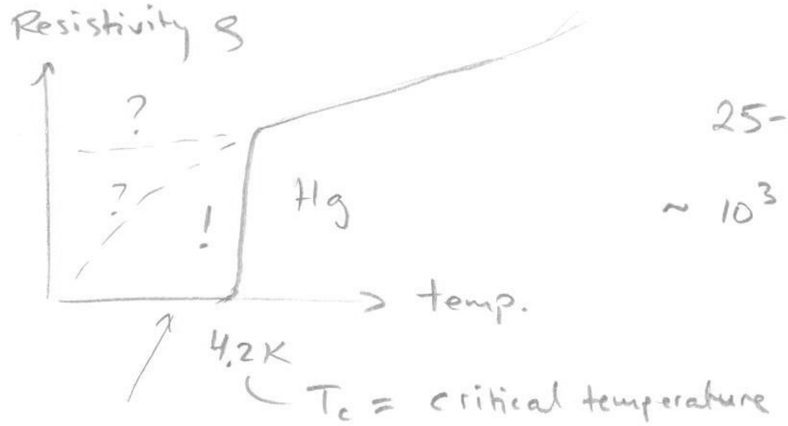
Lecture # 18 — Superconductivity

Discovered in 1911 by Kamerling Onnes + students

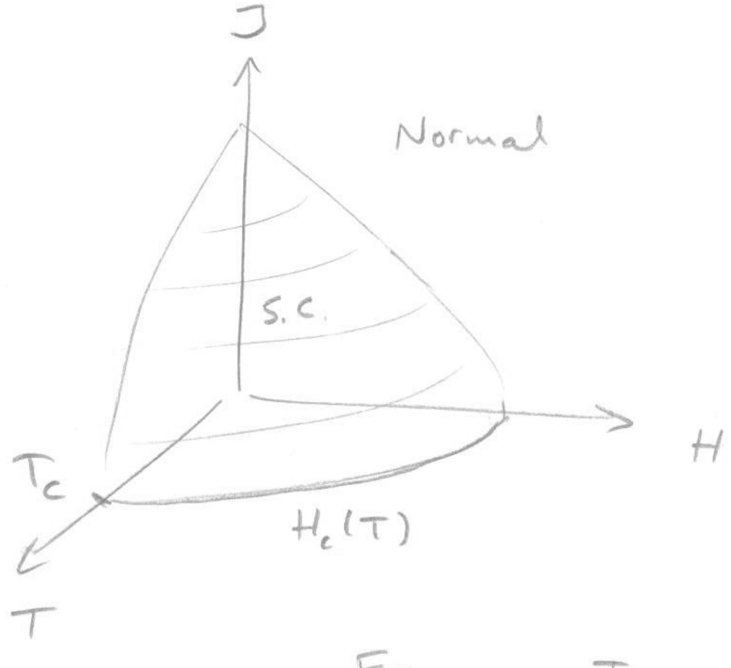


25-30 elements

~ 10^3 alloys, maybe more

× experimentally not measurable ρ

× theoretically, some conditions, $\rho = 0$

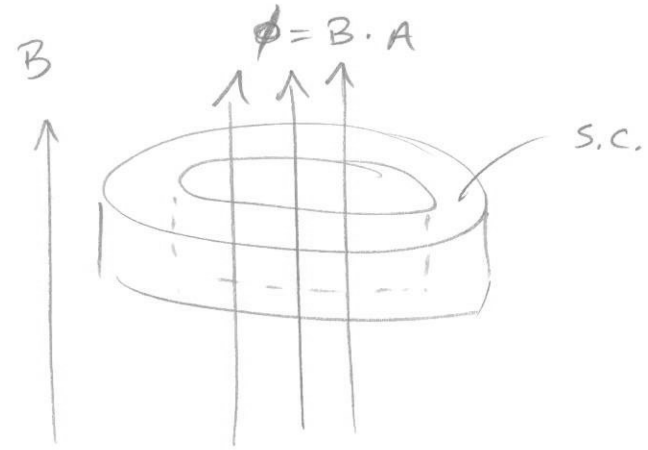


× Macroscopic quantum phenomenon

× Equilibrium thermodynamics

Ex.	T_c	H_c
Al	1.14 K	10 mT
Pb	7.19 K	80 mT
Nb_3Ge	23.2 K	35 T
$YBa_2Cu_3O_{7-\delta}$	92 K	~150 T

