Electroweak Theory

Ingradients:

Particle content:

leptons: $e \quad \mu \quad \tau$ electric charge -1

$\nu_e \quad \nu_\mu \quad \nu_\tau$ " 0

+ anti particles.

Generic notation:

$$l \qquad (m_l \neq 0)$$

$$\nu_l \qquad (m_{\nu_l} \approx 0)$$

we start with $m_l = 0, \; m_{\nu_l} = 0.$

Fields: $l : \psi_l$

$\nu_l : \psi_{\nu_l}$

$$\mathcal{L} = i \bar{\psi}_l \not{\partial} \psi_l + i \bar{\psi}_{\nu_l} \not{\partial} \psi_{\nu_l} \equiv i \bar{\Psi}_l \not{\partial} \Psi_l$$

we have introduced the doublet:

$$\Psi_l = \begin{pmatrix} \psi_{\nu_l} \\ \psi_l \end{pmatrix}$$

$\mathcal{L}$ is invariant under $U(2) \sim SU(2) \times U(1)$ global transf. acting on Ψ_l. But we want to introduce more structure and consider only part of the transformations.

$$\psi_\ell = \psi_\ell^L + \psi_\ell^R$$

$$\psi_{\nu_e} = \psi_{\nu_e}^L + \psi_{\nu_e}^R$$

construct:

$$\Psi_\ell^L = \begin{pmatrix} \psi_\ell^L \\ \psi_{\nu_e}^L \end{pmatrix}$$

We choose not to combine ψ_ℓ^R & $\psi_{\nu_e}^R$ in another double. Then,

$$\mathcal{L} = i\bar{\Psi}_\ell^L \not\partial \Psi_\ell^L + i\bar{\psi}_\ell^R \not\partial \psi_\ell^R + i\bar{\psi}_{\nu_e}^R \not\partial \psi_{\nu_e}^R .$$

consider an SU(2) transformation such that

Ψ_ℓ^L : SU(2) doublet

$\psi_\ell^R, \psi_{\nu_e}^R$: SU(2) singlets.

i.e.,

$$\Psi_\ell^L \longrightarrow U \Psi_\ell^L$$

$$U \in SU(2), \qquad U = e^{i\alpha_j \tau_j/2}$$

$$\bar{\Psi}_\ell^L \Rightarrow \bar{\Psi}_\ell^L U^+$$

This is a symmetry of $\mathcal{L}$.

For α_j infinitesimal,

$$\Psi_\ell^L \to \Psi_\ell'^L = \left(1 + i\alpha_j \frac{\tau_j}{2}\right)\Psi_\ell^L$$

$$\bar{\Psi}_\ell^L \to \bar{\Psi}_\ell'^L = \bar{\Psi}_\ell^L\left(1 - i\alpha_j \frac{\tau_j}{2}\right)$$

Associated conserved currents:

$$\boxed{J_i^\mu = \frac{1}{2}\bar{\Psi}_\ell^L \gamma^\mu \tau_i \Psi_\ell^L} \qquad (i = 1, 2, 3)$$

(Weak isospin currents)

History of isospin: $\begin{pmatrix} n \\ p \end{pmatrix}$ doublet in nuclear physics & the approximate SU(2) symmetry

<u>Weak isospin charges</u>:

$$I_i^W = \int d^3x\ J_i^0 = \int d^3x\ \bar{\Psi}_\ell^L \gamma^0 \tau_i \Psi_\ell^L$$

$$[I_i^W, I_j^W] = i\epsilon_{ijk} I_k^W.$$

$\Rightarrow$ all three charges are not simultaneously measurable. As in the angular momentum case, pick up $I_3^W \Rightarrow$ the components of the doublet are characterized by their I_3^W eigen values: $1/2$ for ψ_{ν_e} and $-1/2$ for ψ_e.

