

Lect. 1 - Drude model

$$\sigma_0 = \frac{ne^2\tau}{m}$$

$$\mathbf{v}_d = -\frac{e\tau}{m}\mathbf{E} = -\mu\mathbf{E}$$

$$R_H = \frac{E_y}{j_x B} = -\frac{1}{ne}$$

$$\omega_c = \frac{eB}{m}$$

$$W = -K \frac{dT}{dx}$$

$$\kappa = \frac{1}{3}c_v l v$$

$$L = \frac{\kappa}{\sigma T}$$

$$Q = -\frac{1}{3} \frac{c_v}{ne}$$

Lect. 2 - Sommerfeld theory

$$f_{FD} = \frac{1}{e^{(\varepsilon-\mu)/k_B T} + 1}$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$\varepsilon_F = \frac{\hbar^2(3\pi^2 n)^{2/3}}{2m}$$

$$D(\varepsilon)d\varepsilon = 2 \frac{V}{8\pi^3} 4\pi k^2 dk$$

$$U = \int_0^\infty \varepsilon D(\varepsilon) f(\varepsilon) d\varepsilon$$

$$\int_0^\infty A(\varepsilon) f(\varepsilon) d\varepsilon = \int_0^\mu A(\varepsilon) d\varepsilon + \frac{\pi^2}{6} (k_B T)^2 A'(\mu)$$

$$c_{v,el} = \frac{\partial u}{\partial T} = \frac{\pi^2}{3} g(\varepsilon_F) k_B^2 T = \frac{\pi^2}{2} n k_B \left(\frac{k_B T}{\varepsilon_F} \right) = \gamma T$$

Lect. 5 - Reciprocal lattice

$$\mathbf{b}_1 = 2\pi \frac{\mathbf{a}_2 \times \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}$$

$$\mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3$$

$$d(hkl) = \frac{2\pi}{|G(hkl)|}$$

$$S_{\mathbf{G}} = \sum_{j=1}^n f_j e^{i\mathbf{G} \cdot \mathbf{d}_j}$$

$$\sin^2 \theta = \frac{\lambda^2}{4a^2} (h^2 + k^2 + l^2)$$

Lect. 6 - Bloch's theorem

$$\psi(\mathbf{r} + \mathbf{R}) = e^{i\mathbf{k} \cdot \mathbf{R}} \psi(\mathbf{r})$$

$$\mathbf{v}_n(\mathbf{k}) = \frac{1}{\hbar} \nabla_{\mathbf{k}} \varepsilon_n(\mathbf{k}) = \frac{1}{\hbar} \frac{\partial \varepsilon_n}{\partial \mathbf{k}}$$

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 \varepsilon}{\partial k^2}$$

$$\frac{d\mathbf{k}}{dt} = \frac{\mathbf{F}}{\hbar}$$

$$g_n(\varepsilon) = \frac{1}{4\pi^3} \int_{S_n(\varepsilon)} \frac{dS}{|\nabla_{\mathbf{k}} \varepsilon_n(\mathbf{k})|}$$

Lect. 7 - NFE, Tightbinding

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{G}} C_{\mathbf{k}-\mathbf{G}} \exp \{i(\mathbf{k}-\mathbf{G}) \cdot \mathbf{r}\}$$

$$C_{\mathbf{k}-\mathbf{G}} = \frac{\sum_{\mathbf{G}'} U_{\mathbf{G}'-\mathbf{G}} C_{\mathbf{k}-\mathbf{G}'}}{\varepsilon - \varepsilon_{\mathbf{k}-\mathbf{G}}^0}$$

Lect. 9 - Band structure

$$\hbar \frac{d\mathbf{k}}{dt} = -e(\mathbf{v}_g \times \mathbf{B})$$

$$\Delta \left(\frac{1}{B} \right) = \frac{2\pi e}{\hbar} \cdot \frac{1}{A_e}$$

$$\varepsilon = \left(n + \frac{1}{2} \right) \hbar \omega_c + \frac{\hbar^2 k_z^2}{2m}$$

Lect. 10 - Classification of solids

$$\phi(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right]$$

$$u^{\text{coul}}(r) = -\alpha \frac{e^2}{r}$$

Lect. 12 - Lattice vibrations

$$\omega(K) = \sqrt{\frac{4C}{M}} \left| \sin \frac{Ka}{2} \right|$$

Lect. 13 - Lattice vibrations II

$$n_s(\mathbf{K}) = \frac{1}{e^{\hbar\omega_s(\mathbf{K})/k_B T} - 1}$$

$$u = \frac{1}{V} \sum_{\mathbf{K}s} \frac{\hbar\omega_s(\mathbf{K})}{e^{\hbar\omega_s(\mathbf{K})/k_B T} - 1}$$

$$c_v = \frac{\partial u}{\partial T} = 3nk_B$$

$$g_D(\omega)d\omega = 3 \frac{1}{(2\pi)^3} 4\pi K^2 dK$$

Lect. 14 - Defects in crystals

$$n_v = N_0 e^{-U_0/k_B T}$$

Lect. 16 - Semiconductors

$$g_c(\varepsilon) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} (\varepsilon - \varepsilon_c)^{1/2}$$

$$n_c(T) = \int_{\varepsilon_c}^{\infty} \frac{1}{e^{(\varepsilon - \mu)/k_B T} + 1} g_c(\varepsilon) d\varepsilon$$

$$N_c(T) \approx \frac{1}{4} \left(\frac{2m_e k_B T}{\pi \hbar^2} \right)^{3/2}$$

$$\sigma = n_c e \mu_e + p_v e \mu_h$$

$$n_c p_v = n_i^2 = N_c P_v e^{-\varepsilon_g/k_B T}$$

Lect. 17 - Magnetism

$$\chi = -\frac{e^2 \mu_0 Z N}{6m} \langle r^2 \rangle$$

$$g \approx 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$\bar{\mu} = -g\mu_B \mathbf{J}$$

$$U = -\bar{\mu} \cdot \mathbf{B} = m_J g \mu_B B$$

$$M = \frac{\mu_B N}{V} \tanh \left(\frac{\mu_B B}{k_B T} \right)$$

$$\chi = \frac{g^2 J(J+1)}{3} \frac{\mu_0 \mu_B^2 n}{k_B T}$$

$$\chi = \mu_0 \mu_B^2 D(\varepsilon_F)$$

$$T_c = \frac{\mu_0 \mu_B^2 \lambda n}{k_B}$$

Lect. 18 - Superconductivity

$$\frac{c_s - c_n}{c_n} = 1.43$$

$$\nabla \times \mathbf{j}_s = -\frac{n_s e^2}{m} \mathbf{B}$$

$$\frac{\Delta(0)}{k_B T_c} = 1.76$$

$$T_c \sim \theta_D e^{-1/N_0 V_0}$$

Units etc.

- $\mu_0 = 4\pi \cdot 10^{-7} \text{ [N/A}^2\text{]}$
- $[\mathbf{T}] = [\text{N/Am}]$
- $k_B T$ has unit $[\text{J}] = [\text{Nm}]$
- μ_B has unit $[\text{Nm/T}] = [\text{Am}^2]$
- B has unit $[\text{T}]$
- H and M have unit $[\text{A/m}]$