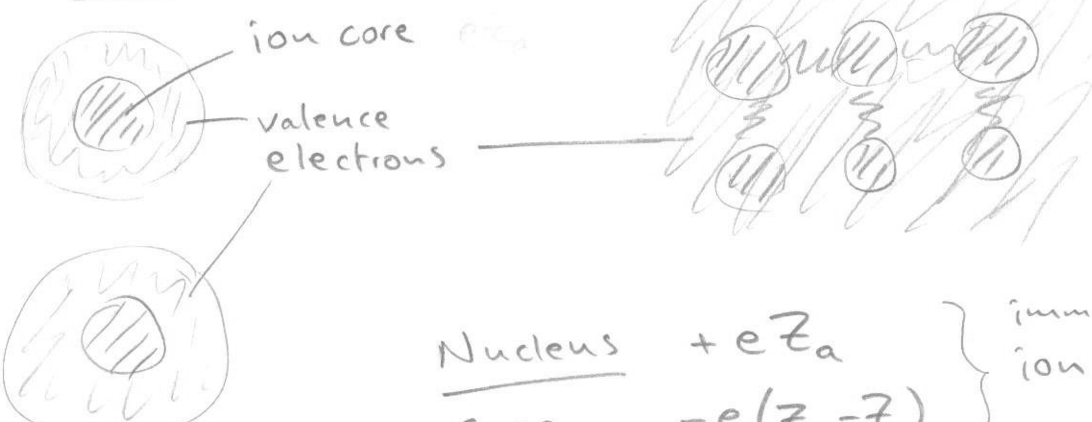
Lecture #1 Drude model ~1900

Simple model of metallic state $\Longrightarrow$ metallic properties

- Treat electrons as independent, classical particles
- Immobile background of metallic ions (Continuum)

Gas $\longrightarrow$ Solid



$$\left.\begin{array}{ll} \text{Nucleus} & +eZ_a \\ \text{Core} & -e(Z_a - Z) \end{array}\right\} \text{immobile ion}$$

$$\text{Valence} \quad -eZ$$

Explains partly

- Electrical conductivity
- Hall effect
- Thermal conductivity
- Thermoelectric effect

conc. of free el.

$$n = \frac{N}{V} = N_A \cdot \frac{Z \cdot \rho_m}{A}$$

free el./atom

density (g/cm^3)

atomic mass $(g/mole)$

$6.023 \cdot 10^{23}$ atoms/mole

Basic assumptions

1. Free electrons (conduction electrons)

2. The conduction electrons move as the molecules in an ideal gas

   Drude: $\langle \frac{mv^2}{2} \rangle = \frac{3}{2} k_B T$ (not correct)

3. Scattering in random directions by collisions with background or each other

4. Instantaneous collisions

Probability for collision during time dt is $\frac{dt}{\tau}$

Defines τ relaxation time

Same probability at any time

Set $P(t)$ = probability for not colliding during time t.

$$P(0) = 1 \qquad P(t_1) \cdot P(t_2) = P(t_1 + t_2)$$

$$P(t+dt) = P(t) \cdot P(dt) = P(t) \cdot \underbrace{\left(1 - \frac{dt}{\tau}\right)}_{\text{probability for not colliding during } dt}$$

$$\frac{P(t+dt) - P(t)}{dt} = -P(t) \cdot \frac{1}{\tau}$$

$$\frac{dP}{dt} = -\frac{1}{\tau} P$$

$$\boxed{P(t) = e^{-t/\tau}}$$

interpretation: τ: average time between collisions for an electron

DC electrical conductivity

sample dependent

$\boxed{\text{Ohm's law}}$ $\quad U = R \cdot I$

$$\vec{E} = \rho \cdot \vec{j}$$

$\frac{U}{L}$ $\quad$ material characteristic $[\Omega\text{cm}]$ $\quad$ $\frac{I}{A}$

