- Participating in the two compulsory seminars and doing satisfactorily the preparatory work (three sets of hand-in problems)
- Completing at least 50% on the written exam
- The final grade will be determined based on the result on the written exam, with the possibility of a bonus point for well done hand-ins

Schedule

FK 5024 - Particle and nuclear physics, astrophysics and cosmology (7.5 hp)

Preliminary Schedule, Autumn 2018

LB - Lars Bergström, DM - David Milstead, JC - Jan Conrad, AB - Anthony Bonfils and FT – Francesco Torsello. Prefixes "M" and "L" refers to the books by Martin and Liddle respectively.

Compulsory seminars are in bold.

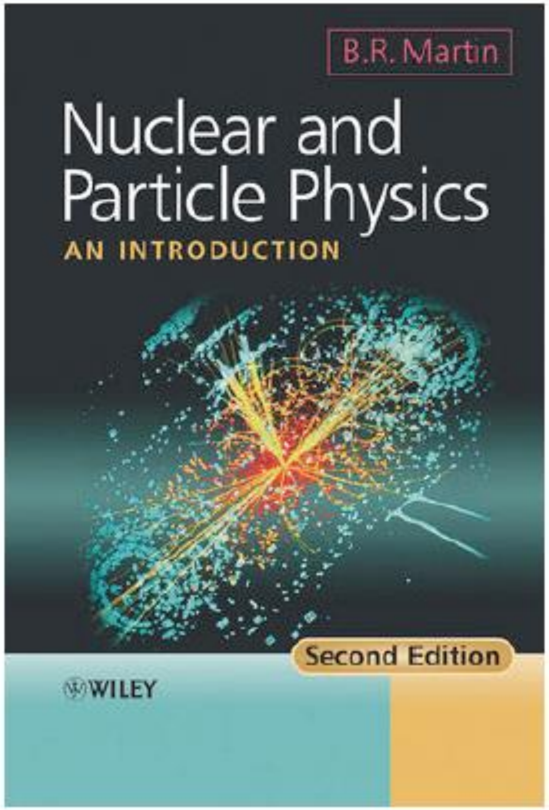
Week	Day	Time	Room		Topic	Preparation
36	Tue 4 Sep	10:15 - 12:00	FP41	DM	Introduction and SR + QM	M B.1
	Tue 4 Sep	13:15 –15:00	FP41	AB	Tutorial: Lorentz transformations and relativistic kinematics (sol'n)	M B.2-B.3
	Thu 6 Sep	10:15 - 12:00	FP41	DM	Particles and forces	M 1.1, 1.5, 1.6-1.6.2, 1.7
	Thu 6 Sep	13:15 - 15:00	FP41	AB	Tutorial (sol'n)	
37	Tue 11 Sep	10:15 - 12:00	FP41	DM	Feynman diagrams, leptons and quarks	M 1.4, 3.1-3.1.3, 3.2, 5.1, 5.2, 9.3.1
	Tue 11 Sep	13:15 - 15:00	FP41	DM	Quarkonium, etc	
	Thu 13 Sep	10:15 - 12:00	FP41	DM	Experimental particle physics and beyond the standard model	M 3.3.2, 4.3-4.4, 9.4, 9.5.1-9.5.2, 9.6.2-9.6.3, 3.1.4
	Thu 13 Sep	13:15 - 15:00	FP41	AB	Tutorial (sol'n)	
38	Tue 18 Sep	10:15 - 12:00	FP41	JC	Nuclear shapes and sizes	M 2.1-2.3, skip 2.1.3
	Tue 18 Sep	13:15 - 15:00	FP41	AB	Tutorial	
	Thu 20 Sep	10:15 - 12:00	FP22	JC	Decays and reactions	M 2.4-2.9
	Thu 20 Sep	13:15 - 15:00	FP41	AB	Tutorial; **Deadline for hand-in problems I**	
39	Tue 25 Sep	10:15 - 12:00	FP41	JC	Nuclear models	M 7.1-7.5 (skip 7.3.3)
	Thu 27 Sep	10:15 - 12:00	FP41	AB	Tutorial/Fermi model	M 7.2, Problems 7.1, 7.3, 7.4. White dwarfs in connection with the Fermi model.
	Thu 27 Sep	13:15 - 15:00	FP41	JC	Alpha, beta and gamma decays	M 7.6-7.8
40	Tue 2 Oct	10:15 - 12:00	FP41	JC	Some applications of nuclear physics	M 8.1-8.4
	Tue 2 Oct	13:15 - 15:00	FP41	AB	Tutorial	Problems 7.7, 7.11, 7.12, 8.1 (first part), 8.2, 8.3, 8.10, 8.12.
	Thu 4 Oct	10:15 - 12:00	FP41	JC	Stellar fusion and neutron stars	M 8.2.1-8.2.3
	Thu 4 Oct	13:15 - 15:00	FP41	FT	Review of python programming	
41	Tue 9 Oct	10:15 - 12:00	FP41	LB	Stars, galaxies and the expanding universe	L 1-2, 5.1-5.2, 6.1, 3.1-3.3, 3.6
	Tue 9 Oct	13.15 - 15:00	FP41	JC/FT	**Seminar on applications of nuclear physics** + Tutorial; **Deadline for hand-in problems II**	Problems 2.1, 2.3-2.6
	Thu 11 Oct	10:15 - 12:00	FP41	LB	Simple cosmological models	L 3.4-3.5, 4 (skip 4.4-4.5), 5.3-5.5
	Thu 11 Oct	13:15 - 15:00	FP41	FT	Radiation physics	L 2.5.2
42	Tue 16 Oct	10.15 - 13:00	FP41	LB	The content and dynamics of the universe	L 6-9
	Tue 16 Oct	13.15 - 15:00	FP41	LB	The early universe and the cosmic microwave background	L 10-12 (skip 10.4)
	Thu 18 Oct	10.15 - 12:00	FP41	FT	Tutorial	Problems 4.1, 4.2, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6
	Thu 18 Oct	13.15 - 15:00	FP41	FT	Tutorial; **Deadline for hand-in problems III**	Problems from chapters 6-8
43	Tue 23 Oct	10.15 - 13:00	FP41	LB	Observations, interpretations and open questions	L 4.4, 4.5, 13.1
	Tue 23 Oct	13.15 - 15:00	FP41	FT	Tutorial	Problems from chapter 9-12
	Thu 25 Oct	10.15 - 12:15	FP41	LB	Review	
	Thu 25 Oct	13.15 - 15:00	FP41	FT	**Seminar on problem solving**	Problem areas selected by poll
44	Thu 1 Nov	08.00 - 13:00	FR4		Exam	