Flux trapping



Change B

$\Downarrow$

induced screening currents due to Faraday law

$$\oint \bar{E} \cdot d\bar{l} = -\frac{d\phi}{dt}$$

$B = \mu_0 (H + M)$

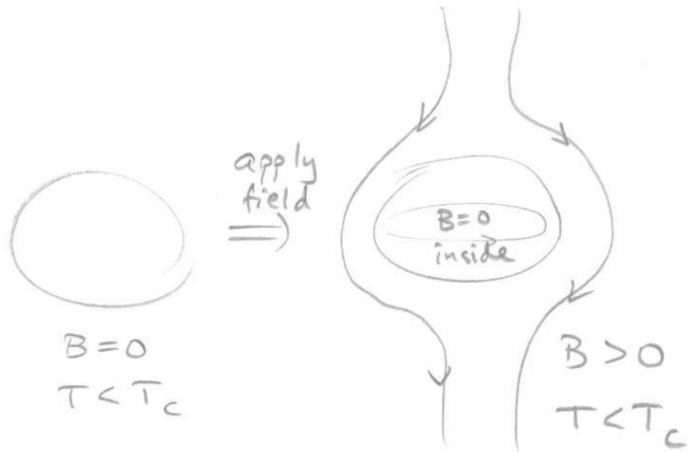
$\chi = \frac{M}{H}$

$E = 0$ in bulk s.c.
(zero resistivity)

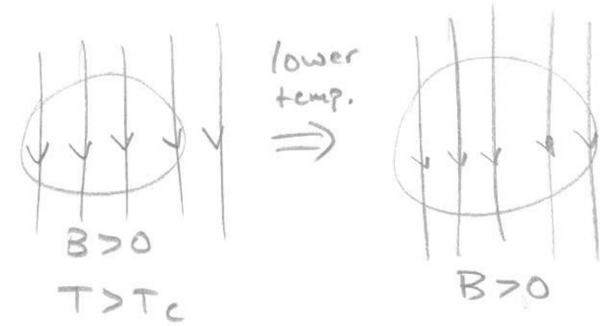
$\Rightarrow \quad \phi = \text{constant}$

Meissner effect

Perfect conductivity :
(and superconductors)



Perfect conductors :



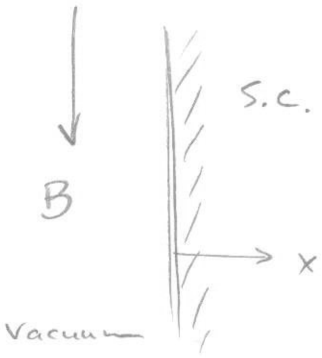
But :

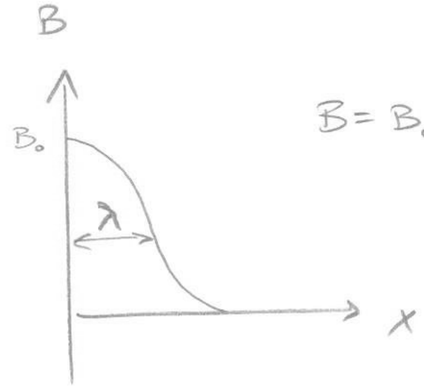
Meissner effect
(superconductor) :

Perfect diamagn

$\chi = -1$

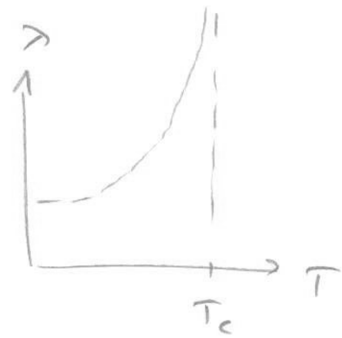
Penetration depth





$$B = B_0\, e^{-x/\lambda}$$

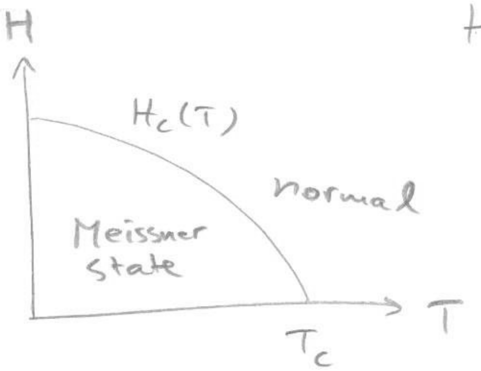
$$\lambda(T) \simeq \underbrace{\lambda(0)}_{\sim 100\,\text{Å}} \frac{1}{\sqrt{1-(T/T_c)^4}}$$



