RA, SH, PET, LH and FT Fysikum, Stockholms universitet

Suggested solutions for
Nuclear & Particle physics, Astrophysics & Cosmology, FK5024
9.00-14.00, 2016-10-28

If you have questions, please consult the relevant teacher. PET for problems 1 and 5, SH for 2 and 4, and RA for 3 and 6.

1. The semi-empirical mass formula (SEMF) can be used to calculate nuclear masses to a rather good precision over a wide range of mass numbers A. The SEMF gives the binding energy, $B(A, Z)$, for a nucleus with atomic number Z and mass number A as:

$$B(A,Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(Z-A/2)^2}{A} + a_p A^{-1/2}\,,$$

where $a_v = 15.56\,\text{MeV}$, $a_s = 17.23\,\text{MeV}$, $a_c = 0.697\,\text{MeV}$, $a_a = 93.14\,\text{MeV}$ and $a_p = 12\,\text{MeV}$ for even-even nuclei, $a_p = -12\,\text{MeV}$ for odd-odd nuclei and $0\,\text{MeV}$ for odd A.

(a) Calculate the energy release in the following induced fission reaction (the incoming neutron can be assumed to have a very low energy).

$$\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{89}_{35}\text{Br} + {}^{145}_{57}\text{La} + 2\text{n}$$

(2p)

(b) Which term in the SEMF gives the largest contribution to the energy release in (a)? Which interaction does this term correspond to? (2p)

1a) The energy release equals the Q-value, i.e.

$$Q = \left[M_n + M\left({}^{235}_{92}U\right) - \left\{M\left({}^{89}_{35}Br\right) + M\left({}^{145}_{57}La\right) + 2M_n\right\}\right]c^2 =$$

$$= \left[M_n + \left\{92M_p + 143M_n - B(235,92)/c^2\right\} - \left\{\text{etc for Br and La}\right\}\right]$$

$$= B(89,35) + B(145,57) - B(235,92) =$$

$$= (\text{according to SEMF}) = (\text{All nuclei have A odd})$$

$$= a_v(89 + 145 - 235)$$
$$- a_s(89^{2/3} + 145^{2/3} - 235^{2/3})$$
$$- a_c(35 \cdot 34/89^{1/3} + 57 \cdot 56/145^{1/3} - 92 \cdot 91/235^{1/3})$$
$$- a_a\left(\left(35 - 89/2\right)^2/89 + \left(57 - 145/2\right)^2/145 - \left(92 - \frac{235}{2}\right)^2/235\right)$$
$$+ 0$$
$$= -15{,}56 \text{ MeV} - 163{,}9 \text{ MeV} + 328{,}2 \text{ MeV} + 8{,}9 \text{ MeV}$$

$$\approx \underline{158 \text{ MeV}}$$

b) Term 3 contributes the most to Q. This is the Coulomb term corresponding to the Coulomb interaction (repulsion between the protons).

2. Only one of the following decays has been observed in nature. Explain which and what interaction that generates the decay. For the decays that don't occur, explain which principle or conservation law that forbids this decay. For hadrons the quark composition is given in brackets after the name of the particle.

   (a) $e^- \rightarrow \mu^- + \nu_e + \bar{\nu}_\mu$ (1p)

   This decay violates conservation of energy ($m_e < m_\mu$) so it does not occur in nature.

   (b) B^+ ($\bar{b}u$) $\rightarrow \bar{D}^0$ ($\bar{c}d$) $+ \pi^+$ ($u\bar{d}$) (1p)

   This decay occurs at about 0.5 % of the cases, and is generated by the weak decay.

   ***Note:** unfortunately there was a typographic mistake in this question, the quark-content of the $\bar{D}^0$ meson was incorrectly given as ($\bar{c}d$).*
   After thourough discussions we have decided to leave the question included in the exam. In order to ensure that this does not disadvantage anyone, credits have been awarded for correct statements about the decays to the extent that there are in principle more than one way to obtain a full score on this question, and that a full score is possible even though one of the decays necessarily incorrectly will be stated to be the one which occurs in nature.

   (c) D^+ ($c\bar{d}$) $\rightarrow e^- + e^+ + \pi^+$ ($u\bar{d}$) (1p)

   This decay would need to couple Z or γ^* to a vertex where a c-quark is changed into a u-quark which does not appear in nature.

   (d) $\mu^+ \rightarrow e^+ + \gamma$ (1p)

   This decay violates conservation of mu- and electron-number.

