Lecture 17 — Magnetism

Ch. 31 – Magnetic moments in solids

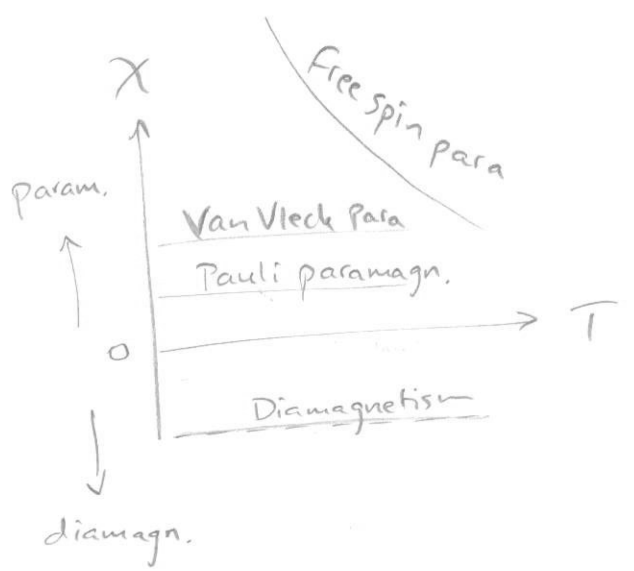
Ch. 32 – Interactions between magnetic moments

Ch. 33 – Magnetic ordering

S.I. : $B = \mu_0 (H + M)$

$$\boxed{\chi = \frac{M}{H}}$$

susceptibility



× spin S

× orbital angular momentum L

× total electronic angular momentum J

Hund's rules

Independent magnetic moments — Ch. 31

- Insulating solids, atomic moments
  - Filled shells $J=0$, $L=S=0$
    - Larmor/Langevin diamagnetism
  - Partially filled shells
    - $J=0$
      - Van Vleck para (unless transition metal)
    - $J \neq 0$
      - Free spin para
        - Curie law
        - iron-group transition metal ions (3d) — quenching $L=0$
        - rare earths (4f)
- independent election approx.
  - cond. el. orbital motion — Landau diamagn.
  - conduction electrons spin — Pauli paramagnetism

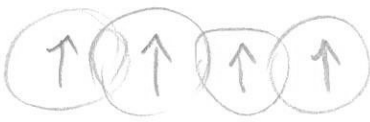
Ch.32 Interaction between magnetic moments

MAIN Source : Electrostatic electron-election interaction

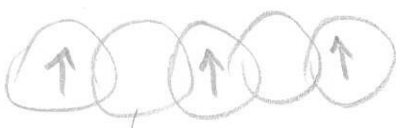
(not dipole-dipole / magnetic
not spin-orbit)

Pauli exclusion principle

Direct exchange



Super exchange



non-magnetic ions

Indirect exchange

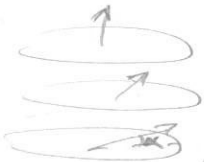
Conduction electrons

Kondo effect

Ch.33

Magnetic ordering

Helix



Ferri

↑↓↑↓↑

Antiferro

↑↓↑↓

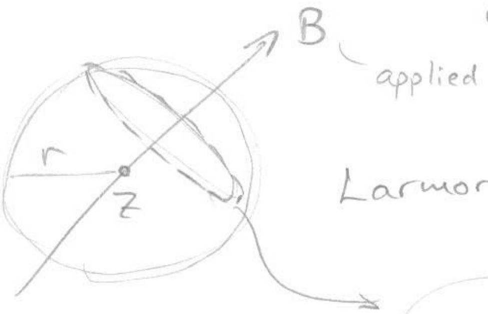
Ferromagnetic

↑↑↑↑

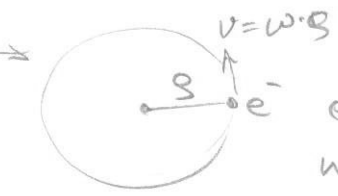
Domains

Larmor / Langevin diamagnetism

Classical picture



Larmor frequency $\omega = \dfrac{eB}{2m}$



electron with radius r_0

Z = atomic number (total # of el.)