The following sections can be read descriptively: M 2.1.2-2.2.2 (results only), 2.8 (examples gamma decay), 2.9 (examples reactions), 7.4, 7.7, 7.8 (selection rules for gamma transitions), while detailed reading is recommended for the remaining sections.

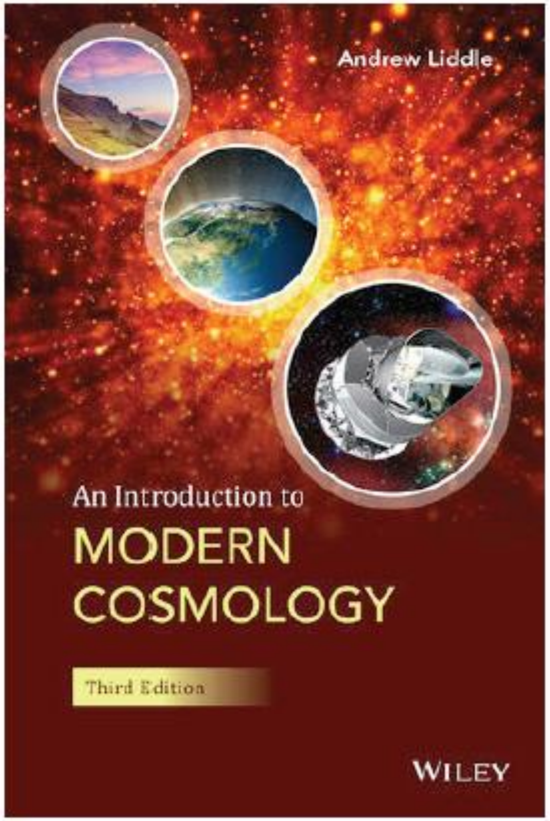
Important dates

- Thursday, September 20: Deadline for hand-in problems, set I
- Tuesday, October 9: Seminar I
- Tuesday, October 9: Deadline for hand-in problems, set II
- Thursday, October 18: Deadline for hand-in problems, set III
- Thursday, October 25: Seminar II
- Thursday, November 1: Exam

Texts



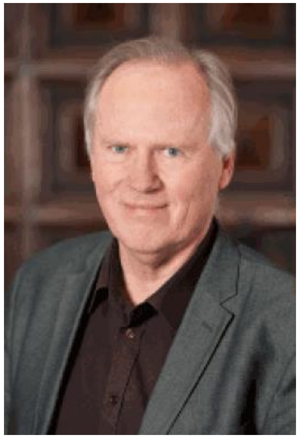
Nuclear and Particle Physics: An Introduction, 2nd Edition (B.R. Martin)



An Introduction to Modern Cosmology, 3rd Edition (Andrew Liddle)

Teachers

Lecturers

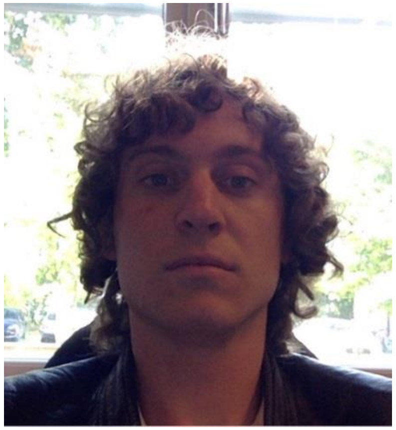


Lars Bergström
(astro)

Jan Conrad
(nuclear)

David Milstead
(particle)

Exercises and tutorials



Anthony Bonfils

Franceso Torsello

Special relativity

Two postulates

(1) The laws of physics are the same in all inertial frames of reference.

