Condensed Matter Physics – FK7060, Jan. 16, 2018.

Lecture 1 – Drude model of metals

Reading

Ashcroft & Mermin, Ch. 1, pp. 2 – 25.

Content

- Drude model
- DC conductivity
- Hall effect, magnetoresistance
- AC conductivity
- Thermal conductivity

Central concepts

- **Drude model**

  Metallic ions (jellium), free valence electrons.

  Electron gas density

$$n = N/V = \frac{N_A Z \rho_m}{A}$$

  Radius of sphere with one conduction electron corresponding to electron density n

$$r_s = \left(\frac{3}{4\pi n}\right)^{1/3}$$

  Basic assumptions:

  1. Interactions neglected between collisions (independent electron approx., free electron approx.)
  2. Instantaneous collisions
  3. Probability of collision $1/\tau$, relaxation time τ
  4. Local thermal equilibrium. Electron velocity after collision from local distribution function (temperature dependent)

- **DC conductivity**

  Ohm's law

$$\mathbf{j} = \sigma \mathbf{E}$$

  The current density $\mathbf{j} = -ne\mathbf{v}_d$, where $\mathbf{v}_d = \langle \mathbf{v} \rangle$ is the average velocity (drift velocity). $\mathbf{v}_d$ becomes zero for $\mathbf{E} = 0$ and is, in general, very small compared to the agerage electron speed $v = \langle |\mathbf{v}| \rangle$.

  Conductivity

$$\sigma_0 = \frac{ne^2\tau}{m}$$

  Mean free path

$$l = v\tau$$

Equation of motion has frictional damping term (1.12)

$$\frac{d\mathbf{p}(t)}{dt} = \mathbf{f}(t) - \frac{\mathbf{p}(t)}{\tau}$$

Here, $\mathbf{p}(t) = m\mathbf{v}_d(t)$. The steady-state solution where $d\mathbf{p}/dt = 0$ gives ($\mathbf{B} = 0$)

$$\mathbf{v}_d = -\frac{e\tau}{m}\mathbf{E} = -\mu\mathbf{E}$$

Force on electron

$$\mathbf{f} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- **Hall effect, magnetoresistance**

  Hall coefficient

$$R_H = \frac{E_y}{j_x B} = -\frac{1}{ne}$$

  Cyclotron frequency

$$\omega_c = \frac{eB}{m}$$

  Hall angle

$$\tan\phi = \frac{E_y}{E_x} = -\omega_c\tau$$

  Magnetoresistance (field-independent in Drude model)

$$\rho(H) = \frac{E_x}{j_x}$$

- **AC conductivity**

  Frequency-dependent conductivity (1.29)

$$\sigma = \frac{\sigma_0}{1 - i\omega\tau}$$

  Dielectric constant (1.35)

$$\epsilon(\omega) = \epsilon_0 + \frac{i\sigma(\omega)}{\omega}$$

  Plasma frequency (1.38)

$$\omega_p = \left(\frac{ne^2}{\epsilon_0 m}\right)^{1/2}$$

  Plasmon: plasma oscillation

- **Thermal conductivity**

  Thermal current density (Fourier's law)

$$\mathbf{j}^q = -\kappa\boldsymbol{\nabla}T \qquad W = -K\frac{\mathrm{d}T}{\mathrm{d}x}$$

  Thermal conductivity

$$\kappa = \frac{1}{3}c_v l v$$

  Lorenz number (Wiedemann-Franz law)

$$L = \frac{\kappa}{\sigma T}$$

  Seebeck effect, thermopower

$$\mathbf{E} = Q\boldsymbol{\nabla}T$$

  with thermopower (1.59)

$$Q = -\frac{1}{3}\frac{c_v}{ne}$$