Electron density $n = \dfrac{1}{2\pi\varrho \cdot (\pi r_0^2)}$

Current density: $j = -nev$

Current: $$I = j \cdot A = -nev \cdot A =$$
$$= -\frac{1}{2\pi\varrho \cdot (\pi r_0^2)} \cdot e \cdot \underbrace{(\omega\varrho)}_{v} \cdot \underbrace{\pi r_0^2 \cdot Z}_{A} =$$
$$= -\frac{1}{2\pi} \cdot e \cdot \frac{eB}{2m} \cdot Z = -\frac{e^2 B Z}{4\pi m}$$

Magnetic moment (1 atom): $$\bar{\mu} = I \cdot (\text{enclosed area}) = I \cdot \pi \langle \varrho^2 \rangle =$$
$$= -\frac{e^2 B Z}{4m} \langle \varrho^2 \rangle$$

$$\langle r_x^2 \rangle = \langle r_y^2 \rangle = \langle r_z^2 \rangle = \frac{1}{3} \langle r^2 \rangle$$
$$\langle \varrho^2 \rangle = \frac{2}{3} \langle r^2 \rangle$$

N atoms: Magnetization $$M = \bar{\mu} N = -\frac{e^2 B Z}{6m} \langle r^2 \rangle N$$

Susceptibility $\chi \approx \dfrac{\mu_0 M}{B}$

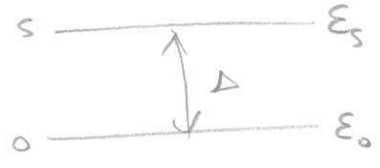
$$\boxed{\chi = -\frac{e^2 \mu_0 Z N}{6m} \langle r^2 \rangle}$$

- $|\chi| \sim 10^{-6}$
- No temp. dep.

Van Vleck paramagnetism

$J = 0$ but $L, S \neq 0$

Quantummechanical 2-energy level



$$\chi = \frac{2\mu_0 N}{\Delta} \left| \langle s | \mu_z | 0 \rangle \right|^2$$

- Positive
- Temp. indep.

Free spin paramagnetism

Magn. moment $\bar{\mu} = -g\mu_B \bar{J}$, $\bar{J} = \bar{L} + \bar{S}$

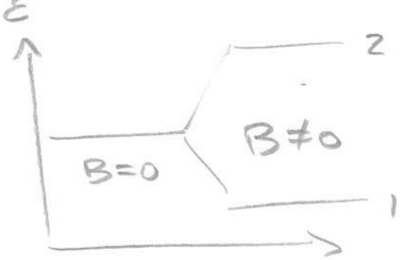
Landé factor $\mu_B = \frac{e\hbar}{2m}$

$$\boxed{g \approx 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}}$$

Energy in magnetic field $U = -\bar{\mu}\cdot\bar{B}$

In general: $\boxed{U = m_J \cdot g \cdot \mu_B B}$ $m_J = -J, \ldots, J-1, J$

$$\underbrace{\left.\begin{matrix} S=\frac{1}{2} \\ L=0 \end{matrix}\right\} J = \frac{1}{2}}_{\Rightarrow g=2} \Longrightarrow m_J = \pm\frac{1}{2} \Longrightarrow U = \pm\frac{1}{2} g\mu_B B = \pm \mu_B B$$



$B \parallel z$

cont. Free spin para ...

m_J	$\bar{\mu}$	ε	# spins
$+\frac{1}{2}$	$-\mu_B \hat{z}$	$\mu_B B$	N_2
$-\frac{1}{2}$	$+\mu_B \hat{z}$	$-\mu_B B$	N_1

