STOCKHOLMS UNIVERSITET
FYSIKUM

Examination in Condensed Matter Physics I, FK3004, 7.5 hp
Friday, June 11, 2010, 09.00-15.00.

Allowed help:
- periodic table and fundamental constants (distributed)
- formula sheet (distributed)
- pocket calculator, BETA / mathematics handbook or similar

Instructions:
All solutions should be easy to read and have enough details to be followed. The use of nontrivial formulas from the formula sheet should be explained. *Summarize each problem* before its solution, so that the solution becomes self-explained. State any assumptions or interpretation of a problem formulation.

Good luck! / A.R.

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1. **a)** A certain metal has a monoatomic *fcc* structure with a lattice parameter $a_{\text{fcc}} = 3.638\,\text{Å}$ at $T = 1080\,\text{K}$. At the higher temperature $T = 1680\,\text{K}$, the structure has changed to *bcc*, with a lattice parameter $a_{\text{bcc}} = 2.936\,\text{Å}$. How much does the density change when going from 1080 K to 1680 K? (2p)
b) Define the reciprocal lattice and explain what the first Brillouin zone is. (2p)

2. A polycrystalline sample with bodycentered tetragonal structure was studied with monochromatic x-ray, $\lambda = 1.5405$ Å. The four lowest Bragg angles were measured to $\theta = 21.00°$, $22.06°$, $28.78°$, and $32.09°$.
a) Give an expression for a general reciprocal lattice vector $\mathbf{G}(hkl)$ for the tetragonal lattice, which has lattice vectors $a\hat{x}$, $a\hat{y}$, and $c\hat{z}$. (0.5p)
b) Start with the Laue condition $\Delta\mathbf{k} = \mathbf{G}$ and deduce the quadratic form for a tetragonal lattice. (1.5p)
c) For *bcc* structures, the allowed reflexes have $h + k + l = 2n$, where n is an integer. Motivate that this is also the case for the bodycentered tetragonal structure. (0.5p)
d) Determine the lattice parameters a and c. (1.5p)

3. A two-dimensional free electron gas is contained in a square of area A. Show that the temperature dependence of the Fermi level (chemical potential) μ is given by

$$\mu(T) = k_B T \cdot \ln\left[\exp\left(\frac{n\pi\hbar^2}{mk_B T}\right) - 1\right]$$

where m is the electron mass and n is the number of electrons per area A. (4p)
Hint:

$$\int_0^\infty \frac{1}{\exp\left[(E-\mu)/k_B T\right] + 1} dE = k_B T \cdot \ln\left[1 + \exp\left(\mu/k_B T\right)\right]$$

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4. Consider a one-dimensional crystal with 1 atom per primitive cell and a lattice parameter $a = 2\,\text{Å}$. The lattice vibrations in this crystal are harmonic with interaction only between nearest neighbors. In the long wavelength limit, i.e., for $K = 2\pi/\lambda \rightarrow 0$, the propagation velocity of the lattice waves is $115\,\text{m/s}$.
a) We know that lattice vibrations are quantized. In which interval of energy are the possible phonon energies found for an infinitely long crystal? Specify lower and upper boundaries in meV. (2p)
b) Now assume that the crystal is having a length $L = 100\,\text{nm}$. Estimate the lowest possible phonon energy (in meV). (2p)

5. a) Discuss the experimental observation and interpretation of the de Haas – van Alphen effect. (2p)
b) Suppose that you are studying an unknown material. You are carrying out the following measurements:

A. Resistivity as a function of temperature.

B. Hall effect.

C. Optical absorption.

D. X-ray diffraction.

Explain how you would use the results of each of these measurements to improve your understanding of what kind of material you have. (2p)

6. a) Show how to obtain the Curie law, i.e., that the magnetic susceptibility $\chi \propto 1/T$, for a free spin paramagnet with $J = 1/2$. (2.5p)
b) Describe two type of crystal defects and their possible, practical importance. (1.5p)