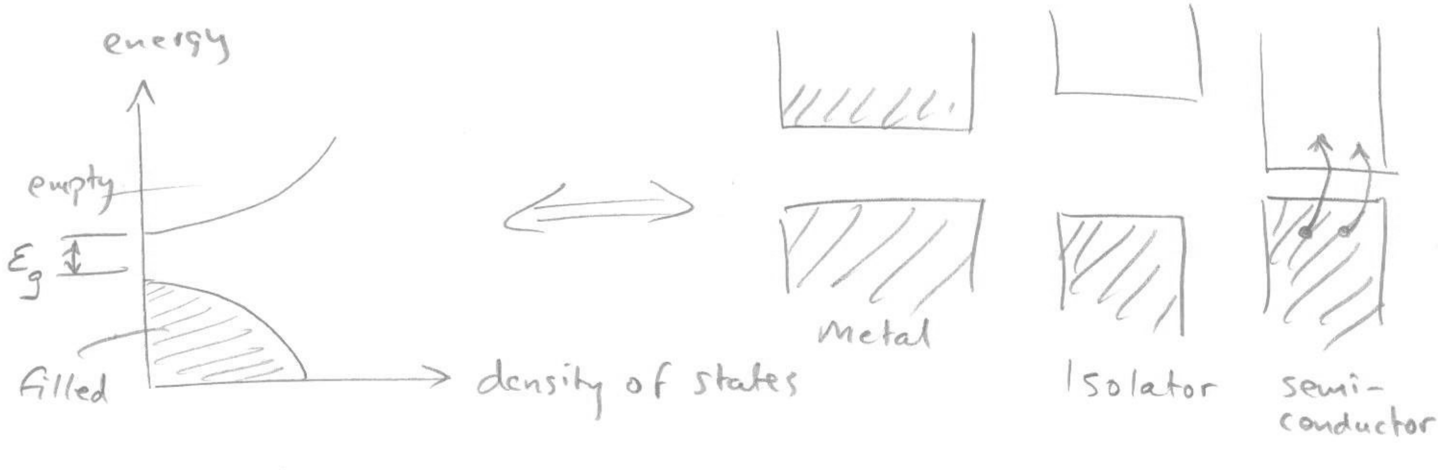
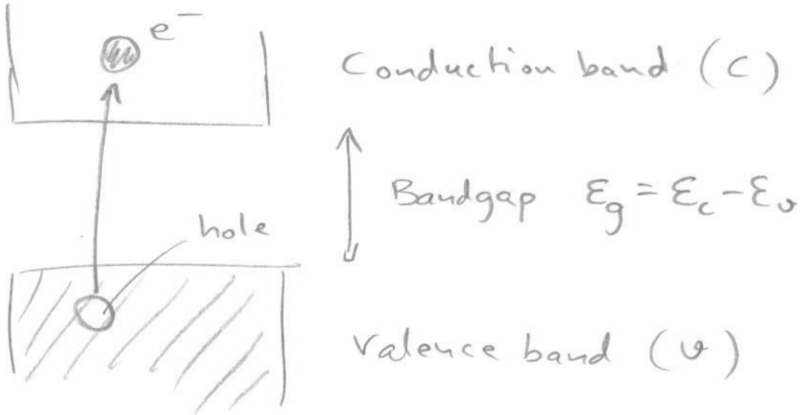
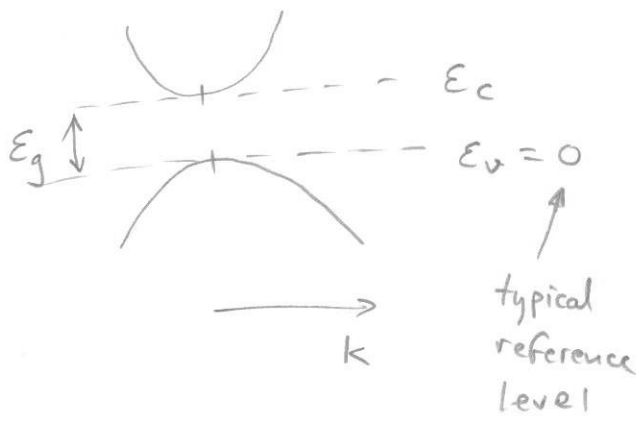
# Lecture #16 Semiconductors




