Condensed Matter Physics – FK7060, Feb. 22, 2018.

Lecture 14 – Semiconductors

Reading

Ashcroft & Mermin, Ch. 28 (p. 562-570, 572-580)

Content

- Energy gap, valence band, conduction band
- Effective mass
- Density of states
- Carrier concentration
- Intrinsic semiconductors
- Law of mass action
- Donor level, acceptor level

Central concepts

- **Energy gap, valence band, conduction band**

  Semiconductors have a energy gap ε_g smaller than $\sim$ 2 eV between the highest filled band, called the *valence band* and the next empty band, the *conduction band*. This allows a small fraction of electrons to be thermally or optically excited across the gap. Since there still is a gap, the resistivity of semiconductors increases with decreasing temperature in contrast to regular metals.

- **Effective mass**

  The constant energy surface close to the band edges can be typically described as an ellipsiod in k-space. Expressing the curvature with an effective mass m_e for electrons in the conduction band and m_h for holes in the valence band, one has

$$\varepsilon_{\text{C.B.}}(k) = \varepsilon_c + \frac{\hbar^2}{2m_e}(\mathbf{k} - \mathbf{k}_c)^2$$

$$\varepsilon_{\text{V.B.}}(k) = \varepsilon_v - \frac{\hbar^2}{2m_h}(\mathbf{k} - \mathbf{k}_v)^2$$

- **Density of states**

  The density of states of the conduction and valence bands are similar to the free electron density of states, except for the electron and hole effective masses replacing the free electron mass, and the energies measured from the band edges:

$$g_c(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(\varepsilon - \varepsilon_c)^{1/2}$$

$$g_v(\varepsilon) = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(\varepsilon_v - \varepsilon)^{1/2}$$

- **Carrier concentration**

  The number of carriers n and p present at temperature T in the conduction and valence bands are given

$$n_c(T) = \int_{\varepsilon_c}^{\infty} \frac{1}{e^{(\varepsilon-\mu)/k_BT} + 1} g_c(\varepsilon) \mathrm{d}\varepsilon$$

$$p_v(T) = \int_{-\infty}^{\varepsilon_v} \frac{1}{e^{(\mu-\varepsilon)/k_BT} + 1} g_v(\varepsilon) \mathrm{d}\varepsilon$$

  If $|\varepsilon - \mu| >> k_BT$ this can be written as

$$n_c(T) = N_c(T) e^{-(\varepsilon_c-\mu)/k_BT}$$

$$p_v(T) = P_v(T) e^{-(\mu-\varepsilon_v)/k_BT}$$

  where

$$N_c(T) = \int_{\varepsilon_c}^{\infty} g_c(\varepsilon) e^{-(\varepsilon-\varepsilon_c)/k_BT} \mathrm{d}\varepsilon \approx \frac{1}{4}\left(\frac{2m_ck_BT}{\pi\hbar^2}\right)^{3/2}$$

$$P_v(T) = \int_{-\infty}^{\varepsilon_v} g_v(\varepsilon) e^{-(\varepsilon_v-\varepsilon)/k_BT} \mathrm{d}\varepsilon \approx \frac{1}{4}\left(\frac{2m_hk_BT}{\pi\hbar^2}\right)^{3/2}$$

- **Intrinsic semiconductors**

  An intrinsic semiconductor means a semiconductor in the state where impurities / doping is not controlling the carrier concentrations n_c and p_v. In this case

$$n_c = p_v = n_i$$

  where n_i is given by

$$n_i = \sqrt{N_cP_v} e^{-\varepsilon_g/2k_BT}$$

- **Law of mass action**

  The carrier concentrations n_c and p_v are controlled mainly of the value of the chemical potential μ, which depends strongly on doping. However, the product n_cp_v only depends on the energy gap and temperature

$$n_cp_v = n_i^2 = N_cP_v e^{-\varepsilon_g/k_BT}$$

  The product n_cp_v is thus not sensitive to the presence of doping.

- **Donor level, acceptor level**

  The intrinsic semiconductor can be doped with impurities, called *donors* and *acceptors*, having more or fewer electrons than the semiconductor. The binding energy of the extra electron to a donor impurity is small compared with the energy gap, so that the donor level will lie just below the conduction band edge. Similarly, the acceptor level lies just above the valence band edge (the hole is bound when the level is empty).

- **Conductivity**

  For electrons, we had earlier

$$j = \sigma E = -nev_d$$

  where the drift velocity

$$v_d = -\mu E$$

  and μ is the mobility.

  For semiconductors, the electrical conductivity becomes

$$\sigma = n_ce\mu_e + p_ve\mu_h$$