resistivity

$$\vec{j} = \sigma \cdot \vec{E}$$

conductivity $= \frac{1}{\rho}$

$$\vec{j} = -ne\langle\bar{v}\rangle$$

$\hookrightarrow$ average velocity

$F = m \cdot a$

$a = \frac{F}{m} = \frac{-eE}{m}$

Effect of electric field on electron:

$$\bar{v}(t) = \bar{v}_0 - \frac{e\bar{E}}{m} \cdot t$$

$\bar{v}_0$: random $\rightarrow$ no net contribution

$$\boxed{\langle\vec{v}\rangle = \int_0^\infty \underbrace{\frac{dt}{\tau} e^{-t/\tau}}_{\text{probability for scattering at time } t} \left(-\frac{e\bar{E}}{m} t\right)}$$

i.e. $P(t) - P(t+dt)$

$$\vec{v}_d = \langle\bar{v}\rangle = -\frac{e\vec{E}}{m}\tau$$

$$\Longrightarrow \quad \vec{j} = \underbrace{\frac{ne^2\tau}{m}}_{\sigma} \vec{E}$$

Measure resistivity to obtain τ

$$\tau = \frac{m}{\rho n e^2}$$

Mean free path $\quad l = v\tau$

$\hookrightarrow v_0$ average speed

Study average momentum of the electrons

$$\vec{p}(t) = m\vec{v}_d(t)$$

$$\boxed{\text{Eq. } 1.12} \qquad \frac{d\vec{p}(t)}{dt} = \underbrace{-\frac{\vec{p}(t)}{\tau}}_{\substack{\text{frictional}\\ \text{damping}}} + \overbrace{\vec{f}(t)}^{\text{external force}}$$

"losing memory of direction"

ex : Stationary: $\frac{d\vec{p}(t)}{dt} = 0$

$$\Longrightarrow \frac{mv_d}{\tau} = -e\vec{E} \longrightarrow \vec{v}_d = -\underbrace{\frac{e\tau}{m}}_{\substack{\mu \\ \text{mobility}}}\vec{E}$$

$$\boxed{\mu = \frac{e\tau}{m}}$$

more general...

Force $\vec{f} = -e(\vec{E} + \vec{v} \times \vec{B})$

Only magnetic field - static case:

perpendicular

$$\frac{m\vec{v}_d}{\tau} = -e\vec{v}_d \times \vec{B} \Longrightarrow \underline{\underline{\vec{v}_d = 0}}$$

Hall effect

Electrical + Magnetic field

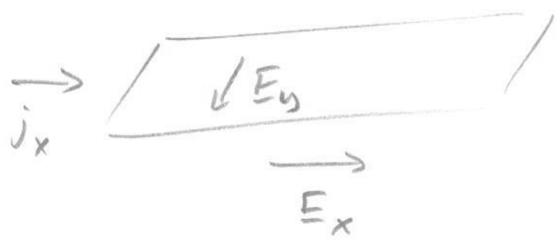
Stationary solution

$$\frac{m\vec{v}_d}{\tau} = \vec{f} = -e\left(\vec{E} + \vec{v}_d \times \vec{B}\right)$$

$$\vec{E} = (E_x, E_y, E_z) \qquad \vec{B} = (0, 0, B)$$

$$\vec{v}_d = (v_{d,x}, v_{d,y}, v_{d,z}) = (v_x, v_y, v_z)$$

$$\begin{cases} \dfrac{m v_x}{e\tau} = -E_x - v_y \cdot B \\[2ex] \dfrac{m v_y}{e\tau} = -E_y + v_x B \\[2ex] \dfrac{m v_z}{e\tau} = -E_z \end{cases} \Longrightarrow \quad j_z = -nev_z = \underbrace{\frac{ne^2\tau}{m}}_{\sigma_0} E_z$$



With $j_y = 0$ $(v_y = 0)$:

$$\begin{cases} j_x = \underbrace{\dfrac{ne^2\tau}{m}}_{\sigma_0} E_x \\[2ex] E_y = v_x B \Longrightarrow -neE_y = j_x B \end{cases}$$