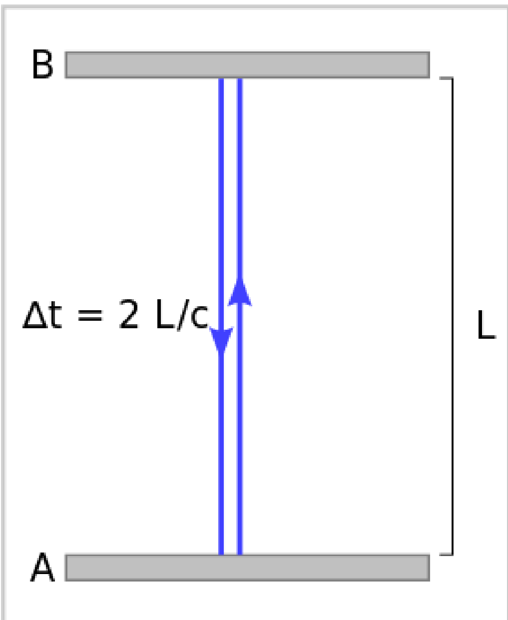
(2) The speed of light in free space has the same value c in all inertial frames of refrence.

Two simple but extremely powerful assumptions.

First postulate

What do we mean by the statement that the laws of physics are invariant to a change of inertial reference frame ?

Second postulate



A photon bounces between two mirrors A, B

Separation= L

Observer at rest wrt apparatus $S \equiv (x, y, z)$:

Observer measures the time Δt for light to bounce:

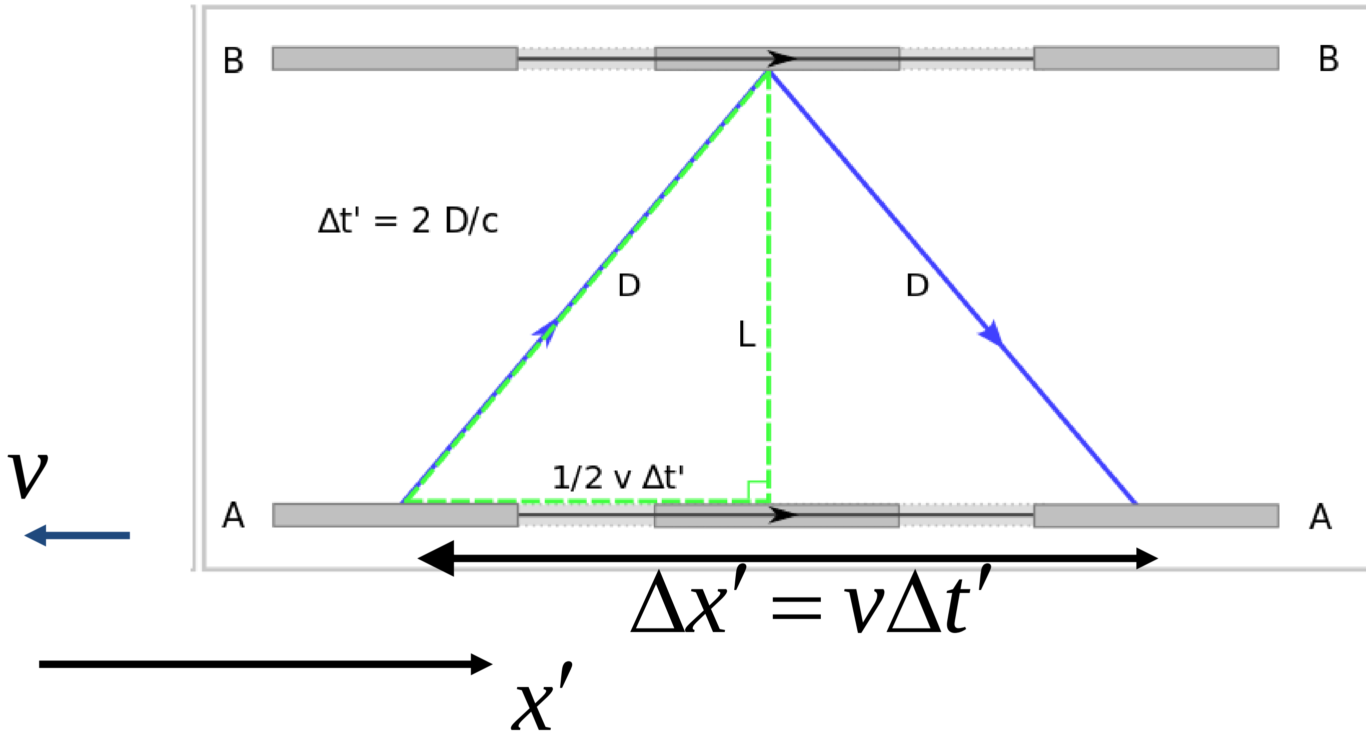
$A \rightarrow B \rightarrow A$.