$\varepsilon_g \sim 1\,\text{eV}$

Typically parabolic bands

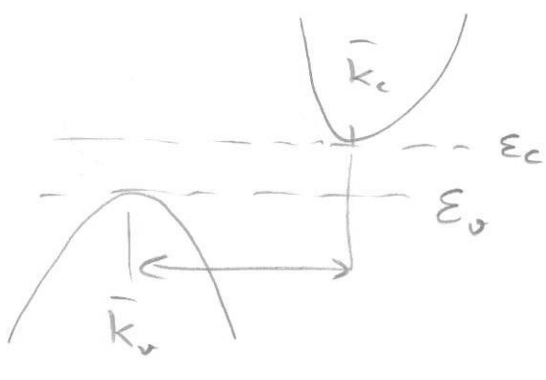



C.B.: $\varepsilon(\bar{k}) = \varepsilon_c + \frac{\hbar^2}{2m_e}(\bar{k}-\bar{k}_c)^2$

V.B: $\varepsilon(\bar{k}) = -\frac{\hbar^2}{2m_h}(\bar{k}-\bar{k}_v)^2$

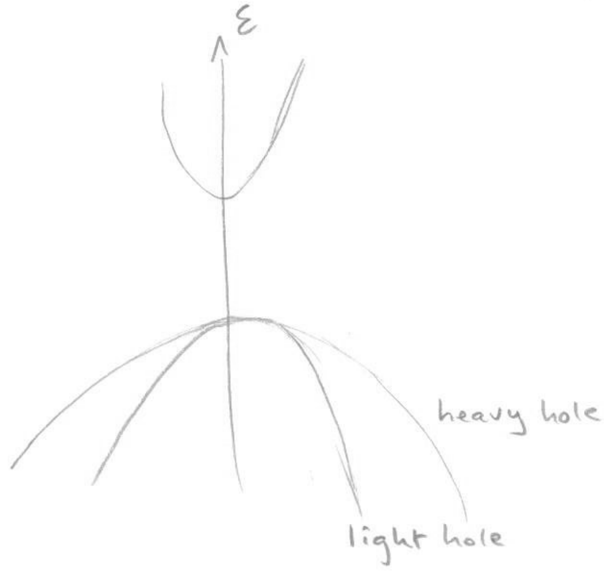
Si: $m_e \approx 0.26\, m$
$m_h \approx 0.5\, m$
free electron mass

## Indirect gap



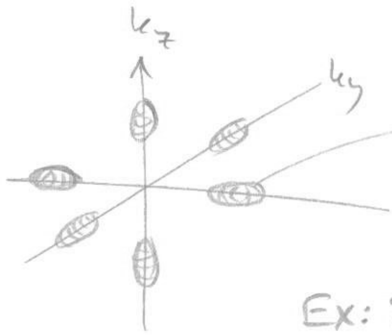

ex. Si

## Degeneracy




$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon}{\partial k^2}$$

Constant-energy surface
near conduction band edge:




Ex: Si

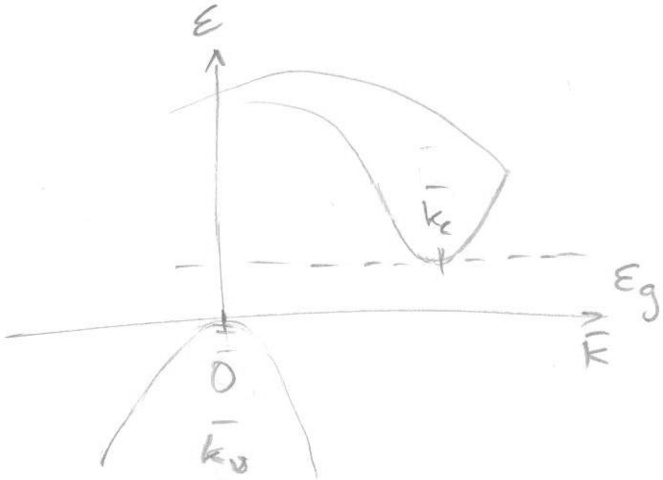
$$\varepsilon(k) = \frac{\hbar^2}{2m}\left[\frac{(k_x-k_c)^2}{0.916} + \frac{k_y^2+k_z^2}{0.191}\right]$$