In analogy with spin, we define

$$I^W = 2\left(I_1^W - i\, I_2^W\right)$$
$$I^{W\dagger} = 2\left(I_1^W + i\, I_2^W\right).$$

They carry I_3^W charges -1 and $+1$ respectively,

$$[I_3^W, I^W] = -I^W, \qquad [I_3^W, I^{W\dagger}] = +I^{W\dagger}.$$

We can carry out a similar construction for the currents and define:

$$J^\mu = 2\left(J_1^\mu - i\, J_2^\mu\right), \qquad J^{\dagger\mu} = 2\left(J_1^\mu + i\, J_2^\mu\right)$$

Using the explicit expressions for τ_j and Ψ_ℓ^L, one gets:

$$J^\mu = \bar{\psi}_\ell\, \gamma^\mu (1-\gamma_5)\, \psi_{\nu_e}$$
$$J^{\dagger\mu} = \bar{\psi}_{\nu_e}\, \gamma^\mu (1-\gamma_5)\, \psi_\ell$$
$$J_3^\mu = \frac{1}{2}\left(\bar{\psi}_{\nu_e}^L \gamma^\mu \psi_{\nu_e}^L - \bar{\psi}_\ell^L \gamma^\mu \psi_\ell^L\right)$$

(*) J^μ & $J^{\dagger\mu}$ are "charged" currents (they involve a neutral particle (ν_e) and an electrically charged particle (ℓ). They give rise to interactions in which the charge of the fermion changes. The extra charge is carried away by gauge bosons (we will see this latter). J_3^μ is a "neutral" current.

(*) vector & axial-vector currents:

$$V^\mu = \bar{\psi}_a \gamma^\mu \psi_b \quad \text{(vector)}$$

$$A^\mu = \bar{\psi}_a \gamma^\mu \gamma_5 \psi_b \quad \text{(Axial)}$$

J^μ, $J^{+\mu}$ and J_3^μ all have $V-A$ form (not $V+A$, or $aV+bA$). For J_3^μ, this can be seen by writing it in terms of ψ_e & $\psi_{\nu e}$,

$$J_3^\mu = \frac{1}{4}\left(\bar{\psi}_{\nu_e} \gamma^\mu (1-\gamma_5) \psi_{\nu e} - \bar{\psi}_e \gamma^\mu (1-\gamma_5) \psi_e\right)$$

This was an experimental discovery. The electroweak theory is engineered to produce this result.

Electromagnetic current:

$\mathcal{L}$ is also invariant under $U(1)_{em}$,

$$\psi_e \to e^{i(-e)\theta(x)} \psi_e$$

$$\psi_{\nu e} \to \psi_{\nu e}$$

leading to the conserved current,

(24)

$$J^\mu_{em} = (-e)\, \bar{\psi}_\ell \gamma^\mu \psi_\ell$$

$$= (-e)\left(\bar{\psi}^L_\ell \gamma^\mu \psi^L_\ell + \bar{\psi}^R_\ell \gamma^\mu \psi^R_\ell \right)$$

The conserved charge is

$$Q_{em} = (-e) \sum_{\vec{p}} \sum_{r=1}^{2} \sum_{\ell} \left(c^\dagger_{r,\ell}(\vec{p})\, c_{r,\ell}(\vec{p}) - d^\dagger_{r,\ell}(\vec{p})\, d_{r,\ell}(\vec{p}) \right)$$

Recall:

$$c^\dagger_{1,\ell}|0\rangle = |\ell^-, R\rangle \quad , \quad d^\dagger_{1,\ell} = |\ell^+, R\rangle$$

$$c^\dagger_{2,\ell}|0\rangle = |\ell^-, L\rangle \quad , \quad d^\dagger_{2\ell} = |\ell^-, R\rangle$$

and

$$Q_{em}\,|\ell^-, R\rangle = (-e)\,|\ell^-, R\rangle$$

$$Q_{em}\,|\ell^-, L\rangle = (-e)\,|\ell^-, L\rangle .$$

But,

$$Q_{em}\,|\nu_e, R\rangle = 0$$

$$Q_{em}\,|\nu_e, L\rangle = 0.$$

Note that $U(1)_{em}$ is not compatible with $SU(2)_w$: $SU(2)_w$ treats ψ_{ν_e} & ψ_ℓ on the same footing & mixes them up, But $U(1)_{em}$ transforms only ψ_ℓ. How do we relate J^μ_3 and J^μ_{em} ?