Difference in start and end position: $\Delta x = 0$

Time for $A \rightarrow B \rightarrow A$: $\Delta t = \dfrac{2L}{c}$

Define $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$

$$(\Delta s)^2 = 4L^2$$



Observer moving to the left wrt apparatus with speed v :

$\Delta x' = v\Delta t'$

$$\Delta t' = \frac{2D}{c} = \frac{2\left(L^2 + \left(\frac{1}{2}\Delta x'\right)^2\right)^{\frac{1}{2}}}{c} \Rightarrow (c\Delta t')^2 - 4\left(L^2 + \left(\frac{1}{2}\Delta x'\right)^2\right) = 0$$

$$\Rightarrow (c\Delta t')^2 - (\Delta x')^2 = 4L^2 = (c\Delta t)^2 - (\Delta x)^2$$

The quantity $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ is a Lorentz invariant

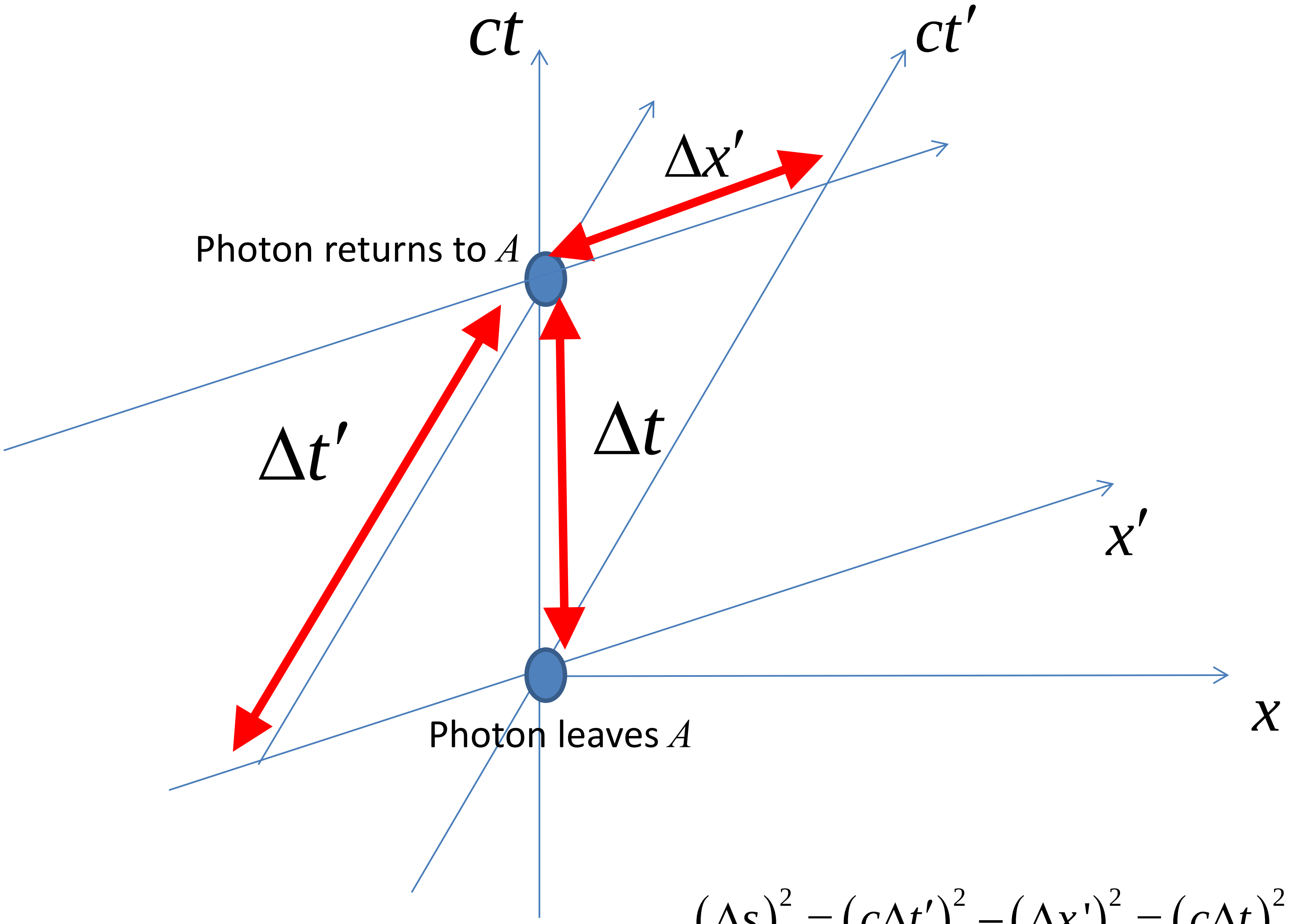
Second postulate

Postulating that the speed of light is constant in all inertial frames

$\rightarrow$

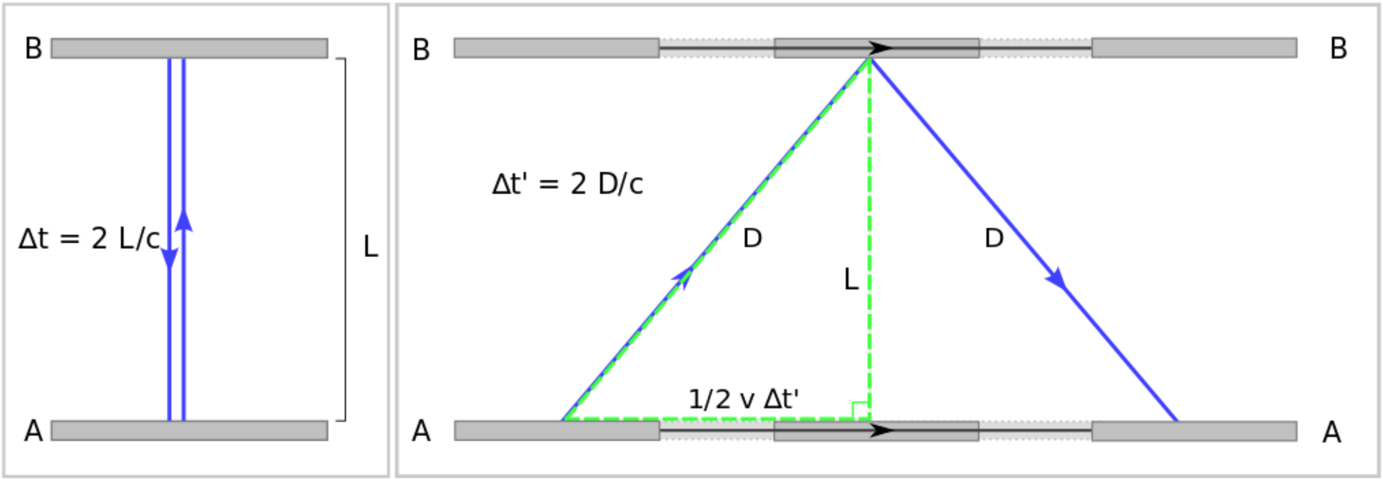
Instead of an invariant time or space interval there is an invariant space-time : $\Delta s = \sqrt{\left(c\Delta t\right)^2 - \left(\Delta x\right)^2}.$

Minkowksi diagram



$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2 = (c\Delta t)^2$$

Time dilation



$$\Delta t = \frac{2L}{c} \Rightarrow c^2 (\Delta t)^2 = 4L^2$$

$$\Delta t' = \frac{D}{c} = \frac{2\left(L^2 + \left(\frac{1}{2} v \Delta t'\right)^2\right)^{\frac{1}{2}}}{c} = \frac{\left(4L^2 + (v\Delta t')^2\right)^{\frac{1}{2}}}{c}$$

If the speed of light is the same for each frame:

$$\Rightarrow \Delta t' = \frac{\left(c^2 (\Delta t)^2 + (v\Delta t')^2\right)^{\frac{1}{2}}}{c} \Rightarrow \Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\Rightarrow$ Time dilation.

