QFT problem set 1

Due date: Friday, Dec. 04, 2020

1. Given a state
$$|n_k\rangle = \frac{1}{\sqrt{n_k!}}(a_k^\dagger)^{n_k}|0\rangle\,,$$
show that $\langle n_k|n_k\rangle = 1$. Furthermore, given $|n_k+1\rangle = c a_k^\dagger |n_k\rangle$, find c so that $|n_k+1\rangle$ is normalized to unity. (*2 points*)

   (Hint: It is convenient to first evaluate the commutator $[a_k, (a_k^\dagger)^{n_k}]$.)

2. (a) For a real scalar field, starting with a trail solution $\phi(x) = e^{ik_\mu x^\mu}$ show that the Klein-Gordon equation has the general solution
$$\phi(x) = \sum_{\mathbf{k}} \left(\tilde{\phi}(\mathbf{k})\,\mathrm{e}^{-ikx} + \tilde{\phi}^\dagger(\mathbf{k})\,\mathrm{e}^{ikx}\right),$$
where $kx = k_\mu x^\mu$ and $k^0 = \omega_\mathbf{k} = \sqrt{m^2+\mathbf{k}^2}$, taking $\hbar = c = 1$. (*2 points*)

   (b) Show that, using the normalization $\tilde{\phi}(\mathbf{k}) = \left(\frac{1}{2V\omega_\mathbf{k}}\right)^{1/2} a(\mathbf{k})$, one obtains the standard commutation relations for $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$. (*2 points*)

3. For the real scalar field, starting from the general expression for the conserved Noether charge, find the expressions for the Hamiltonian H and the momenta P_i. Compute the quantities H and P_i in terms of the creation and annihilation operators, showing the steps in your calculation. (*3 points*).