Exam, FK5024, Nuclear & particle physics, astrophysics & cosmology, October 26, 2017

08:00 – 13:00, Room FR4 (Oskar Klein Auditorium)

No tools allowed except calculator (provided at the exam) and the attached formula sheet.

1. (4 p) Consider the following decays/reactions (the particles are not bound or virtual). Discuss which of these are possible to observe and draw a Feynman diagram in that case. If a process is impossible state a conservation law forbidding it.
(a) $Z^0 \to e^+ + e^-$
(b) $p \to n + e^+ + \nu_e$
(c) $\mu^- \to e^- + \mu^+ + e^-$
(d) $\tau^+ \to e^+ + \nu_\tau + \nu_e$

2. (4 p) In a scattering process, an electron interacts with a quark in the proton via the exchange of a virtual photon with four-momentum of the photon P_γ (one can show that $P_\gamma^2 < 0$ for scattering). If the proton's four-momentum is P, show that the fraction x of the proton's momentum carried by the struck quark is

$$x = \frac{-P_\gamma^2}{2P \bullet P_\gamma}.$$

(The symbol $\bullet$ denotes the 4-scalar product.) You can assume that the proton, and thus the quarks, are travelling at a relativistic speed and that particle rest masses can be neglected.

3. (4 p) The maximum positron kinetic energy in the spectrum of positrons emitted in the nuclear decay $^{11}\text{C} \to {}^{11}\text{B}$ is 0.96 MeV. Use this information and the known mass of ^{11}B, 10.2551 GeV/c^2, to compute the mass of ^{11}C.

4. Consider a parent nucleus with Z+2 protons, undergoing α-decay into a daughter nucleus with Z protons. The charge of the α-particle in units of e is $z = 2$.
(a) (1 p) Write down the expression for the Coulomb potential $V(r)$ of the α-particle at a distance r from the daughter nucleus.
(b) (1 p) Given the binding energy $B = 34$ MeV and $Z = 90$, find the value of the radius a where the α-particle is classically confined using the formula $B = V(a)$. Given the value $Q = 6$ MeV for the reaction, find the value of the radius b for which the α-particle has tunneled away of the Coulomb potential, using the formula $Q = V(b)$.
(c) (2 p) Compute the Gamow factor

$$G \approx \sqrt{\frac{2mc^2}{Q}} \frac{zZ}{137} \left(\frac{\pi}{2} - 2\sqrt{x} \right),$$

where $x = a/b = Q/B$, and $mc^2 = 3.73\,\mathrm{GeV}$ is the rest mass of the α-particle. Estimate the (small) probability to penetrate the barrier, using the formula $P = \exp(-2G)$.

5. Explain briefly the following concepts:
(a) (1 p) Radiation-matter equality.
(b) (1 p) Dark matter.
(c) (2 p) The geometry term proportional to k and the Λ term in the Friedmann equation.

6. (4 p) (a) (1 p) For an equation of state $p/c^2 = w \cdot \rho$, how does ρ depend on the scale factor a?
(b) (2 p) The early Universe could in principle be dominated by cosmic strings of length $l = a(t) \cdot l^0$ where $a(t)$ is the scale factor and l^0 is a constant (i.e., the strings get longer by the scale factor). The energy density in a string is $\lambda \cdot l$ with the string tension λ being a constant. The total energy density of $i = 1, 2, 3, \ldots$ strings of lengths $l_i = a(t) l_i^0$ in a physical volume V is thus

$$\rho_s = \sum_i \frac{\lambda l_i}{V}.$$

What is the equation of state for these cosmic strings?
(c) (1 p) How does the scale factor depend on time for cosmic strings?

Good Luck!

Useful equations

Friedmann equation:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m+\rho_r) - \frac{kc^2}{a^2} + \frac{\Lambda}{3} = H_0^2\left[\Omega_M(1+z)^3+\Omega_R(1+z)^4+\Omega_K(1+z)^2+\Omega_\Lambda\right]$$

Fluid eqation:

$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$

Acceleration equation:

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda}{3}$

"Deceleration parameter":

$q_0 = -\left(\frac{\ddot{a}(t_0)}{a(t_0)}\right)\frac{1}{H_0^2}$

Equation of state:

$p/c^2 = w \cdot \rho$

Coulomb potential:

