EXAM SOLUTIONS/ANSWERS

STOCKHOLMS UNIVERSITET
FYSIKUM

Examination in Condensed Matter Physics I, FK7042/FK3004, 7.5 hp
Wednesday, June 13, 2012, 09.00-14.00.

Allowed help:
- periodic table and fundamental constants (distributed)
- formula sheet (distributed)
- pocket calculator, BETA / mathematics handbook or similar

Instructions:
All solutions should be easy to read and have enough details to be followed. The use of nontrivial formulas from the formula sheet should be explained. *Summarize each problem* before its solution, so that the solution becomes self-explained. State any assumptions or interpretation of a problem formulation.

Good luck! / A.R.

---

1. The atoms in a lattice can be modelled as hard spheres. $f = 0.68$, $f = 0.52$
a) Calculate the filling fraction of such atoms arranged in *bcc* and simple cubic (*sc*) structure, respectively. (2p)
b) What are the coordination numbers for the atoms in these structures? (0.5p) 8 , 6
c) The *hcp* structure is close-packed. Should this correspond to a lower or higher coordination number? Motivate! (0.5p) higher
d) The diamond structure does not itself correspond to a Bravais lattice, but can be described as a cubic Bravais lattice with a basis of 8 atoms. However, another Bravais lattice exists that could be used together with a smaller cell / basis to generate the diamond structure. Find the cell volume for this smallest possible cell expressed in the conventional (cubic) lattice parameter *a*. Motivate clearly. (1p) $a^3/4$

SEE EXAM 2011-06-10

2. The Drude model is a simple model of the metallic state that treats electrons as independent, classical particles.
a) Define and interpret the relaxation time τ in the Drude model, and find the probability for an electron not to collide during a time *t*. (2p) $P(t) = e^{-t/\tau}$, average time between collisions
b) Show how the introduction of τ together with the Drude assumption of scattering in random directions leads to Ohm's law. (2p)

SEE EXAM 2012-03-15

$$\left.\begin{array}{l} j = -ne\langle \bar{v}\rangle \\ \bar{v} = \bar{v}_0 - \frac{e\bar{E}}{m}t , \quad \langle \bar{v}_0\rangle = 0 \end{array}\right\} \; j = \underbrace{\frac{ne^2\tau}{m}}_{\sigma} E$$

3. a) Show that the volume v_g of the reciprocal lattice primitive cell is $v_g = (2\pi)^3/v_c$, where v_c is the volume of the direct lattice primitive cell. Hint: $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$. (1.5p)
b) Describe what a Brillouin zone is. (1p)
c) Iron (Fe) at room temperature has *bcc* structure with a lattice parameter $a = 2.87$ Å. Find the maximum *k* value of the first Brillouin zone in the $< 1\,0\,0 >$ direction for iron. (1.5p)

SEE EXAM 2011-03-25

a)

$$\left.\begin{array}{l} v_c = \bar{a}_1 \cdot (\bar{a}_2 \times \bar{a}_3) \\ v_g = \bar{b}_1 \cdot (\bar{b}_2 \times \bar{b}_3) \\ \bar{b}_3 = 2\pi \frac{\bar{a}_1 \times \bar{a}_2}{v_c} \\ + \text{hint} \end{array}\right\} \Rightarrow \text{solution}$$

b) BZ = Wiegner-Seitz cell in rec. lattice

continues on backside...

c) Rec. lattice of bcc is fcc, side $\frac{4\pi}{a}$

$$k_{max}(100) = \frac{1}{2}\frac{4\pi}{a} = \frac{2\pi}{a}$$

4. Aluminium (Al) has *fcc* structure with a lattice parameter $a = 4.05$ Å.

a) Determine an expression for the *k*-volume of the 1:st Brillouin zone for Al. (1p) $V_g = \frac{32\pi^3}{a^3}$

b) Describe the basic assumptions of the Debye model and find an expression for the volume of the Debye sphere for Al. (2p) $\frac{4}{3}\pi k_D^3 = 32\pi^3/a^3 \Rightarrow k_D = (24\pi^2)^{1/3}/a$

c) Compare the Debye sphere radius with the distance from the center Γ of the Brillouin zone to the zone boundary in the [100] direction and the [111] direction for Al. (1p) $k_{max}^{[100]} = \frac{2\pi}{a}$ $k_{max}^{[111]} = \frac{\sqrt{3}\pi}{a}$

SEE EXAM 2010-03-27

5. a) Discuss the experimental observation and interpretation of the de Haas – van Alphen effect. (2p) $\Delta\left(\frac{1}{B}\right) = \frac{e}{h} \cdot \frac{1}{A}$

b) Suppose that you are studying an unknown material. You are carrying out the following measurements:

A. Resistivity as a function of temperature. Metal, insulator, bandgap, superconductor?

B. Hall effect. Sign and number of charge carriers

C. Optical absorption. Bandgap, defects

D. X-ray diffraction. Crystal structure, lattice parameters

Explain how you would use the results of each of these measurements to improve your understanding of what kind of material you have. (2p)

SEE EXAM 2010-06-11

6. a) The paramagnetic susceptibility χ of rare-earth ions at high temperature is proportional to the square of the effective Bohr magneton number p and inversely proportional to temperature. The ions Ce^{3+}, Gd^{3+}, and Dy^{3+} have the electron configurations $4f^1 5s^2 p^6$, $4f^7 5s^2 p^6$, and $4f^9 5s^2 p^6$. One of these ions has a measured Bohr magneton number close to 8.0. Which one? (1.5p)

b) Discuss how magnetic ions interact. (1p)

c) Superconductors can be divided into two groups, type-I and type-II, depending on their behavior in magnetic fields. Describe the magnetic field – temperature phase diagram of the two groups. Also briefly discuss the Meissner effect and explain what a vortex is. (1.5p)

SEE EXAM 2008-06-05 (problem 5a, 5c)

— " — problem 6a

a) $\chi = \frac{g^2 J(J+1)}{3} \frac{\mu_0 \mu_B^2 n}{k_B T}$, $p = g\sqrt{J(J+1)}$ $\Rightarrow \chi \propto \frac{p^2}{T}$

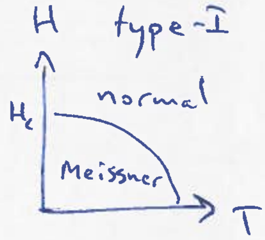
$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

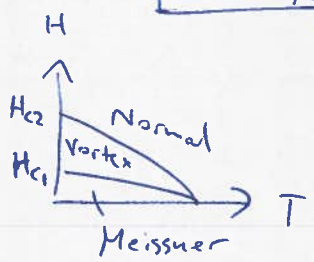
$4f^1$: $g = \frac{6}{7}$, $J = \frac{5}{2}$, $p = 2.54$

$4f^7$: $g = 2$, $J = \frac{7}{2}$, $p = 7.94$

$4f^9$: $g = \frac{4}{3}$, $J = \frac{15}{2}$, $p = 10.65$

c)





Meissner effect: Expulsion of field...

Vortex:

$\phi_0 = \frac{h}{2e}$ ...

b)