Critical field

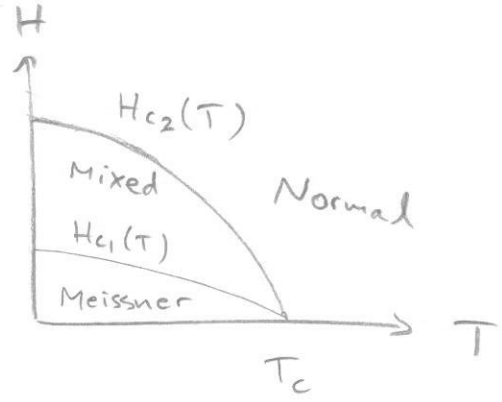
$$H_c(T) = H_c(0)\left[1-(T/T_c)^2\right]$$

type-I



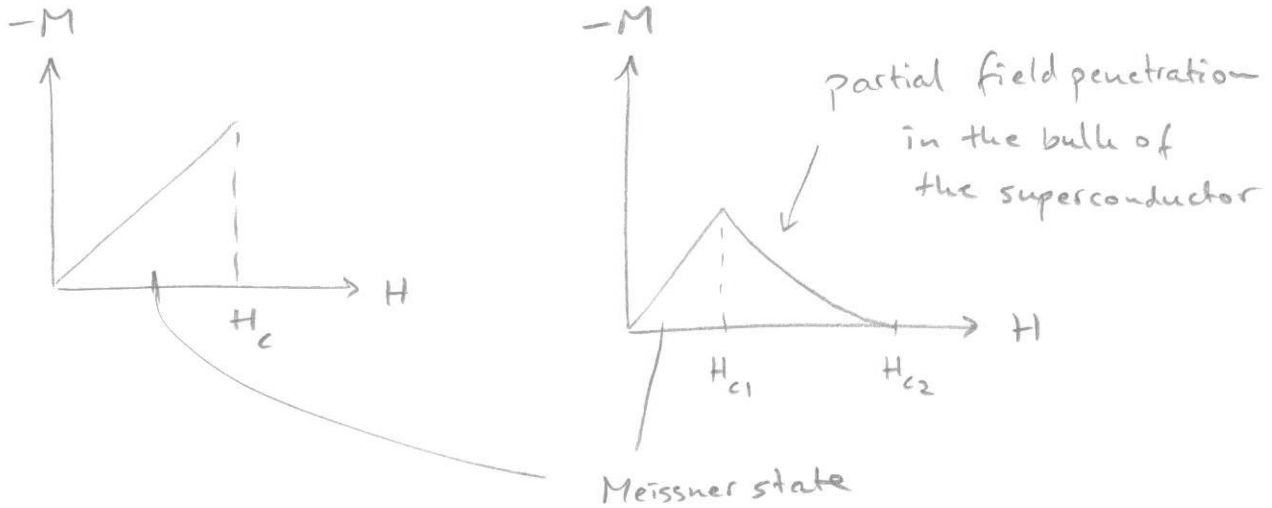
× "Clean" metals (elements)

type-II



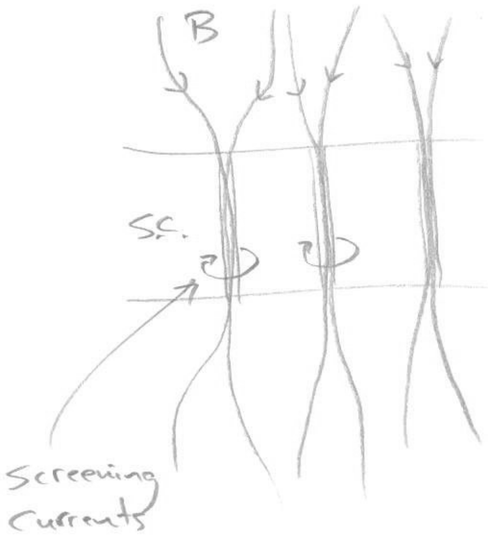
× Dirty metals
× Alloys
× High-T_c superconductors

type-I type-II



Mechanism: B-field penetrates the s.c. in thin "tubes", brought to the normal state.