(Ellipsoid)

Average for different directions and
6 band edges

$$\Downarrow$$

Effective mass $\frac{m_e}{m} = 0.26$



Approximation: parabolic bands, spherical energy surfaces

$$\Longrightarrow \quad \mathcal{E}(\bar{k}) = \mathcal{E}_c + \frac{\hbar^2}{2m_e}(\bar{k}-\bar{k}_c)^2 \qquad (\text{C.B.})$$

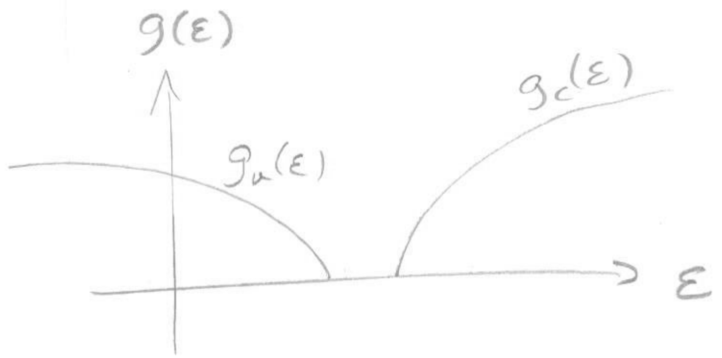
$$\mathcal{E}(\bar{k}) = \mathcal{E}_v - \frac{\hbar^2}{2m_h}(\bar{k}-\bar{k}_v)^2 \qquad (\text{V.B})$$

Density of states

$$g_c(\mathcal{E}) = A_c \cdot \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(\mathcal{E}-\mathcal{E}_c)^{1/2} \qquad (\mathcal{E}>\mathcal{E}_c)$$

number of conductionbandedges in the B.Z. (6 in Si)

$$\boxed{g_v(\mathcal{E}) = A_v \cdot \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(\mathcal{E}_v-\mathcal{E})^{1/2}} \qquad (\mathcal{E}<\mathcal{E}_v)$$




Compare free electrons: (2.61)

$$g(\mathcal{E}) = \frac{m}{\hbar^2\pi^2}\sqrt{\frac{2m\mathcal{E}}{\hbar^2}} = \frac{1}{2\pi^2}\cdot\left(\frac{2m}{\hbar^2}\right)^{3/2}\cdot\sqrt{\mathcal{E}}$$

# Carrier concentration

$n =$ # electrons/vol. for conductionband

$p =$ # holes/vol. for valenceband

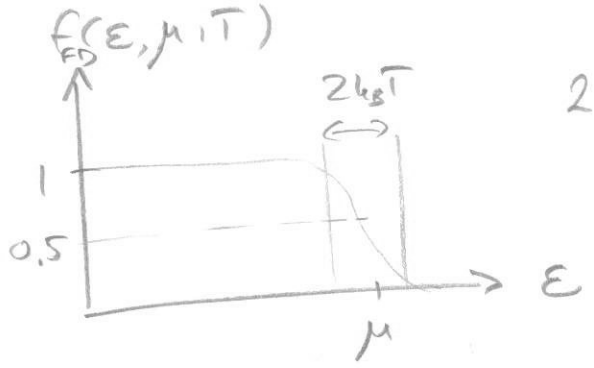
$n(\varepsilon)d\varepsilon =$ # between $\varepsilon$ and $d\varepsilon$

i.e. $\underset{n_c(T)}{n} = \int_{\varepsilon_c}^{\infty} n(\varepsilon)d\varepsilon \qquad \underset{p_v(T)}{p} = \int_{-\infty}^{\varepsilon_v} p(\varepsilon)d\varepsilon$

$$n_c(T) = \int_{\varepsilon_c}^{\infty} \underbrace{\frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}}_{f_{FD}} g_c(\varepsilon)\, d\varepsilon$$