Define $\boxed{\omega_c = \dfrac{eB}{m}}$ $\qquad -\dfrac{ne^2\tau}{m} E_y = \omega_c \tau \cdot j_x$

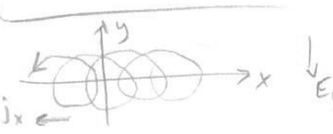
Cyclotron frequency

= freq for motion in B-field

$\omega = \frac{2\pi}{T}$

$F = \frac{mv^2}{r} = ev.B$, $v = \omega . r$

$$\boxed{j_x = -\left(\frac{\sigma_0}{\omega_c \tau}\right) E_y}$$



Define <u>Hall coefficient</u>

$$R_H = \frac{E_y}{j_x \cdot B} \implies \boxed{R_H = -\frac{1}{ne}}$$

Drude problem: × Experimentally, this is only order of magnitude
Even sign may change!

× Field dependence

Magnetoresistance $\rho(H) - \rho(0) = \Delta\rho(H) \neq 0$ in general

– not accounted for in Drude model

<u>AC conductivity</u>

$$\begin{cases} \dfrac{d\vec{p}}{dt} = -\dfrac{\vec{p}}{\tau} - e\vec{E} \\ \vec{E}(t) = \mathrm{Re}(\vec{E}(\omega)e^{-i\omega t}) \end{cases}$$

Set $\vec{p}(t) = \mathrm{Re}\left(\vec{p}(\omega)\, e^{-i\omega t}\right)$

$$\implies -i\omega p(\omega) = -\frac{\vec{p}(\omega)}{\tau} - e\vec{E}(\omega)$$

$$\vec{j} = -\frac{ne\vec{p}}{m} \implies \vec{j} = \underbrace{\frac{ne^2\tau}{m}}_{\sigma_0} \cdot \frac{1}{1-i\omega\tau}\vec{E}(\omega)$$

$$\boxed{\sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}}$$

Propagation of electromagnetic radiation

If $\vec{E}$ does not vary much over distance $\sim l$ (mean free path)

i.e. $\lambda \gg l$

$\downarrow$

$10^3 - 10^4$ Å visible light

then one can apply

- $\times$ Maxwell equations
- $\times$ $\vec{j} = \sigma(\omega)\vec{E}$

$\Longrightarrow$ Wave equation but with dielectric constant

$$\boxed{\varepsilon(\omega) = \varepsilon_0 + \frac{i\sigma(\omega)}{\omega}}$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

High ω: $\omega\tau \gg 1$

$$\varepsilon(\omega) = \varepsilon_0 - \frac{\cancel{i}\sigma_0}{\cancel{i}\omega^2\tau}$$

$$\varepsilon(\omega) = \varepsilon_0 - \frac{ne^2}{m\omega^2}$$

$$\boxed{\varepsilon(\omega) = \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right) \qquad \omega_p = \left(\frac{ne^2}{\varepsilon_0 m}\right)^{1/2}}$$

Plasma frequency

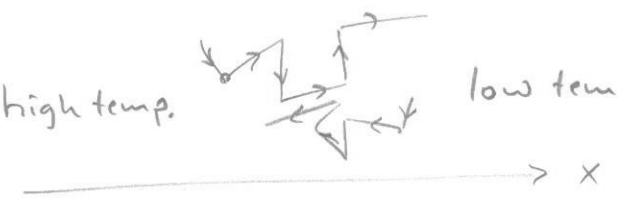
ε real, negative : solutions decay $\rightarrow$ no propagation of radiation

$\Longleftrightarrow \omega < \omega_p$

__But__ $\omega > \omega_p \Longrightarrow$ $\boxed{\text{transparent metal}}$

Thermal conductivity

$\sim 90\%$ of thermal cond. in metals

high temp.  low temp.

