Joakim Edsjö
Fysikum, Stockholms Universitet
Tel: 08-55 37 87 26

# Exam in Analytical Mechanics, 5p

**August 24, 2001**

9–15

5 problems on 6 hours. Each problem gives a maximum of 5 points.
Write your name on each sheet of paper!
If you want your result by e-mail, write your e-mail address on the first page.
*Allowed books:* Physics Handbook and attached collection of formulae.

**1.** Consider a rigid body.

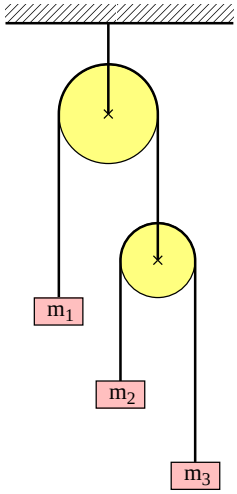
a) Show that the angular frequency vector $\omega$ is unique, i.e. that it is independent of which reference point in the body that is chosen. (3p)

b) Show that for the components of the tensor of inertia the following relation holds true, $I_{zz} \leq I_{xx} + I_{yy}$. For what kind of bodies do we have equality? (2p)

**2. If you have passed on the hand-in exercises, you don't have to do this problem. You will get full points for it anyway.**

A double Atwood machine (see figure) consists of three masses $m_1$, $m_2$ and $m_3$ connected with massless threads. The threads run over two massless pulleys that can rotate without friction around their symmetry axes. The masses are effected by the gravitational force downwards in the figure.

Write down the equations of motion for the system and solve these. The motion of the three masses can be assumed to take place vertically only. (5p)



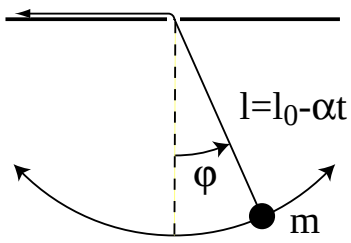

**3.** a) Define Poisson brackets and describe how they can be used to investigate if a transformation $(\underset{\sim}{q}, \underset{\sim}{p}) \rightarrow (\underset{\sim}{Q}, \underset{\sim}{P})$ is canonical. (3p)

b) The Hamiltonian for a particle system with two degrees of freedom is given by

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + a_1 q_1^2 + a_2 q_2^2$$

Determine a condition on $a_1$ and $a_2$ such that $q_1 p_2 - q_2 p_1$ is a constant of motion. (2p)

**4.** Consider a planar mathematical pendulum with the length $l$ and mass $m$. The thread goes through a hole and is pulled through this hole by an external force with a constant speed $\alpha$. The length of the pendulum thus decreases with time and can be written $l(t) = l_0 - \alpha t$.
Determine the Hamiltonian and the energy for the system. Is the Hamiltonian a constant of motion? Is the energy a constant of motion? Discuss your results. (5p)

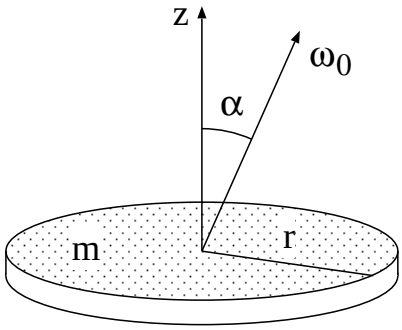



**5.** A frisbee is thrown away with a rotational velocity of $\nu$ turns/second. The rotation axis makes an angle $\alpha$ with the symmetry axis which is perpendicular to the plane of the frisbee.

a) Show that the symmetry axis, the angular momentum vector and the angular velocity vector all lie in the same plane. (3p)

b) Determine with which angular velocity the symmetry axis rotates around the angular momentum vector. (2p)

The frisbee can be approximated with a thin solid circular disk with mass $m$ and radius $r$.




**Good luck!**

*Solutions will eventually be posted as well as be available on* `http://www.physto.se/~edsjo/teaching/am/index.html`.