$n_c(\varepsilon)\, d\varepsilon = f_{FD}\, g_c(\varepsilon) d\varepsilon$

$p_v(\varepsilon)\, d\varepsilon = (1-f_{FD})\, g_v(\varepsilon) d$




$2k_BT \sim 50$ meV at R.T.

For holes: $1 - f_{FD} = 1 - \dfrac{1}{e^{(\varepsilon-\mu)/k_BT}+1} =$

$$= \frac{e^{(\varepsilon-\mu)/k_BT} \cancel{+1} \cancel{-1}}{e^{(\varepsilon-\mu)/k_BT}+1} = \frac{1}{1+e^{-(\varepsilon-\mu)/k_BT}} = \underbrace{\frac{1}{e^{(\mu-\varepsilon)/k_BT}+1}}$$

just as for electrons but with energy scale in opposite direction

If $|\varepsilon - \mu| \gg k_B T$ i.e. typically if $\varepsilon > \varepsilon_c$ of $\varepsilon < \varepsilon_v$

$$\frac{1}{e^{(\varepsilon-\mu)/k_B T} + 1} \approx e^{-(\varepsilon-\mu)/k_B T} \qquad \varepsilon > \varepsilon_c$$

$$\frac{1}{e^{(\mu-\varepsilon)/k_B T} + 1} \approx e^{-(\mu-\varepsilon)/k_B T} \qquad \varepsilon < \varepsilon_v$$

$$n_c(T) = \int_{\varepsilon_c}^{\infty} \frac{1}{e^{(\varepsilon-\mu)/k_B T} + 1} \, \underbrace{g_c(\varepsilon)}_{\sim\sqrt{\varepsilon-\varepsilon_c}} \, d\varepsilon =$$

$$= \int_{\varepsilon_c}^{\infty} e^{-(\varepsilon-\mu)/k_B T} \, \underbrace{e^{-(\varepsilon_c-\mu)/k_B T} \, e^{+(\varepsilon_c-\mu)/k_B T}}_{\text{extend}} \, g_c(\varepsilon)\, d\varepsilon =$$

move out

$$= e^{-(\varepsilon_c-\mu)/k_B T} \underbrace{\int_0^{\infty} e^{-(\varepsilon-\varepsilon_c)/k_B T} \, \underbrace{g_c(\varepsilon)}_{\sim\sqrt{\varepsilon-\varepsilon_c}} \, d(\varepsilon-\varepsilon_c)}_{\text{function of } x = \varepsilon - \varepsilon_c}$$

$N_c(T)$ Not depending on energy

$$\boxed{\begin{aligned} n_c(T) &= N_c(T)\, e^{-(\varepsilon_c-\mu)/k_B T} \\ p_v(T) &= P_v(T)\, e^{-(\mu-\varepsilon_v)/k_B T} \end{aligned}}$$

where
$$N_c(T) = \frac{1}{4}\left(\frac{2 m_c k_B T}{\pi \hbar^2}\right)^{3/2} \cdot A_c \qquad (m_c = m_e)$$

$$P_v(T) \quad - \; - \; m_v \; - \; \cdot A_v \qquad (m_v = m_h)$$

## Law of mass action

We had $n = N_0 \, e^{-(\varepsilon_c - \mu)/k_B T}$

$$p = P_0 \, e^{-(\mu - \varepsilon_v)/k_B T}$$

n and p determined by chemical potential $\mu$

$$n \cdot p = N_0 P_0 \, e^{-(\varepsilon_c - \varepsilon_v)/k_B T} = N_0 P_0 \cdot e^{-\varepsilon_g/k_B T}$$