Weak isospin charge assigments

$$J_3^\mu = \frac{1}{2}\left(\bar{\psi}^L_{\nu_e}\gamma^\mu\psi^L_{\nu_e} - \bar{\psi}^L_l\gamma^\mu\psi^L_l\right)$$

In analogy with J^μ_{em}, we have

$$I_3^W = \frac{1}{2}\sum_{\vec{p}}\sum_l\Big[\left(c^\dagger_{2,\nu_e}c_{2\nu_e} - d^\dagger_{2,\nu_e}d_{2,\nu_e}\right)$$
$$-\left(c^\dagger_{2,l}c_{2,l} - d^\dagger_{2,l}d_{2,l}\right)\Big]$$

(c_1 & d_1 correspond to ψ^R)

$$c^\dagger_{2,\nu_e}|0\rangle = |\nu_e, L\rangle$$
$$c^\dagger_{2l}|0\rangle = |l^-, L\rangle.$$
$$I_3^W|\nu_e, L\rangle = \frac{1}{2}|\nu_e, L\rangle$$
$$I_3^W|l^-, L\rangle = -\frac{1}{2}|l^-, L\rangle$$
$$I_3^W|l^-/\nu_e, R\rangle = 0$$

Relation between I_3^W and Q_{em} (or J_3^μ & J^μ_{em}):

$$J_3^\mu = \frac{1}{2}\left(\bar{\psi}^L_{\nu_e}\gamma^\mu\psi^L_{\nu_e} - \psi^L_e\gamma^\mu\psi^L_e\right)$$
$$-\frac{1}{2}\psi^L_l\gamma^\mu\psi^L_l + \frac{1}{2}\psi^L_l\gamma^\mu\psi^L_l$$
$$-\psi^R_l\gamma^\mu\psi^R_l + \psi^R_l\gamma^\mu\psi^R_l$$
$$= \frac{J^\mu_{em}}{e} - J^\mu_Y$$

$$J_3^\mu = \frac{J_{em}^\mu}{e} - J_Y^\mu$$

where J_Y^μ stands for

$$J_Y^\mu = -\frac{1}{2}\left(\bar{\psi}_{\nu_e}^L \gamma^\mu \psi_{\nu_e}^L + \bar{\psi}_\ell^L \gamma^\mu \psi_\ell^L\right)$$
$$- \bar{\psi}_\ell^R \gamma^\mu \psi_\ell^R$$

since $\partial_\mu J_{em}^\mu = 0$, $\partial_\mu J_3^\mu = 0$, one has,

$$\partial_\mu J_Y^\mu = 0 .$$

what is the symmetry associated with this conservation law? From the structure of J_Y^μ one can read off the following $U(1)_Y$:

$$\left.\begin{array}{l} \psi_{\nu_e}^L \to e^{i(-1/2)\beta}\psi_{\nu_e}^L \\ \psi_\ell^L \to e^{i(-1/2)\beta}\psi_\ell^L \\ \psi_\ell^R \to e^{i(-1)\beta}\psi_\ell^R \\ \psi_{\nu_e}^R \to \psi_{\nu_e}^R \end{array}\right\} \quad \begin{array}{l} \psi_\sigma \to e^{i\beta Y_\sigma}\psi_\sigma \\ Y_{(\nu_e,L)} = Y_{(\ell,L)} = -\frac{1}{2} \\ Y_{(\ell,R)} = -1, \quad Y_{(\nu_e,R)} = 0 \end{array}$$

Y_σ: "weak hypercharge" of the fields.

(*) $U(1)_Y$ is compatible with $SU(2)_W$ since

$\psi^L_{\nu_e}$ and ψ^L_e are transformed in the same way.
Therefore the full symmetry group of the theory is

$$SU(2)_W \times U(1)_Y .$$

The relation

$$J^\mu_3 = J^\mu_{em}/e - J_Y \quad \text{or} \quad I^W_3 = \frac{Q_{em}}{e} - Y$$

tells us that $U(1)_{em}$ is a specific combination of $SU(2)_W$ and $U(1)_Y$ transformations:

$$\alpha_1, \alpha_2 = 0.$$

$$\underbrace{\alpha_3}_{SU(2)_W} = \underbrace{\beta}_{U(1)_Y} = e\underbrace{\theta}_{U(1)_{em}}$$

What about transformation corresponding to the other values of parameters? In the final theory (after we add up some extra ingradients) these other transformations are no longer manifest symmetries: they are spontaneously broken. $U(1)_{em}$ is the only unbroken combination.

	I^W_3	Y	Q_{em}/e
$\psi^L_{\nu e}$	$1/2$	$-1/2$	0
ψ^L_e	$-1/2$	$-1/2$	-1
ψ^R_e	0	-1	-1
$\psi^R_{\nu e}$	0	0	0