Lorentz transformations

Boost in x-direction.

If an observer in S records an event at $(ct, x, y, z\ )$ then an observer in S' records the same event with coordinates:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

v=relative speed between frames in x-direction.

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$ (time dilation) , $x' = \gamma(x - vt)$ (length contraction)

$y' = y$, $z' = z$

Proper time

Consider frame S' moving at speed v wrt a particle.

Frame S is the particle rest frame

$\Delta t' =$ time interval in frame S'

$\Delta t =$ time interval in S ="proper" time

Time dilation $\Rightarrow$ $\Delta t' = \gamma \Delta t \Rightarrow \gamma = \dfrac{\Delta t'}{\Delta t}$

For infinitesimal intervals $\gamma = \dfrac{dt'}{dt}$

Nature's speed limit

Consider the motion of the particle over a space-time interval $|\Delta s|$.

$S:\ c\Delta t, \Delta x \quad , \quad S':\ c\Delta t', \Delta x'$

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$\Delta x = 0 \ , \ (c\Delta t)^2 \geq 0$$

$$\Rightarrow (c\Delta t')^2 - (\Delta x')^2 \geq 0 \quad \Rightarrow c^2 \geq \left|\frac{\Delta x'}{\Delta t'}\right|^2$$

Particle speed in S' $\quad v' = \sqrt{\left|\frac{\Delta x'}{\Delta t'}\right|^2} \leq c$

No particle can move faster than light!

This violates causality.

The speed of light and causality

If information can be sent faster than light a message can be sent to the past $\Rightarrow$ paradox!