$V(r) = \frac{Q_1 Q_2}{4\pi\epsilon_0 r} = \frac{Z_1 Z_2 \alpha \hbar c}{r}$,

where $Q_i = Z_i e$, $\alpha = \frac{1}{137}$ is the fine-structure constant and $\hbar c = 1.973 \cdot 10^{-7}$ eV·m

Constants (W, Z masses and Hubble h are 2017 values)

Name	Symbol	value
Newton's constant	G_N	$6.672 \cdot 10^{-11}$ m^{3}kg^{-1}s^{-2}
Speed of light	c	$2.998 \cdot 10^8$ m s^{-1} or $3.076 \cdot 10^{-7}$ Mpc year^{-1}
Planck's constant	$\hbar = h/2\pi$	$1.055 \cdot 10^{-34}$ m^2 kg s^{-1}
Boltzmann's constant	k_B	$1.381 \cdot 10^{-23}$ J/K or $8.619 \cdot 10^{-5}$ eV/K
Radiation constant	$\alpha_{rad} = \pi^2 k_B^4/15\hbar^3 c^3$	$7.565 \cdot 10^{-16}$ J m^{-3} K^{-4}
Electron rest mass energy	$m_e c^2$	0.511 MeV
Proton rest mass energy	$m_p c^2$	938.3 MeV
Neutron rest mass energy	$m_n c^2$	939.6 MeV
W boson rest mass energy	$m_W c^2$	80.4 GeV
Z boson rest mass energy	$m_Z c^2$	91.2 GeV
Planck energy	$M_{Pl} c^2$	$1.2 \cdot 10^{19}$ GeV
Thomson cross section	σ_e	$6.652 \cdot 10^{-29}$ m^2
Neutron half-life (free neutron)	$t_{\frac{1}{2}}$	611 s
Hubble constant	H_0	$100 \cdot h$ km s^{-1} Mpc^{-1}
	h	0.70 ± 0.03
Inverse Hubble constant	H_0^{-1}	$9.77 h^{-1} \cdot 10^9$ years
Critical density	ρ_c^0	$1.05 h^2 \cdot 10^{-5}$ GeV cm^{-3}

Conversion factors

1 pc = 3.261 light-years = $3.086 \cdot 10^{16}$ m
1 AU = $1.5 \cdot 10^{11}$ m
1 year = $3.156 \cdot 10^7$ s
1 eV = $1.602 \cdot 10^{-19}$ J
1 $M_\odot$ = $1.989 \cdot 10^{30}$ kg

PARTICLE DATA

(Mass in MeV/c^2; Lifetime in Seconds; Charge in Units of Proton Charge.)

QUARKS (Spin $\frac{1}{2}$)

	Flavor	Charge	Mass (speculative)		
			Bare	Effective	
				In baryons	In mesons
First generation	d	$-\frac{1}{3}$	7.5	363	310
	u	$+\frac{2}{3}$	4.2		
Second generation	s	$-\frac{1}{3}$	150	538	483
	c	$+\frac{2}{3}$	1100	1500	
Third generation	b	$-\frac{1}{3}$	4200	4700	
	t	$+\frac{2}{3}$	>23,000		

LEPTONS (Spin $\frac{1}{2}$)

	Lepton	Charge	Mass	Lifetime	Principal decays
First generation	e	−1	0.511003	∞	—
	ν_e	0	~ 0	∞	—
Second generation	μ	−1	105.659	2.197×10^{-6}	$e\nu_\mu\bar{\nu}_e$
	ν_μ	0	~ 0	∞	—
Third generation	τ	−1	1784	3.3×10^{-13}	$\mu\nu_\tau\bar{\nu}_\mu$, $e\nu_\tau\bar{\nu}_e$, $\rho\nu_\tau$
	ν_τ	0	~ 0	∞	—

MEDIATORS (Spin 1)

Mediator	Charge	Mass	Lifetime	Force	
gluon	0	0	∞	strong	
photon (γ)	0	0	∞	electromagnetic	electroweak
$W^\pm$	±1	81,800	unknown	(charged) weak	
Z^0	0	92,600	unknown	(neutral) weak	