Vortices



From above:

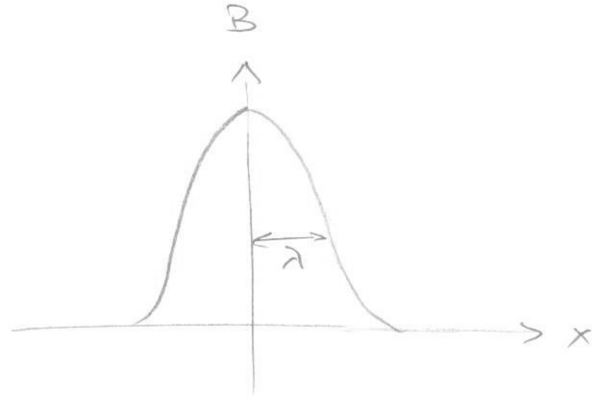
Each tube is associated with the smallest amount of flux allowed by quantum mechanics of s.c., the flux quantum

$$\phi_0 = \frac{h}{2e} \approx 2.07 \cdot 10^{-15} \; T \cdot m^2$$

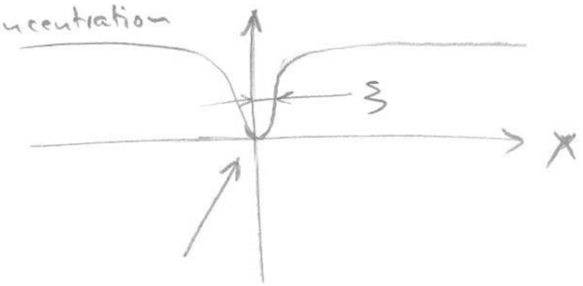
$$\phi = n \cdot \phi_0 = B \cdot A$$

vortices repel each other $\Longrightarrow$ vortex lattice

The vortex



Supercond. electron concentration



Singularity at $x = 0$ $\Longleftrightarrow$ normal state core of size $\sim \xi$

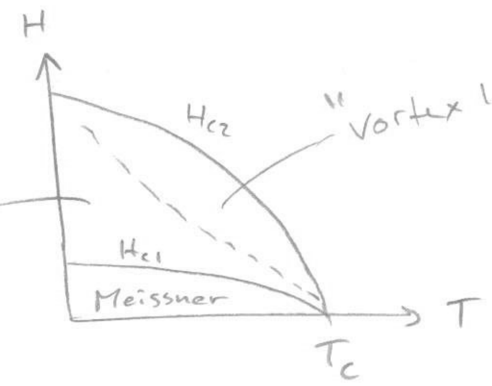
$$\mu_0 H_{c_1} \approx \frac{\phi_0}{4\pi\lambda^2}$$

$$\mu_0 H_{c_2} = \frac{\phi_0}{2\pi \xi^2} \qquad \mu_0 H_c(T) = \frac{\phi_0}{2\sqrt{2}\,\pi\lambda\xi}$$

$$\kappa \equiv \frac{\lambda}{\xi} \qquad H_{c_2} \approx \kappa H_c \approx \kappa^2 H_{c_1}$$

thermodynamic critical field

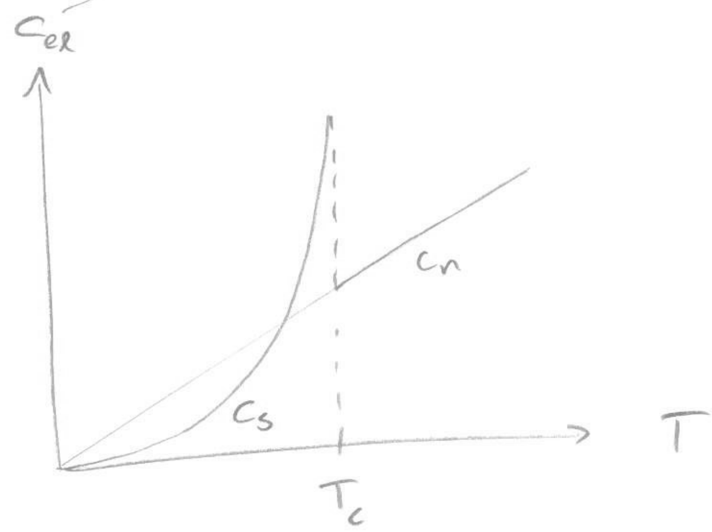
type-II: κ large $\Rightarrow$ $H_{c_2} > H_{c_1}$