Eg you can arrange for your father to be killed before he met your mother. But then how would you be present to send the message ?

Causality cannot be violated - the cause must precede the effect!

Can nature's speed limit be beaten ?

Yes.

Eg dot from a laser pointer, a shadow...

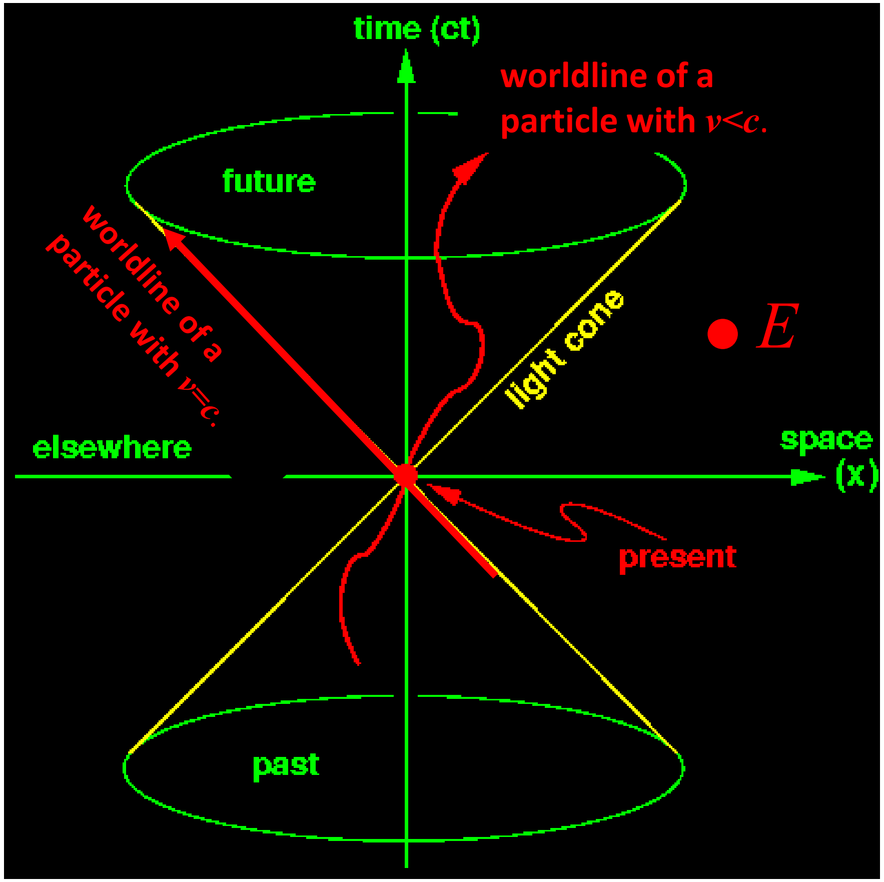
Quantum mechanics allows it within the Copenhagen interpretation.

However, it is impossible to send a message faster than light (we think).

Space-time

Light cone for past, present and future of an object (eg particle).

World-line follows particle's path.



Particle interacts (event) as it moves through space-time.

Space-time between events:

$$\left(\Delta s\right)^2 = \left(c\Delta t\right)^2 - \left(\Delta x\right)^2$$

$$\frac{v}{c} = \frac{\Delta x}{c\Delta t}$$

$\left(\Delta s\right)^2 > 0 \quad (v < c)$ Time-like

$\left(\Delta s\right)^2 = 0 \quad (v = c)$ Light-like

Interactions are causal.

Particle confined to the light cone.

Event E , non-causal eg decay of another particle.

$\left(\Delta s\right)^2 < 0$ space-like

Four vectors

Four vector: $X = (ct, x, y, z) = (ct, \vec{r})$

Consider two four vectors: $X_A,\ X_B$.

$$X_A = (ct_A, x_A, y_A, z_A) \quad ; \quad X_B = (ct_B, x_B, y_B, z_B)$$

Eg $X_A \equiv$ space-time of production of a particle in frame S.

$X_B \equiv$ space-time of decay of a particle in frame S

Four vectors from sum/subtraction:

$$X_C = X_A + X_B = \left(c\left(t_A + t_B\right), x_A + x_B, y_A + y_B, z_A + z_B\right)$$

$$X_D = X_A - X_B = \left(c\left(t_A - t_B\right), x_A - x_B, y_A - y_B, z_A - z_B\right)$$

Four vectors

Scalar product:

$$X_A \bullet X_B = c^2 t_A t_B - x_A x_B - y_A y_B - z_A z_B$$

The scalar product is a lorentz invariant:

$$\left(X_A \bullet X_B\right)_S = \left(X_A \bullet X_B\right)_{S'}$$

Use this property to derive earlier result:

$$\left(X_A - X_B\right) \bullet \left(X_A - X_B\right) =$$

$$\left(c\left(t_A - t_B\right), x_A - x_B, y_A - y_B, z_A - z_B\right) \bullet \left(c\left(t_A - t_B\right), x_A - x_B, y_A - y_B, z_A - z_B\right) =$$

$$\left(c\Delta t\right)^2 - \left[\left(\Delta x\right)^2 + \left(\Delta y\right)^2 + \left(\Delta z\right)^2\right] = \left(\Delta s\right)^2 \Rightarrow$$ invariant.