$N_1 + N_2 = N$

Bolzmann statistics

$$\frac{N_1}{N} = \frac{e^{-(-\mu_B B)/k_B T}}{e^{-\mu_B B/k_B T} + e^{\mu_B B/k_B T}} = \frac{e^{x}}{e^{x} + e^{-x}} \qquad x = \frac{\mu_B B}{k_B T}$$

$$\frac{N_2}{N} = \frac{e^{-x}}{e^{x} + e^{-x}}$$

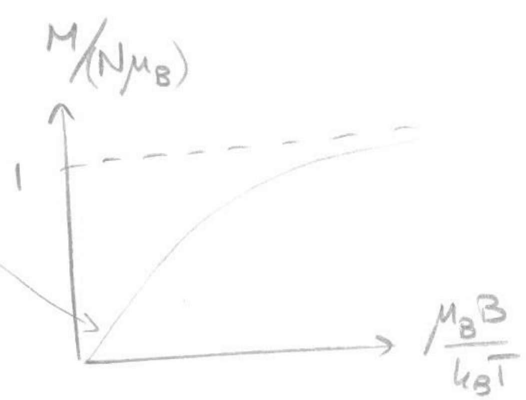
$N_1 > N_2$ since lower energy

$$M = \mu_B (N_1 - N_2) = \mu_B N \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \underline{\underline{\mu_B N \tanh\left(\frac{\mu_B B}{k_B T}\right)}}$$

$\frac{\mu_B B}{k_B T} \ll 1$: $\tanh x \approx x$

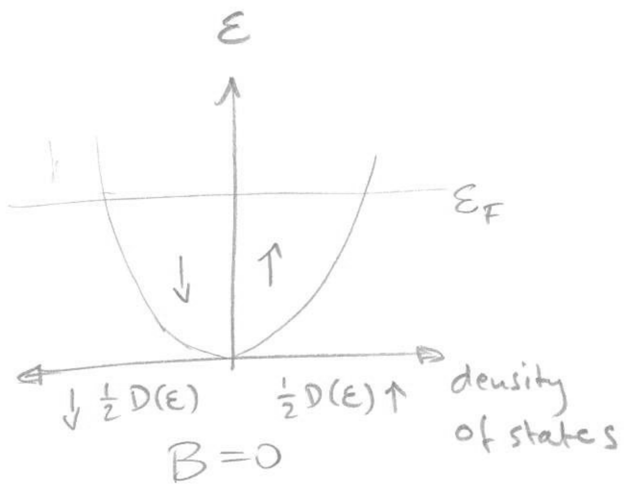
$$\boxed{M = N \cdot \mu_B^2 \frac{B}{k_B T}}$$

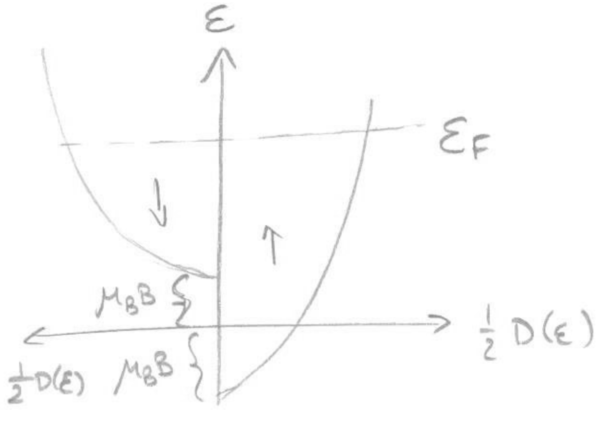
$$\boxed{\chi = \frac{\mu_B^2 N}{k_B} \cdot \frac{1}{T}}$$



$\boxed{\text{Pauli paramagnetism}}$ – Conduction electrons

Spin





$\uparrow B \neq 0$

(exaggerated: $\overset{10\,\mathrm{T\ field}}{\mu_B B} \sim 0.5\,\mathrm{meV}$, $\varepsilon_F \sim 5\,\mathrm{eV}$)

$$M = \mu_B \left(n(\uparrow) - n(\downarrow) \right) = \mu_B \int_{-\infty}^{\infty} d\varepsilon \left[f_{FD}(\varepsilon) \cdot \tfrac{1}{2} D(\varepsilon + \mu_B B) - f_{FD}(\varepsilon) \cdot \tfrac{1}{2} D(\varepsilon - \mu_B B) \right]$$

$$= \mu_B \int_{-\infty}^{\infty} f_{FD} \cdot \frac{1}{2} \cdot \left\{ \cancel{D(\varepsilon)} + \mu_B B \cdot D'(\varepsilon) - \left(\cancel{D(\varepsilon)} - \mu_B B \cdot D'(\varepsilon) \right) \right\} d\varepsilon =$$

