FK5024 hand-in problems – deadline 24:00 Thursday September 20th

(1) A proton and an electron collide head-on.
Show that the square of the four-momentum transferred between the proton and electron can be written as: $q^2 = -4EE' \cos^2\left(\frac{\theta}{2}\right)$. Here E is the electron energy before the collision, E' is the electron energy after the collision and θ is the angle between the electron after the collision and direction of the incoming proton. You may neglect the masses of electron and proton.

(2) (i) Express 2.05×10^{-27} kg in units of GeV/c^2.
 (ii) Express 3.4×10^{-24} kgms^{-1} in units of GeV/c.
 (iii) Express 6.2 keV/c in SI units.
 (iv) Express 5.2 GeV/c^2 in SI units.

(3) Consider the following decays/reactions. Discuss which of these are possible to observe and draw a Feynman diagram in that case. If a process is impossible state a conservation law forbidding it.
 (i) $\rho \to e^+ + e^-$
 (ii) $p \to e^+ + \pi^-$
 (iii) $K^+ \to \mu^+ + \nu_\mu + \pi^0$
 (iv) $\mu^+ + \mu^- \to \pi^- + \pi^+$

(4) Electrons and muons are collided head on at a collider. The reaction $e^- + \mu^+ \to \nu_e + \bar{\nu}_\mu$ takes place. The electrons each have an energy of 35 GeV. The muons have an energy of 50 GeV.
(a) Calculate the centre-of-mass energy of the collision.
(b) The electron neutrino has an energy of 25 GeV and moves at an angle of 20^o with respect to the incoming electron's direction. Calculate the muon neutrino's momentum.
(c) Draw a Feynman diagram of the process $e^- + \mu^+ \to \nu_e + \bar{\nu}_\mu$.

(5) A new particle is discovered. The width of the mass distribution is 200 MeV.
(i) Estimate the particle lifetime.
(ii) Does the particle decay dominantly by strong, weak or electromagnetic processes ?

(6) Assuming that there are two only neutrino flavours α and β and that there are two mass states i and j , the probability that a neutrino which was in flavour state α at time $t=0$ has transformed to a neutrino in flavour state β at a later time t is given by:

$$PP_{\nu_\alpha \to \nu_\beta} = \left[\sin 2\theta \, \sin\frac{(E_j - E_i)t}{2\hbar}\right]^2$$

where E_i and E_j are energy eigenvalues and θ is a mixing angle.

Show that $P_{\nu_\alpha \to \nu_\beta} \sim \left[\sin 2\theta \, \sin\frac{L}{L_0}\right]^2$ where L is the distance travelled by the neutrino at time t

and $L_0 = \frac{4E\hbar c}{(m_j^2 - m_i^2)c^4}$ where $E \sim E_i \sim E_j$ and m_i and m_i are the mass eigenvalues.