Four momentum

Invariant scalar product

Evaluate $P \bullet P$ for object with four-momentum P in an arbitrary frame

$$P \bullet P = \left(\frac{E}{c}, \vec{p}\right) \bullet \left(\frac{E}{c}, \vec{p}\right) = \frac{E^2}{c^2} - p^2$$

But $P \bullet P$ can also evaluated for frame when object is at rest:

$$E = mc^2, p = 0$$

$$P \bullet P = (mc, 0) \bullet (mc, 0) = m^2 c^2$$

$$\Rightarrow m^2 c^2 = \frac{E^2}{c^2} - p^2$$

$$\Rightarrow E^2 = \left(mc^2\right)^2 + (pc)^2$$

Velocity, momentum, energy

Energy $E = \gamma mc^2$ $m =$ invariant rest mass

Momentum $\vec{p} = \gamma m\vec{v}$

$\tau =$ proper time ; $\gamma = \dfrac{dt}{d\tau}$

$dX = \left(cdt, dx, dy, dz\right)$

Four velocity U

$$U = \frac{dX}{d\tau} = \left(\frac{cdt}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau}\right)$$

$$\Rightarrow U = \left(c\frac{dt}{dt}\frac{dt}{d\tau}, \frac{dx}{dt}\frac{dt}{d\tau}, \frac{dy}{dt}\frac{dt}{d\tau}, \frac{dz}{dt}\frac{dt}{d\tau}\right) = \gamma\left(c, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) = \left(\gamma c, \gamma\vec{v}\right)$$

Four momentum $P = Um = \left(mc\gamma, m\vec{v}\right) = \left(\dfrac{E}{c}, \vec{p}\right)$

Lorentz transformations

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$ (time dilation) , $$x' = \gamma\left(x - vt\right)$$ (length contraction)

Write in matrix formulation

$$X' = \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} \quad , \quad \Lambda = \begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix} \quad , \quad X = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$X' = \Lambda X \qquad \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Lorentz transformations

General result for 4-vectors, eg also for 4-momentum

$$P' = \begin{pmatrix} \frac{E'}{c} \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} \quad , \quad \Lambda = \begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \quad P = \begin{pmatrix} \frac{E^{'}}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$P' = \Lambda P \qquad \begin{pmatrix} \frac{E'}{c} \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Frames of reference

Two main frames used:

(1) Centre-of-mass frame i.e. total $\vec{p} = 0$

Eg Process: $A + B \rightarrow X$; $X =$ particle or system of particles.

What is the centre-of-mass energy of the system ?

Rest mass of $A + B$ or X system.

$$E_{CM}^2 = \left(c\vec{P}_A + c\vec{P}_B\right)^2 = \left(c\vec{P}_X\right)^2 = m_X^2 c^4$$

$$E_{CM}^2 = c^2\left(\vec{P}_A + \vec{P}_B\right)^2 = c^2\left(\vec{P}_A^2 + \vec{P}_B^2 + 2\vec{P}_A \bullet \vec{P}_B\right) = c^2\left[m_A^2 c^2 + m_B^2 c^2 + \left(\frac{E_A E_B}{c^2} - \vec{p}_A \bullet \vec{p}_B\right)\right]$$

(2) Fixed target frame. Eg $A + B \rightarrow X$, B is at rest before the interaaction.

Usefulness of working in a fixed frame

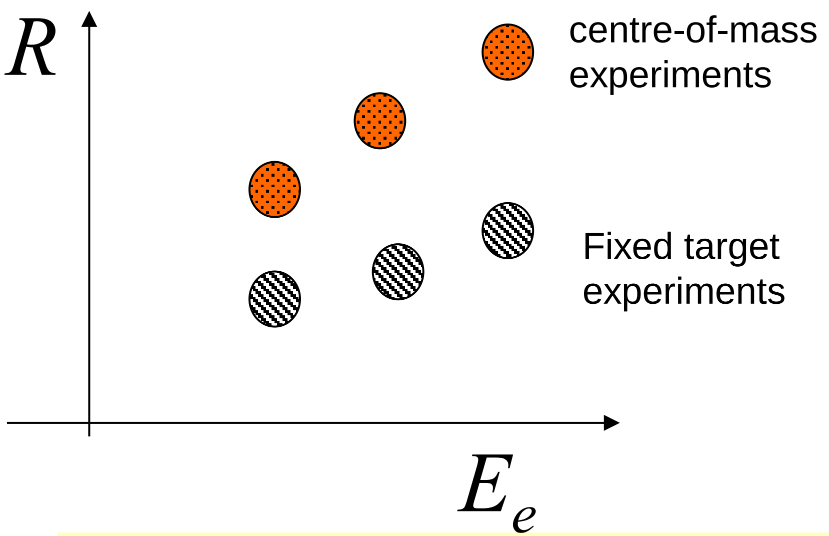
Eg many different experiments measure the ratio R of the processes:

