

**Examination in Condensed Matter Physics I, FK7042 / FK3004, 7.5 hp**

Thursday, March 20, 2014, 09.00-14.00.

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**Allowed help:**

- periodic table and fundamental constants (distributed)
- formula sheet (distributed)
- pocket calculator, BETA / mathematics handbook or similar

**Instructions:**

All solutions should be easy to read and have enough details to be followed. The use of nontrivial formulas from the formula sheet should be explained. *Summarize each problem* before its solution, so that the solution becomes self-explained. State any assumptions or interpretation of a problem formulation and define any introduced variables.

Good luck! / A.R.

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**1.** The *bcc* structure can be described as a simple cubic (*sc*) Bravais lattice with a 2-point basis of  $\mathbf{0}$  and  $(a/2)(\hat{x} + \hat{y} + \hat{z})$ , where  $a$  is the lattice parameter of the cubic Bravais lattice.

**a)** Can the *bcc* structure be described as a Bravais lattice with a single-point basis? If so, specify the primitive lattice vectors (in cartesian coordinates). If not, motivate why the structure would not be a Bravais lattice. (2p)

**b)** Suppose that we keep the nearest neighbor distance constant. Find the ratio of the two structural densities,  $\rho_{\text{bcc}}/\rho_{\text{sc}}$ , under this condition. (2p)

**2.** The free electron model in 2D is important not only for two-dimensional systems, but also for materials with nearly cylindrical Fermi surfaces.

**a)** Derive an expression for the density of states  $g(\varepsilon)$  for free electrons in two dimensions. (1p)

**b)** Find a corresponding expression for the Fermi energy  $\varepsilon_F$  in 2D. Express the answer in terms of the electron density  $n$ . (1p)

**c)** Assume a square lattice with 1 electron per atom and a lattice parameter  $a = 2.4 \text{ \AA}$ . Calculate the average energy (in eV) of free electrons in 2D. (2p)

**3.** A polycrystalline sample with bodycentered tetragonal structure was studied with monochromatic x-ray,  $\lambda = 1.5405 \text{ \AA}$ . The four lowest Bragg angles were measured to  $\theta = 21.00^\circ, 22.06^\circ, 28.78^\circ$ , and  $32.09^\circ$ .

**a)** Give an expression for a general reciprocal lattice vector  $\mathbf{G}(hkl)$  for the tetragonal lattice, which has lattice vectors  $a\hat{x}$ ,  $a\hat{y}$ , and  $c\hat{z}$ . (0.5p)

**b)** Start with the diffraction condition  $\Delta\mathbf{k} = \mathbf{G}$  and deduce the quadratic form for a tetragonal lattice. (1.5p)

**c)** For *bcc* structures, the allowed reflexes have  $h + k + l = 2n$ , where  $n$  is an integer. Motivate that this is also the case for the bodycentered tetragonal structure. (0.5p)

**d)** Index the structure and determine the lattice parameters  $a$  and  $c$ . (1.5p)

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4. A measurement of the heat capacity  $C_v$  of silver resulted in the following data:

| $T$ (K) | $C_v$ (mJ/mol K) |
|---------|------------------|
| 1.32    | 1.28             |
| 1.62    | 1.85             |
| 2.26    | 3.40             |
| 3.12    | 6.92             |

a) Explain what information you could obtain from the data. (1p)

b) Analyze the data to draw as many conclusions as possible! Make suitable assumptions. (3p)

5. a) For direct optical absorption in a semiconductor, the excitation of an electron is assumed to occur vertically in the band structure ( $\epsilon$  vs.  $k$ ). Estimate if this is a reasonable model. (2p)

b) Explain the following concepts: Bloch function, localized states. (2p)

6. a) Show that the magnetic susceptibility of free conduction electrons is proportional to the density of states at the Fermi energy. (2.5p)

b) Discuss the classification of insulators. (1.5p)