Condensed Matter Physics – FK7060, Mar. 6, 2018.

Lecture 18 – Superconductivity

Reading

Ashcroft & Mermin, Ch. 34 (read for general understanding)

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Central concepts

- **Critical temperature**

  Superconductors are in the superconducting state only below a certain temperature, the *critical temperature* T_c. Materials are divided into low-T_c or low-temperature superconductors, with a T_c below about 30 K, and high-T_c or high-temperature superconductors (HTSC), with higher T_c's. The highest critical temperature of a compound at normal pressure is currently about 135 K. The known HTSC's are typically ceramic-like compounds composed of three to five elements, almost all containing oxygen and copper, while low-T_c materials with some few exceptions are elements and metal-alloys.

- **Meissner effect**

  In the superconducting state, current is (except in certain cases) conducted without resistance. If a material with this property of perfect conductivity is exposed to a magnetic field, persistent screening currents at the surface will be induced to screen out the magnetic field, since $E = 0$ in the superconductor and, according to the Faraday law of induction,

  $$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi}{dt}$$

  so that the enclosed flux is kept constant (or zero if initially zero). However, superconductivity is not the same as perfect conductivity. Superconductors that are placed in a (weak enough) magnetic field at $T > T_c$ and then cooled down to below T_c *expel* the magnetic field, so that B is always zero in the bulk. Since

  $$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

  this is corresponds to *perfect diamagnetism* with $\chi = M/H = -1$.

- **Critical field, type-I, type-II**

  Just as the temperature cannot be too high for superconductivity to occur, too strong magnetic fields also destroy superconductivity by making the regular, normal state energetically favorable. The superconductors are classified into two groups depending on their behavior in magnetic fields. All bulk superconductors display the Meissner effect at low enough magnetic fields.

  For *type-I* superconductors, the Meissner state remains (for favorable geometries) up to a critical field $H_c(T)$, where superconductivity suddenly disappears.

  For *type-II* superconductors, the Meissner state only remains up to a *lower critical field* H_{c1}, above which magnetic field partially penetrates the superconductor, until the field reaches an *upper critical field* H_{c2}, where the transition to the normal state is finally occuring.

- **Fluxoid quantization**

  Flux enclosed by a (macroscopic) superconducting ring is quantized in amounts of the flux quantum,

$$\phi_0 = \frac{h}{2e}$$

- **Vortex, mixed state**

  When magnetic field starts to penetrate type-II superconductors at H_{c1}, this happens because it becomes energetically favorable to let certain parts of the system become normal, instead of just increasing the screening currents and associated kinetic energy. In type-I superconductors the boundaries between normal and superconducting states have positive energy, so that such surfaces are avoided. In type-II superconductors, however, this energy is negative, and the flux penetrating in the so called *mixed state* between H_{c1} and H_{c2} is divided into the smallest possible bundles, i.e., the flux quantum. The resulting thin filaments of flux are called *vortices*, the name coming from the screening currents surrounding them.

  Although the superconductor in the mixed state is still in its superconducting state, it may not always conduct current without resistance. This is because moving vortices induce electrical fields that may drive currents in their normal cores.

- **Specific heat, thermal conductivity**

  The electronic specific heat shows a discontinuity at T_c,

$$\frac{c_s - c_n}{c_n} = 1.43$$

  The electronic thermal conductivity in the superconducting state is very low. Only a (temperature dependent) fraction of the conduction electrons are available to transport entropy / heat.

- **Cooper pairs, energy gap**

  Electrons in the superconducting state can form *Cooper pairs*. Such a pair of coupled electrons takes the character of a boson, which condenses into a ground state, described by a macroscopic wave function. The condensation is enabled through an attraction between the normally repulsive electrons, usually mediated through electron-phonon interaction. This attraction gives rise to a pair-binding energy of a few meV.

  When many Cooper pairs are allowed to form in the superconducting state, the pairing opens a gap 2Δ in the normal electron density of states around the Fermi energy. This gap prevents small excitations such as scattering, and thus leads to superconductivity. The presence of a common, macroscopic wave function prevents the destruction of an individual pair wave function without destroying the entired paired state, to a high energy cost.

- **Tunneling, Josephson effect**

  Tunneling through a thin insulator from a metal to a superconductor is described by the Giaever effect. It is found that there is a potential threshold $V = \Delta/e$ before a tunneling current flows.

Tunneling between two superconductors can occur with single electrons, but also with paired electrons if the barrier is thin. Such Cooper pair tunneling is described by the Josephson effects. In the *DC Josephson effect*, a supercurrent may flow across the junction in the absence of any applied electrical field. In the presence of a magnetic field the tunneling current is given by

$$I = I_0 \frac{\sin(\pi\phi/\phi_0)}{\pi\phi/\phi_0}$$

where ϕ is the total magnetic flux in the junction. In the *AC Josephson effect*, an oscillatory supercurrent of frequency

$$\omega_J = \frac{2eV}{\hbar}$$

is induced by applying a DC voltage V.

- **London equations**

  The first London equation describes the relation between supercurrent and electrical field,

  $$\frac{d}{dt}\mathbf{j_s} = \frac{n_s e^2}{m}\mathbf{E}$$

  where $j_s = -n_s e\mathbf{v_s}$ is the supercurrent and n_s is the density of superconducting electrons.

  The second London equation desctibes the relation between supercurrent and magnetic field,

  $$\nabla \times \mathbf{j_s} = -\frac{n_s e^2}{m}\mathbf{B}$$

  Together with the Maxwell equation, $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$, this equation gives

  $$\nabla^2 \mathbf{B} = \frac{1}{\lambda^2}\mathbf{B}$$

  where

  $$\lambda = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

  is the London penetration depth.

- **Ginzburg-Landau theory**

  The Ginzburg-Landau theory describes the macroscopic properties of a superconductor by combining thermodynamics with a complex order parameter

  $$\psi = |\psi| e^{i\phi}$$

- **BCS theory**

  The BCS theory is a microscopic theory of superconductivity, describing how to approximate the macroscopic quantum state of the system of attractively interacting electrons. In its simplest form, it relates the zero-temperature energy gap with T_c, according to

  $$\frac{\Delta(0)}{k_B T_c} = 1.76$$

  and gives an estimate of T_c,

  $$T_c \sim \theta_D e^{-1/N_0 V_0}$$

  where θ_D is the Debye frequency, $N_0 = g(\varepsilon_F)/2$ is the density of states for one spin direction, and V_0 is an effective coupling / attractive interaction parameter.