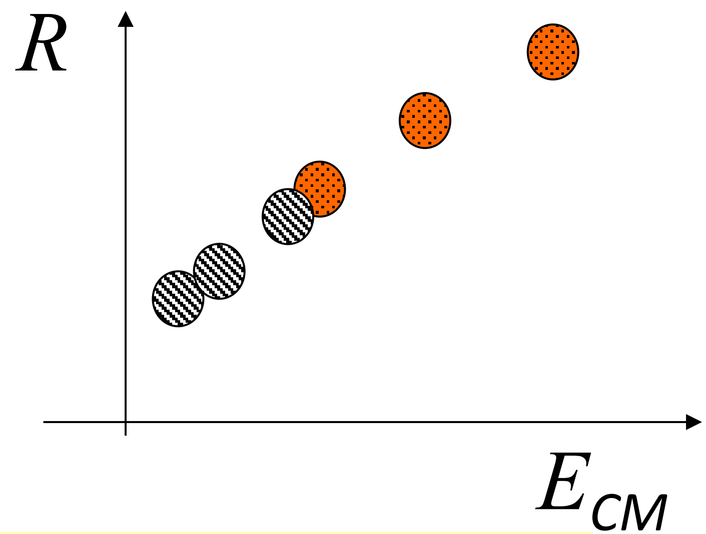
(a) $e+p \rightarrow e+p+n+\bar{n}$ (b) $e+p \rightarrow e+p$

Some experiments are fixed target (electron beam on stationary proton) and the others take place in the centre-of-mass (colliding beams).

The beam energies are often different between the experiments.

To compare the experiments' results, plot their results as a function of centre-of-mass energy.





Could similarly choose a frame in which the target proton is at rest and plot the results as a function of pion beam energy. Just fix a frame!

Formulae

Lorentz transformation:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{dt}{d\tau}$$

$t' = \gamma\left(t - \frac{vx}{c^2}\right)$ (time dilation) , $x' = \gamma\left(x - vt\right)$ (length contraction)

$y' = y$, $z' = z$

Space-time four vector: $X = \left(ct, \vec{r}\right)$

Velocity four-vector: $U = \left(\gamma c, \gamma \vec{v}\right)$

Energy, momentum:

$E = \gamma mc^2$; $E^2 = \left(mc^2\right)^2 + \left(pc\right)^2$; $p = \gamma mv$

Momentum four-vector $P = \left(\frac{E}{c}, \vec{p}\right)$

Units

Typical units used in this course.

Energy: 1 eV= 1.602×10^{-19} J :

1 keV = 10^3 eV (ionisation energy) 1 MeV= 10^6 eV (nuclear binding energy) ,

1 GeV= 10^9 GeV (accelerator energy)

Distance 1 fm = 10^{-15} m (size of a proton)

Momentum units : eV/c Mass units : eV/c^2

Conversion

Convert from MKS-SI units :

Eg Momentum 5×10^{-23} kgms^{-1} $= X$ eV$/c$ $= X \dfrac{1.602\times10^{-19}\,\text{J}}{3\times10^{8}\,\text{ms}^{-1}} = X \times 0.534 \times10^{-27}$ kgms^{-1}

$$\Rightarrow X = \frac{5\times10^{-23}}{0.534\times10^{-27}} = 9.36\times10^{4}\ \text{eV}/c = 93.6\ \text{keV}/c$$

Convert to MKS-SI units :

$$\text{Eg mass} = 1.2\ \frac{\text{GeV}}{c^2} = 1.2\times \frac{10^{9}\times1.602\times10^{-19}\ \text{J}}{\left(3\times10^{8}\right)^{2}\ \text{m}^{2}\text{s}^{-2}} = 2.136\times10^{-27}\ \text{kg}$$

Natural units

You may encounter natural units in some texts.

Convention $c = 1, \quad \hbar = 1$

Eg $E = \hbar\omega \rightarrow E = \omega \quad , \quad E^2 = (pc)^2 + (mc^2)^2 \rightarrow E^2 = p^2 + m^2$

Counter intuitive eg frequency in units of GeV.

Conversion table:

Quantity	Natural units	SI
Energy	1 GeV	1.602×10^{-10}J
Mass	1 GeV	1.78×10^{-27}kg
Momentum	1 GeV	5.34×10^{-19}kgms^{-1}
Time	1 GeV^{-1}	6.58×10^{-25}s
Frequency	1 GeV	1.52×10^{24}s^{-1}
Distance	1 GeV^{-1}	1.97×10^{-16}m

Summary

- Course information given
- Special relativity
  - Two postulates
  - Lorentz transformations
  - Four vectors
- Units