Always valid
(regardless of doping)

$$\boxed{n \cdot p = \text{const}\left(\varepsilon_g/k_B T\right)}$$

product independent of $\mu$

Intrinsic semiconductors – impurities not important (doping)

$$n = p = n_i$$

$$n_i^2 = N_0 P_0 \, e^{-\varepsilon_g/k_B T}$$

$$\boxed{n_i = \sqrt{N_0 P_0} \, e^{-\varepsilon_g/2k_B T}}$$

# Chemical potential – intrinsic case

$n = p = n_i$ gives

Eq: $$N_c(T) \cdot e^{-(\varepsilon_c - \mu)/k_BT} = P_v(T)\, e^{-(\mu - \varepsilon_v)/k_BT}$$

$$N_c(T) = \frac{1}{4}\left(\frac{2m_c k_BT}{\pi\hbar^2}\right)^{3/2} \cdot A_c \qquad P_v(T) = \frac{1}{4}\left(\frac{2m_v k_BT}{\pi\hbar^2}\right)^{3/2} A_v$$

$$e^{(\mu - \varepsilon_c + \mu - \varepsilon_v)/k_BT} = \frac{P_v(T)}{N_c(T)} = \left(\frac{m_v}{m_c}\right)^{3/2} \cdot \underbrace{\left(\frac{A_v}{A_c}\right)}_{\text{Don't bother}}$$

mix

$$2\mu - \underbrace{(\varepsilon_v + \varepsilon_c)}_{2\varepsilon_v + \varepsilon_g} = k_BT \cdot \ln\left[\left(\frac{m_v}{m_c}\right)^{3/2}\right]$$

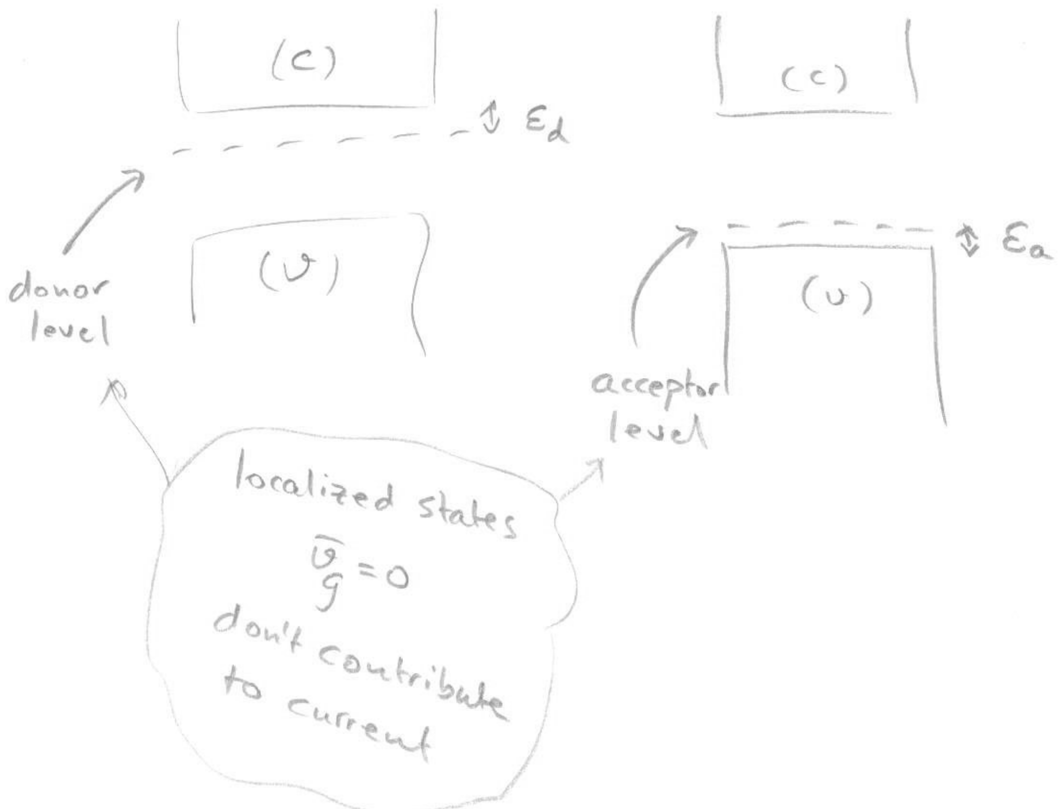
$$\boxed{\mu = \varepsilon_v + \frac{1}{2}\varepsilon_g + \frac{3}{4} k_BT \ln\left(\frac{m_v}{m_c}\right)}$$