3. In the course we saw that the scale factor, a, as a function of time, t, can be written as $a(t) \propto t^{2/3}$ and $a(t) \propto t^{1/2}$ for matter and radiation domination, respectively. Derive similar expressions for:

   (a) An expanding empty universe that does not contain any matter or radiation and where $\Lambda = 0$. What can be said about the geometry for such a Universe, and what needs to be measured if we want to know the relative size 10 Gyr from today? (2p)

   The Friedmann equation for $\Lambda = 0$ and $\rho = 0$ gives

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} \quad \Rightarrow \quad \dot{a} = \pm\sqrt{-k}$$

   that only has a solution for an **open** *($k < 0$) and expanding universe, $\dot{a}(t) > 0$,*

$$a(t) \propto t\,.$$

   Since the Universe in this case is growing linearly with time, the only free parameter in this case is the age of the Universe. The relative scale factor,

a_1/a_0 for $a(t) \propto t$ is

$$\frac{a_1}{a_0} = \frac{t_1}{t_0} = \frac{t_0 + 10\,\text{Gyr}}{t_0}\,.$$

That is, the only thing that needs to be measured is the age of the Universe, t_0, where $t_0 = 1/H_0$.

(b) A flat Λ-dominated universe with $\Lambda > 0$. Although, we live in a universe with $\Omega_M \approx 0.3$ and $\Omega_\Lambda \approx 0.7$ this solution will become relevant in the future, why? (2p)

In this case we have $\rho = 0$ and $k = 0$, and the Friedmann equation becomes

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \quad \Rightarrow \quad a(t) \propto \exp\left(t\sqrt{\Lambda/3}\right)\,.$$

Since Λ is constant, and $\rho_m \propto 1/a^3$, Λ will dominate completely in the future, and this solution will become a good approximation.

4. One of the principal processes that generates energy in stars like the sun is the proton-proton cycle, where hydrogen is converted to helium. This process happens in three steps:

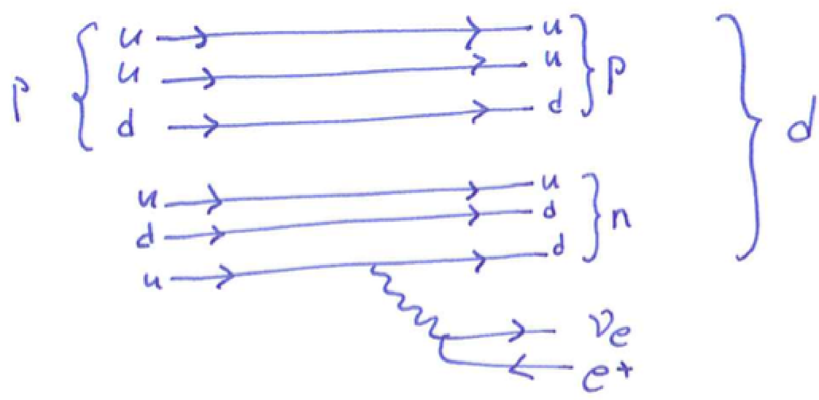
   (a) p + p $\rightarrow$ d + X + Y, where d is a deuterium nucleus (a bound system of a proton and a neutron)

   (b) d + p $\rightarrow$ ${}^3_2$He + γ

   (c) 2 ${}^3_2$He $\rightarrow$ ${}^4_2$He + 2 p

   Consider the first step. Which type of interaction is this. What are the particles X and Y? Draw a Feynman-diagram which describes this process on the quark-lepton level. (4p)

   In this process one proton is converted into a neutron by emission of a positron and an electron-neutrino, mediated by a virtual (off the mass-shell) W-boson.



5. The waste from a thermal nuclear fission reactor contains, apart from uranium, fission products and also transuranic elements. In the table below you see the amounts of some of the long-lived radioactive nuclides taken out from an average-size Swedish reactor every year. The cross sections for neutron capture leading to gamma-emission (i.e. no fission), σ_γ, and leading to fission, σ_f, are also given.

Nuclide	Half-life (yr)	Mass in waste (kg)	n capture σ (10^{-28} m^2) (for thermal neutrons)	
			σ_γ	σ_f
^{239}Pu	24 000	166.0	270	742
^{240}Pu	6 600	76.7	290	0.05
^{99}Tc	210 000	24.7	226	0
^{137}Cs	30	31.8	0.22	0

(a) Assume that the tabulated amounts are taken out from the reactor at a certain time. Calculate the activity (= decays per unit time) of the four nuclides at that time and at a time 10 000 years later. (2p)