"vortex liquid" — vortices move due to thermal fluctuations $\Longrightarrow$ resistance

Ordered vortex solid

Specific heat



$$\left.\frac{c_s - c_n}{c_n}\right|_{T_c} = 1.43 \quad \text{(BCS value)}$$

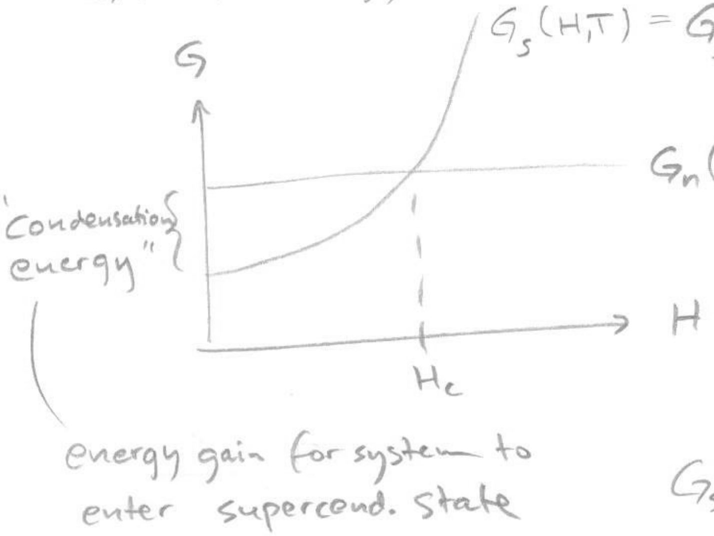
$$c_n = \gamma T \quad (+ A T^3)$$
phonons

$$c_s \sim e^{-\Delta / k_B T}$$

$$C = T \frac{\partial S}{\partial T} \qquad S = \int_0^T \frac{C}{T}\, dT \quad \text{entropy}$$

Superconducting state has lower entropy $\Longleftrightarrow$ more ordered

Gibbs free energy



$$G_s(H,T) = G_s(0,T) + \frac{\mu_0 H^2}{2}$$
energy per volume for "field" in s.c. opposite to applied field to cancel internal field

$$G_n(H,T) = G_n(0,T)$$

$$G_s = G_n$$

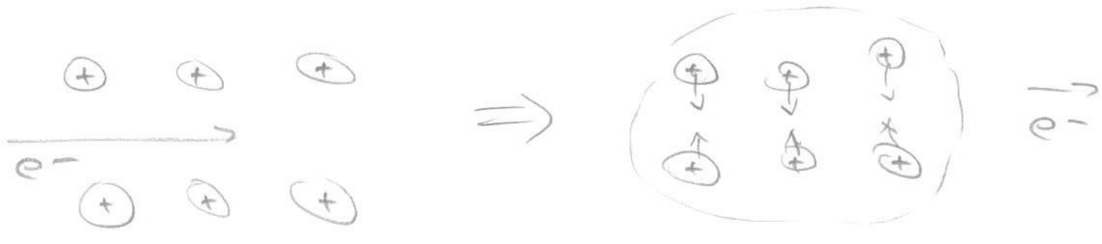
$$\Rightarrow \text{cond. energy} \quad \frac{\mu_0 H_c^2}{2}$$

Cooper pairs (cooper, 1956)

BCS theory 1957

Net attractive interaction between electrons at Fermi surface

Ionic motion (phonons) overscreen Coulomb repulsion



Can attract second electron

$\Rightarrow$ electrons form Cooper pairs of 2 el

All cooper pairs can be described by the same wave fuction / state

Condensation into a macroscopic wave function of all cooper pairs.

