Condensed Matter Physics – FK7060, Feb. 22, 2018.

Lecture 15 – Magnetism

Reading

Ashcroft & Mermin, Ch. 31, 32, 33 (read for general picture)

Content

- Susceptibility
- Larmor / Langevin diamagnetism
- Landé factor
- Free spin paramagnetism
- Hund's rules
- Pauli paramagnetism
- Magnetic ordering

Central concepts

- **Susceptibility**

$$\chi = \frac{\partial M}{\partial H} \approx \frac{M}{H}$$

- **Larmor / Langevin diamagnetism**

  This diamagnetism from all electrons in the atoms is a small, negative susceptibility that is always there, given by

$$\chi = -\frac{e^2 \mu_0 Z N}{6m} \left\langle r^2 \right\rangle$$

  where Z is the total number of electrons in the atom, N is the number of atoms, and $\left\langle r^2 \right\rangle$ is the mean square atomic radius.

- **Landé factor**

$$g = \frac{g_0 + 1}{2} + (g_0 - 1)\frac{S(S+1) - L(L+1)}{2J(J+1)}$$

  where $g_0 \approx 2.0023$ is the electron g-factor. With $g_0 \approx 2$, the Landé factor becomes

$$g \approx 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

- **Free spin paramagnetism**

  The free spin paramagnetism is caused by ions with partially filled shells and nonzero J. Their magnetic moments

$$\bar{\mu} = -g\mu_B \mathbf{J}$$

  are aligned by the magnetic field and misaligned by thermal disorder. The energy in magnetic field is given by

$$U = -\bar{\mu} \cdot \mathbf{B} = m_J g \mu_B B$$

  where m_J goes from $-J$ to J.

For $J = 1/2$, the magnetization becomes

$$M = \frac{\mu_B N}{V} \tanh\left(\frac{\mu_B B}{k_B T}\right)$$

which gives a susceptibility

$$\chi = \frac{\mu_0 \mu_B^2 n}{k_B T}$$

at high temperatures compared to $\mu_B B/k_B$. Here $n = N/V$. This is the *Curie law*. For a general J the susceptibility becomes

$$\chi = \frac{g^2 J(J+1)}{3} \frac{\mu_0 \mu_B^2 n}{k_B T}$$

To check units:

- [T] = [N/Am]
- $k_B T$ has unit [J] = [Nm]
- μ_0 has unit [N/A^2]
- μ_B has unit [Nm/T] = [Am^2]
- B has unit [T]
- H and M have unit [A/m]

- **Hund's rules**

  Hund's rules describe how to find the states lowest in energy. They are

  1. Maximize S
  2. Maximize L
  3. Take $J = |L - S|$ for less than half-filled shells and $J = L + S$ for more than half-filled shells.

  The configuration has a notation ${}^{2S+1}L_J$, where $L = 0, 1, 2, 3 \ldots$ is written as S, P, D, F, ...

- **Pauli paramagnetism**

  The Pauli paramagnetism is caused by conduction electron spin alignment with an applied magnetic field. The magnetization is given by

$$M = \mu_B[n(\uparrow) - n(\downarrow)]$$

  where the number of spins along the magnetic field increases because their states decrease by an energy $\mu_B B$, while the states with spin opposite to the field increase by the same amount. The resulting susceptibility becomes

$$\chi = \mu_0 \mu_B^2 D(\varepsilon_F)$$

- **Magnetic ordering**

  The *Weiss postulate* suggests that magnetic order can be described by an internal field (exchange field/molecular field) proportional to the magnetization

$$B_e = \mu_0 \lambda M$$

  This gives solutions $M > 0$ even for zero applied fields if the temperature is below a certain critical temperature. For free spin para, $J = 1/2$, we have

$$T_c = \frac{\mu_0 \mu_B^2 \lambda n}{k_B}$$