QFT problem set 4

(Fawad Hassan)

Deadline: Monday, March 09, 2020

1. Show that the notions of helicity and chirality coincide for zero mass fermions.

2. (a) Starting from $(D_\mu\Phi)^\dagger D^\mu\Phi$, where Φ is the Higgs doublet in the electroweak theory, show that after spontaneous symmetry breaking, the gauge fields $W, W^\dagger, Z$ become massive while the photon field A remains massless. You can use the unitary gauge for Φ.

   (b) Starting from the Yang-Mills Lagrangian for the $SU(2)_W \times U(1)_Y$ gauge fields W_i^μ and B^μ, work out the Lagrangian for the physical fields $A^\mu, Z^\mu, W^\mu, W^{\dagger\mu}$, including the interaction terms.

   (c) Obtain the Feynman rules for the vertices of the type $WW^\dagger AA$, $WW^\dagger A$ and $WW^\dagger AZ$.

3. Consider the elastic electron-neutrino scattering processes, $\nu_\mu + e^- \to \nu_\mu + e^-$ and $\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$ in electroweak theory. Write down the expressions for the Feynmann amplitudes.

4. The following generalized Higgs-neutrino coupling term can be added to the original electroweak theory (see Mandl and Shaw for details and conventions),

$$-G_{l'l}\bar{\Psi}^L_{l'}(x)\psi^R_{\nu_l}(x)\tilde{\Phi}(x) - G^*_{l'l}\tilde{\Phi}^\dagger(x)\bar{\psi}^R_{\nu_l}(x)\Psi^L_{l'}(x) \qquad (A)$$

   In the unitary gauge, write (A) in terms of $\psi_j = \sum_l U_{jl}\psi_{\nu_l}$, where U is the unitary matrix that diagonalizes the Hermitian coupling matrix G, *i.e.,* $(UGU^\dagger)_{ij} = \lambda_i\delta_{ij}$. What are the masses m_j of the eigenstate neutrinos ν_j associated with the fields $\psi_j(x)$? Draw the Higgs-neutrino (ν_j) interaction vertex and show that it comes with vertex factor $(-i/v)m_j$.

5. Problem 19.2 (page 448) from Mandl and Shaw, 2nd edition (Decay rates are defined in section 16.5 (page 372) of the book).