Experimental observation: Wiedemann-Franz law:

$$\frac{K}{\sigma T} = \text{const. } L$$

(see table 1.6)

Thermal current density

$$\vec{j}^{q} = -K \underbrace{\nabla T}_{\text{temp. gradient}}$$

"energy current density"

After each collision: speed of electron from <u>local T</u>

Mean electronic velocity vanishes locally

but high temp. side will have higher energies

$\Longrightarrow$ flow of thermal energy

$\mathcal{E}(T)$: thermal energy per electron at temp. T

compare:

- $j = -nev$ charge current
- $j^q = nv\mathcal{E}$ thermal current

$\mathcal{E}(T[x-v\tau])$ $\longrightarrow \cdot \longleftarrow$ $\mathcal{E}(T[x+v\tau])$

$\frac{n}{2}$, $v \longrightarrow$ $\qquad$ $\frac{n}{2}$, $\overset{v}{\longleftarrow}$

$$\vec{j}^{q} = \frac{1}{2} nv \Big(\mathcal{E}(T[x-v\tau]) - \mathcal{E}(T[x+v\tau]) \Big)$$

$$\Longrightarrow \vec{j}^{q} = nv^2\tau \cdot \frac{d\mathcal{E}}{dT}\left(-\frac{dT}{dx}\right)$$

$$\frac{d\varepsilon}{dT} = \frac{1}{N}\frac{dU}{dT}$$

$$n\frac{d\varepsilon}{dT} = \frac{N}{V}\frac{d\varepsilon}{dT} = \frac{1}{V}\frac{dU}{dT} = c_v$$ electronic specific heat

$$\Longrightarrow \quad \vec{j}^{q} = v^2\tau \cdot c_v \cdot \left(-\frac{dT}{dx}\right)$$

three dimensions:

$$\langle v_x^2 \rangle = \frac{1}{3}\langle v^2 \rangle$$

$$\vec{j}^{q} = \underbrace{\frac{1}{3}v^2\tau\, c_v}_{\boxed{K = \frac{1}{3}c_v l v}}\left(-\vec{\nabla}T\right)$$

$l = v\tau$

thermal conductivity

$$\frac{K}{\sigma} = \frac{\frac{1}{3}c_v v^2 \cancel{\tau}}{\frac{ne^2\cancel{\tau}}{m}} = \frac{1}{3}\cdot\frac{1}{ne^2}\cdot mv^2\cdot c_v$$

Erroneously, Drude assumed $c_v = \frac{3}{2}nk_B$

and $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$

$$\Longrightarrow \quad \frac{K}{\sigma} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2\cdot T$$

<u>But</u> Problem is that measurements show $c_{v,el} \ll \frac{3}{2}nk_B$

$c_{v,\text{isolators}} \approx c_{v,\text{metals}}$ at R.T.

Thermoelectric effect — Seebeck effect

Electrical field due to temperature gradient

$$\vec{E} = Q \vec{\nabla} T$$

velocity due to temp. gradient:

$$v_q = \frac{1}{2}\Big(v(x - v\tau) - v(x + v\tau) \Big) = -v\tau \frac{dv}{dx} = -\tau \frac{d}{dx}\left(\frac{v^2}{2}\right)$$

$$\vec{v}_q = -\frac{1}{3} \cdot \frac{\tau}{2} \frac{dv^2}{dT} \vec{\nabla} T$$

3D

$$\vec{v}_E = -\frac{e\tau}{m} \vec{E}$$

$$\vec{v}_q + v_{\vec{E}} = 0 \quad \text{(measuring field without flowing current)}$$

$$\Longrightarrow \quad \frac{e\tau}{m} \vec{E} = -\frac{\tau}{6} \frac{dv^2}{dT} \vec{\nabla} T$$

$$\vec{E} = -\frac{1}{3e} \underbrace{\frac{d}{dT}\left(\frac{mv^2}{2}\right)}_{\frac{d\varepsilon}{dT} = \frac{c_v}{n}} \vec{\nabla} T = \underbrace{-\frac{c_v}{3ne}}_{Q} \vec{\nabla} T$$

Problem: Sign of Q not always the same