(b) During recent years the possibility of using transmutation techniques to decrease the long-lived activity in the waste has been discussed. One way could be to put the waste in a flux of thermal neutrons, thereby inducing neutron capture reactions transforming e.g. ^{99}Tc to ^{100}Tc, which is very short-lived (half-life 17 s) and decays to the stable ^{100}Ru. The rate of capture reactions is given by $R = \sigma J N$, where σ is the capture cross section, N the number of target nuclei (^{99}Tc in this example) and J the neutron flux. Calculate the neutron flux needed to, in one month, decrease the amount of ^{99}Tc to half its value. (2p)

5 a) The activity $\mathcal{A}$ is given by the number of radioactive nuclei N as:

$$\mathcal{A} = \lambda N, \quad \text{where } \lambda = \frac{\ln 2}{t_{1/2}}$$

$$N \approx \frac{M}{A\,\text{kg}} \cdot 6 \cdot 10^{26}, \quad \text{where } M \text{ is the mass.}$$

The activity of ^{239}Pu with mass 166,0 kg is therefore:

$$\mathcal{A}_0(^{239}Pu) \approx \frac{\ln 2}{24000\,\text{yr}} \cdot \frac{166{,}0\,\text{kg}}{239\,\text{kg}} \cdot 6 \cdot 10^{26} = 3{,}8 \cdot 10^{14}\,\text{s}^{-1} = 3{,}8 \cdot 10^{14}\,\text{Bq}$$

After 10 000 years

$$\mathcal{A}_{10000}(^{239}Pu) = \mathcal{A}_0(^{239}Pu)\, e^{-\frac{\ln 2}{t_{1/2}} 10000\,\text{yr}} = 2{,}8 \cdot 10^{14}\,\text{Bq}.$$

In a similar way:

$$\mathcal{A}_0(^{240}Pu) = 6{,}3 \cdot 10^{14}\,\text{Bq}, \quad \mathcal{A}_{10000}(^{240}Pu) = 2{,}2 \cdot 10^{14}\,\text{Bq}$$

$$\mathcal{A}_0(^{99}Tc) = 1{,}5 \cdot 10^{13}\,\text{Bq}, \quad \mathcal{A}_{10000}(^{99}Tc) = 1{,}4 \cdot 10^{13}\,\text{Bq}$$

$$\mathcal{A}_0(^{137}Cs) = 1{,}0 \cdot 10^{17}\,\text{Bq}, \quad \mathcal{A}_{10000}(^{137}Cs) \doteq 0{,}00\,\text{Bq}$$

b) Number of ^{99}Tc nuclei destroyed per unit time is:

$$-\frac{dN}{dt} = R = \sigma J_n N$$

differential equation (similar in structure to the radioactive decay equation!).

$$\Rightarrow N(t) = N(t{=}0)\, e^{-\sigma J_n t}$$

Requirement $N(1\,\text{month}) = \frac{1}{2} N(t{=}0) \Rightarrow 1\,\text{month} = \frac{\ln 2}{\sigma J_n}$

$$\Rightarrow J_n = \frac{\ln 2}{\sigma \cdot 1\,\text{month}} = \frac{\ln 2}{22 \cdot 10^{-28}\,\text{m}^2 \cdot 2{,}6 \cdot 10^6\,\text{s}} = \underline{1{,}2 \cdot 10^{19}\,\text{m}^{-2}\text{s}^{-1}}$$

6. Cosmological observations.

   (a) Give two examples of observations that can be used as evidence for the Big Bang model. (2p)

   Observations could for example be

   The expansion of the Universe *The fact that the Universe appears to have been expanding during the cosmic history, suggests that the Universe started in a singularity for $t \to 0$ and $a(t) \to 0$.*

   The CMB *The relic radiation shows that the Universe was in thermal equilibrium in its infancy and provides a natural connection between the early Universe and that the present Universe appears to be homogeneous and isotropic on large scales. Further, it suggests that it the Universe was very hot in the beginning, and that it has been cooling off ever since.*

   BBN *The measured abundance between the lightest elements in the Universe is very close to what is predicted in a hot Big Bang scenario, and cannot be explained by stellar production.*

   (b) Give two examples of observations that suggest the presence of dark matter in the Universe. (2p)

   Some of the observations that we have discussed during the course are

   Rotation curves of galaxies *When studying the rotation curves of galaxies they seem to move much faster at larger radii than what is suggested by the visible matter.*

   Velocity dispersions *The velocity dispersion of individual galaxies in galaxy clusters is higher than what we could expect from studying the visible matter.*

   Temperature of gas in galaxy clusters *The gas that falls in a galaxy cluster during formation will be heated by the gravitational potential and can therefore be used as a measurement of the latter. This suggests a much higher mass than the visible matter.*

It is not enough to just list observations, but you must also explain the connection between the observations and the conclusions.

Good luck!