1 Hand-in problems, set III

Deadline Friday October 19th, at 16:00

(You may send solutions in pdf format to Lars, lbe@fysik.su.se, or leave them in his post box - ask at the Reception to deliver there.)

1. (2 p) Suppose that a new space telescope will be built that measures parallax with an accuracy of $1.0 \cdot 10^{-6}$ arcseconds. How many light-years away will the most distant stars be that can have their distance determined with the parallax method using this telescope?

2. (a) (2 p) Show that the expansion rate of the universe at a given time t,
$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{1}$$
with $\dot{a}(t) \equiv da(t)/dt$ also can be written, changing the time variable t to redshift,
$$z = \frac{a(t_0)}{a(t)} - 1 \tag{2}$$
as
$$H(z) = \frac{-1}{(1+z)}\frac{dz}{dt}. \tag{3}$$
(b) (2 p) One can also show that (see formula sheet on the homepage of the course)
$$H^2(z) = H_0^2\left[\Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_K(1+z)^2 + \Omega_\Lambda\right], \tag{4}$$
where you can use $H_0 = 70$ kms^{-1}Mpc^{-1} for the Hubble constant (i.e., h=0.7), and Ω_M, Ω_R, Ω_Λ and Ω_K are the present energy densities, normalized to the critical density, in mass, radiation, dark energy, and curvature, respectively. From this, show that the "lookback time", i.e. the time taken from emission at redshift z_e (time t_e) to observation at time t_0, can be written
$$t_0 - t_e = \int dz \frac{f(z)}{H(z)} \tag{5}$$
(Hint: Solve for dt from Eq. (3)). Determine the integration limits and the function $f(z)$).

3. (a) (2 p) Write a computer program (e.g. in python) which plots the value of the integrand in Eq. (5), for integer values, $z = 1, 2, \ldots 10$. You may use $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$, a flat universe, $h = 0.7$, and neglect radiation (i.e., $\Omega_K = \Omega_R = 0$). The plot should be logarithmic for the y-axis, linear for the x-axis.
(b) (1 p) Same, for $\Omega_K = \Omega_\Lambda = 0$, but suppose that radiation was very high, $\Omega_R = 0.7$, $\Omega_M = 0.3$.
(c) (1 p) Same, for a matter-only universe, $\Omega_M = 1$.