$\Delta E \, \Delta t \sim h$

$\hbar\omega_D$ virtual phonon

$\Delta t \sim \frac{h}{\hbar\omega_D} \sim \frac{2\pi}{\omega_D}$

$v_F \cdot \Delta t \sim$ range of interaction ~ 100 nm

ξ coherence length

BCS (Bardeen - Cooper - Schrieffer, 1957)

$$\boxed{T_c = \Theta_D \, e^{-\frac{1+\lambda}{\lambda-\mu^*}}}$$

not penetr. depth here

λ = attractive interaction

μ^* = repulsive — " —

Θ_D = Debye temp.

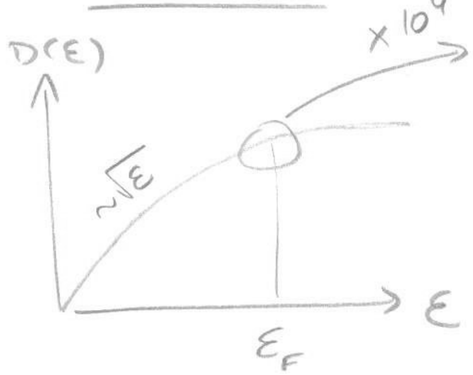
Strong electron-phonon interaction needed

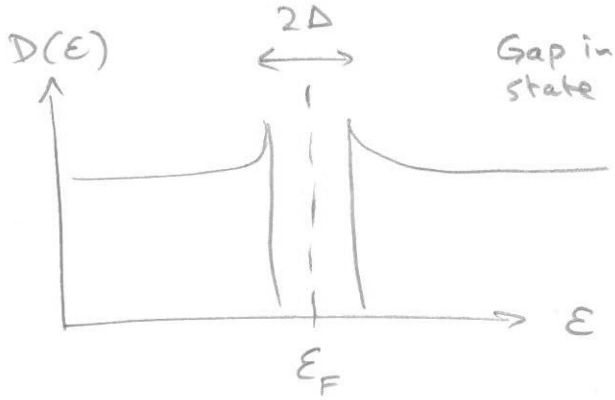
$\Longrightarrow$ "bad" metals better superconductors

Cu, Ag weak interaction $\Longrightarrow$ not superconductors

High-T_c s.c. : Other interaction than electron-phonon?

Energy gap





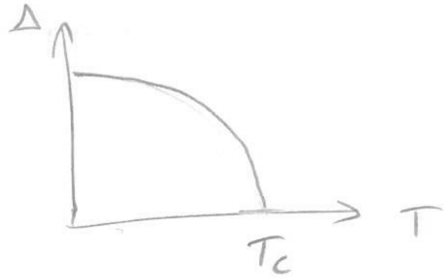
Gap in normal state D.O.S.

ΔG = (# electrons) × (energy gain)

$$\Delta G \approx \left(D(\varepsilon_F)\cdot\Delta\right)\cdot\frac{\Delta}{2} \approx \frac{\Delta^2}{2}D(\varepsilon_F)$$

BCS

$$\Delta(0) = 1.76\, k_B T_c$$



Thermal excitation/splitting of cooper pairs

$\Downarrow$

fewer Cooper pairs

$\Downarrow$

lower energy gain per pair

$\Longrightarrow$ Collective phenomenon

Isotope effect

$$T_c \sim \Theta_D \cdot e^{-\frac{1+\lambda}{\lambda-\mu^*}}$$

$$\omega = \underbrace{\sqrt{\frac{4C}{M}}}_{} \left| \sin \frac{ka}{2} \right|$$

$$\omega_{max} \sim \omega_D \qquad \hbar\omega_D = k_B\Theta_D \sim \frac{1}{\sqrt{M}}$$

$$T_c \sim \frac{1}{\sqrt{M}} \qquad \frac{d\ln T_c}{d\ln M} = -\frac{1}{2}$$

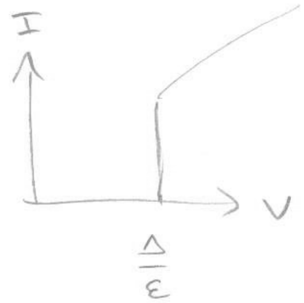
not always exactly $\frac{1}{2}$

different isotopes $\Rightarrow$ slight shifts of T_c

$\Longrightarrow$ phonons

Tunneling

metal
ins.
s.c



* Single particle tunneling (Giaever)
* Electronpair tunneling (Josephson)

$\hookrightarrow$ interference effects

SQUID