$$= \mu_B^2 B \int_{-\infty}^{\infty} f(\varepsilon)\, D'(\varepsilon)\, d\varepsilon = \left\{ \text{part. int.} \right\} =$$

$$= \mu_B^2 B \Big[\underbrace{\Big[f(\varepsilon) D(\varepsilon) d\varepsilon \Big]_{-\infty}^{\infty}}_{=0} - \underbrace{\int_{-\infty}^{\infty} f'(\varepsilon) D(\varepsilon)\, d\varepsilon}_{\approx D(\varepsilon_F) \underbrace{\int_{-\infty}^{\infty} f'(\varepsilon) d\varepsilon}_{-1} = D(\varepsilon_F)} \Big] =$$

$\infty \to$ gives 0

$-\infty \to$ gives 1

$$= + \mu_B^2 D(\varepsilon_F) \cdot B$$

$$\chi = + \mu_B^2 \cdot D(\varepsilon_F)$$

- Temp. independent
- Positive

Hund's rules

Electrons in a given atomic shell arrange so that the ground state has

1. Total value of S maximized
2. L maximized under condition of 1
3. $\begin{cases} J = |L-S| & \text{less than half-filled shell} \\ J = |L+S| & \text{more} \text{ —— " ——} \end{cases}$

Notation $\boxed{{}^{2S+1}L_J}$

$L = 0, 1, 2, 3, 4$

S P D F G H I

ex $[\,]\,3d^2$

$d \Leftrightarrow l = 2$

m_l: +2 [↑], +1 [↑], 0 [ ], −1 [ ], −2 [ ]

$S = 1$

$L = 3$

$$J = L - S = 2$$

Notation: $\boxed{{}^3F_2}$

Rare earth's: as above

3d transition metals: L Lost because of crystal field quenching $L = 0$ $\boxed{J = S}$

Magnetic ordering

Earlier – Free spin para: $M = N \cdot \mu_B \cdot \tanh\left(\frac{\mu_B B}{k_B T}\right)$

Weiss postulate: Internal field / exchange field / Molecular field:

$$B_E = \mu_0 \underset{\approx}{\lambda} \cdot M$$

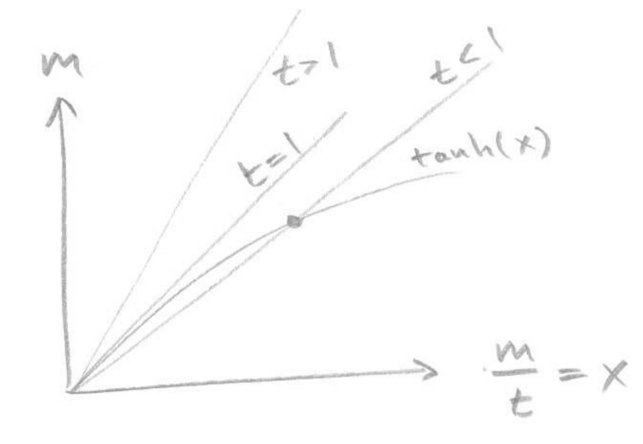
$$\Rightarrow \quad M = N \mu_B \tanh\left(\frac{\mu_B (B_a + \mu_0 \lambda M)}{k_B T}\right) \qquad (B_a = \mu_0 H)$$

Solutions $M > 0$ for $B_a = 0$?

Set $\boxed{m = \frac{M}{N \mu_B}}$ $\quad m = \tanh\left(\frac{\mu_B \mu_0 \lambda M}{k_B T}\right) = \tanh\left(\frac{\mu_B^2 \mu_0 \lambda N}{k_B T} m\right)$

Equation $\boxed{m = \tanh\left(\frac{m}{t}\right)}$ where $\boxed{t = \frac{k_B T}{\mu_0 \mu_B^2 \lambda N}}$

reduced temp.



Non-trivial solution for $\boxed{t < 1}$

$$t = 1 \Leftrightarrow T = \boxed{T_c = \frac{\mu_0 \mu_B^2 \lambda N}{k_B}}$$