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Hand-in exercises in Analytical Mechanics

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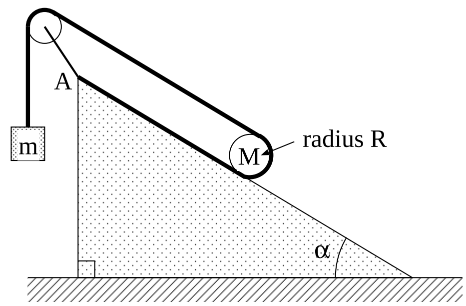
The last date for handing in the exercises is given in the course plan, on the course web site www.physto.se/~edsjo/teaching/am/ and in the right-hand margin for each problem. The hand-in exercises are not compulsory, but if you pass on them you will get one problem for free on the exam.

1. A thin cylindrical shell with mass M and radius R can roll on a fixed wedge with angle α (see figure). Around the cylinder, a thin thread (with negligible mass) is wrapped. One end of the thread is attached to the point A, whereas the other end is attached to a mass m. The thread runs over a frictionless and massless wheel at A. The masses m and M are affected by gravity downwards in the figure. **Th 1/2**



a) Derive and solve the equation of motion for the mass m. (3p)

b) Determine the angle α for which the system is in equilibrium. (2p)

2. a) Consider the two-body problem with a gravitational potential (i.e. Kepler's problem), **Th 8/2**

$$U(r) = -\frac{A}{r} \qquad ; \qquad A > 0$$

Derive the solutions, $r(\varphi)$ and prove Kepler's frist law, i.e. that the curves are ellipses when the total energy of the relative motion is $E < 0$. (2p)

b) Prove Kepler's third law, i.e. that

$$\frac{a^3}{T^2} = \text{konstant}$$

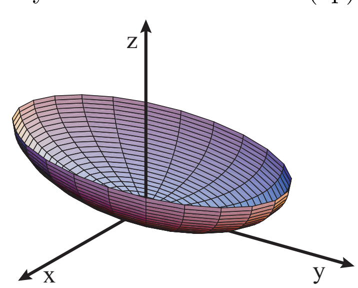
where a is length of the semimajor axis and T is the period. (2p)

c) Under what circumstances is Kepler's third law valid exactly? (1p)

3. A bath tub has the shape of a half ellipsoid, where the height z is given by **Th 15/2**

$$z = c - c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$$

where a, b and c are constants. You have just taken a bath and emptied the tub when you drop the soap in the bath tub. The soap will den perform small oscillations around the equilibrium point at the bottom of the tub. Determine the angular frequencies of these oscillations! The friction between the soap and the tub is negligible. (5p)

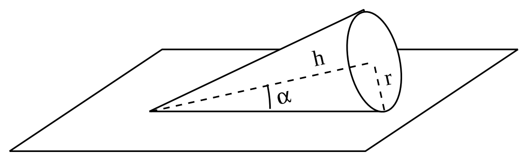


4. a) Define what a canonical transformation is and describe how a generating function can be used to generate the transformation. (2p) **Th 22/2**

b) Start from Hamilton's variational principle $\delta \int [\sum_i p_i \dot{q}_i - H(\underset{\sim}{q}, \underset{\sim}{p}, t)]dt = 0$ and show that a generating function $S(\underset{\sim}{q}, \underset{\sim}{P}, t)$ can generate a canonical transformation. Also derive the variable transformations between the old variables $\{\underset{\sim}{q}, \underset{\sim}{p}\}$ and the new variables $\{\underset{\sim}{Q}, \underset{\sim}{P}\}$. (3p)

Hint: Note that $\frac{d}{dt}\sum_i Q_i P_i$ can be added or subtracted from the Hamiltonian without changing the equations of motion.

5. A homogenous cone with mass m, height h and top angle 2α is rolling without slipping on a plane. The angular velocity is ω_0. Calculate the kinetic energy. (5p) **Tu 13/3**



Hint: The part of the cone that at each moment is in contact with the plane is temporarily at rest. This means that the angular velocity vector, ω at each moment has to lie along this line.

Good luck!