$T \to 0$ : $\mu = \varepsilon_v + \frac{1}{2}\varepsilon_g$ (middle of gap)

## Doping

Adding small amounts of electron donors / acceptors

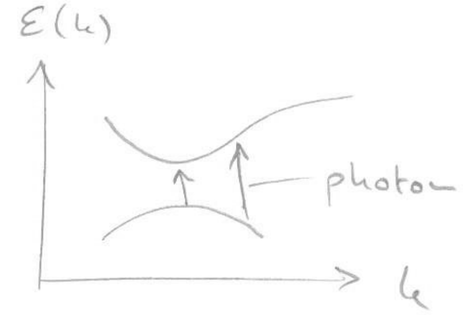
el doping / hole doping



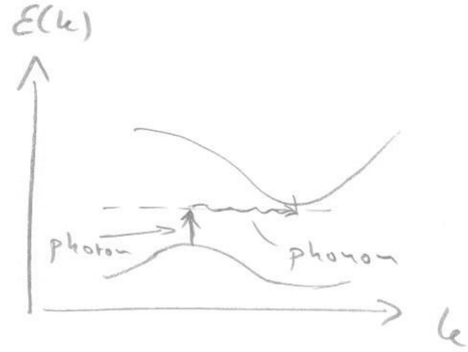

$$\boxed{n \cdot p = n_i \cdot p_i}$$

Charge balance $\quad n + N_a^- = p + N_d^+$

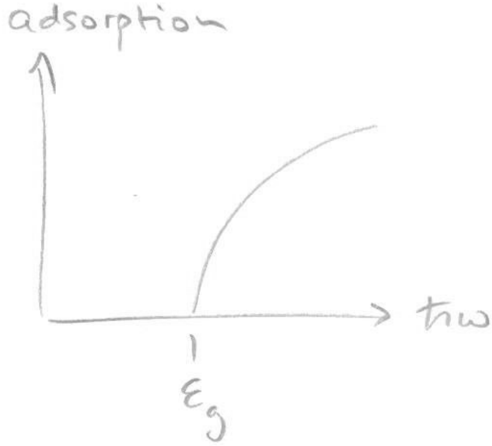
# Adsorption of electromagnetic radiation

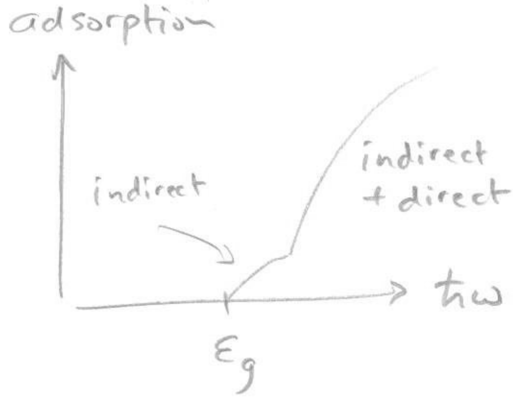



Direct
optical adsorption




Indirect
adsorption
(little energy to phonon too)

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**Conductivity** $\sigma = n e \mu_e + p e \mu_h$

intrinsic : $\sigma \sim e^{-\mathcal{E}_g / 2 k_B T}$

**Hall effect** $n \gg p : \quad R_H = -\frac{1}{ne}$

$p \gg n : \quad R_H = +\frac{1}{pe}$