BARYONS (Spin $\frac{1}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
N p	uud	+1	938.280	∞	—
N n	udd	0	939.573	900	$pe\bar{\nu}_e$
Λ	uds	0	1115.6	2.63×10^{-10}	$p\pi^-$, $n\pi^0$
Σ^+	uus	+1	1189.4	0.80×10^{-10}	$p\pi^0$, $n\pi^+$
Σ^0	uds	0	1192.5	6×10^{-20}	$\Lambda\gamma$
Σ^-	dds	−1	1197.3	1.48×10^{-10}	$n\pi^-$
Ξ^0	uss	0	1314.9	2.90×10^{-10}	$\Lambda\pi^0$
Ξ^-	dss	−1	1321.3	1.64×10^{-10}	$\Lambda\pi^-$
Λ_c^+	udc	+1	2281	2×10^{-13}	not established

BARYONS (Spin $\frac{3}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
Δ	uuu, uud, udd, ddd	+2, +1, 0, −1	1232	0.6×10^{-23}	$N\pi$
Σ^*	uus, uds, dds	+1, 0, −1	1385	2×10^{-23}	$\Lambda\pi$, $\Sigma\pi$
Ξ^*	uss, dss	0, −1	1533	7×10^{-23}	$\Xi\pi$
Ω^-	sss	−1	1672	0.82×10^{-10}	ΛK^-, $\Xi^0\pi^-$, $\Xi^-\pi^0$

PSEUDOSCALAR MESONS (Spin 0)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
$\pi^\pm$	$u\bar{d}$, $d\bar{u}$	+1, −1	139.569	2.60×10^{-8}	$\mu\nu_\mu$
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	134.964	8.7×10^{-17}	$\gamma\gamma$
$K^\pm$	$u\bar{s}$, $s\bar{u}$	+1, −1	493.67	1.24×10^{-8}	$\mu\nu_\mu$, $\pi^\pm\pi^0$, $\pi^\pm\pi^\pm\pi^\mp$
K^0, $\bar{K}^0$	$d\bar{s}$, $s\bar{d}$	0, 0	497.72	K_S^0 0.892×10^{-10}	$\pi^+\pi^-$, $\pi^0\pi^0$
				K_L^0 5.18×10^{-8}	$\pi e\nu_e$, $\pi\mu\nu_\mu$, $\pi\pi\pi$
η	$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$	0	548.8	7×10^{-19}	$\gamma\gamma$, $\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0$
η'	$(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$	0	957.6	3×10^{-21}	$\eta\pi\pi$, $\rho^0\gamma$
$D^\pm$	$c\bar{d}$, $d\bar{c}$	+1, −1	1869	9×10^{-13}	$K\pi\pi$
D^0, $\bar{D}^0$	$c\bar{u}$, $u\bar{c}$	0, 0	1865	4×10^{-13}	$K\pi\pi$
$F^\pm$ (now $D_s^\pm$)	$c\bar{s}$, $s\bar{c}$	+1, −1	1971	3×10^{-13}	not established
$B^\pm$	$u\bar{b}$, $b\bar{u}$	+1, −1	5271	14×10^{-13}	$D + ?$
B^0, $\bar{B}^0$	$d\bar{b}$, $b\bar{d}$	0, 0	5275		
η_c	$c\bar{c}$	0	2981	6×10^{-23}	$KK\pi$, $\eta\pi\pi$, $\eta'\pi\pi$

VECTOR MESONS (Spin 1)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
ρ	$u\bar{d}$, $d\bar{u}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$	+1, −1, 0	770	0.4×10^{-23}	$\pi\pi$
K^*	$u\bar{s}$, $s\bar{u}$, $d\bar{s}$, $s\bar{d}$	+1, −1, 0, 0	892	1×10^{-23}	$K\pi$
ω	$(u\bar{u} + d\bar{d})/\sqrt{2}$	0	783	7×10^{-23}	$\pi^+\pi^-\pi^0$, $\pi^0\gamma$
ϕ	$s\bar{s}$	0	1020	20×10^{-23}	K^+K^-, $K^0\bar{K}^0$
J/ψ	$c\bar{c}$	0	3097	1×10^{-20}	e^+e^-, $\mu^+\mu^-$, 5π, 7π
D^*	$c\bar{d}$, $d\bar{c}$, $c\bar{u}$, $u\bar{c}$	+1, −1, 0, 0	2010	$>1 \times 10^{-22}$	$D\pi$, $D\gamma$
Υ	$b\bar{b}$	0	9460	2×10^{-20}	$\tau^+\tau^-$, $\mu^+\mu^-$, e^+e^-